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A SURVEY OF STUDENTS' ABILITY OF IDENTIFYING ERRORS IN WRONG SOLUTIONS FOR THE MATHEMATICAL PROBLEMS RELATED TO THE MONOTONICITY OF FUNCTIONS

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Abstract:
The monotonicity of a function plays an important role in the general mathematics curriculum in Vietnam, because it is considered as an effective tool for solving mathematical problems involved with the monotonic intervals of functions, their extreme, absolute maximum value and absolute minimum value. Normally, students commit errors in solving these problems because of their complexity and difficulty. In addition, specific characteristics of knowledge also make children make mistakes. The sample consisted of 362 students, and they had the task of identifying errors in false assumptions. From the results of the survey, it was found that when dealing with the monotonicity of the functions, students were still misled.

Keywords: monotonicity of a function, error, reason, mathematics education

1. Introduction

1.1 Errors in solving mathematical problems and their reasons

Errors in solving mathematical problems are unavoidable for any student, so this also motivates many authors to study them. Indeed, they carried studies of students' errors

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on specific mathematical topics and gave reasons for those errors. The results showed that there were many different errors and their reasons were also varied.

A Vietnamese mathematical educator, Thuc (2009) said that correcting errors was important in developing students' thinking, reinforcing their knowledge and skills. Furthermore, he argued that students' errors were due to various reasons such as: negligence, carelessness, lack of knowledge and limited mathematical language.

The researcher, Loc (2014, 2015) pointed out students' errors in solving mathematical problems due to compliance with the Didactic contract, a theory in mathematics education in France.

The author, Newman (1977) offered a model of error analysis regarding the five elements: reading, comprehension, transformation, skill processing, and coding. In addition, some students' errors in solving mathematical problems were due to the difficulties, namely lack of understanding of the appropriate procedures, the complexity of the problem, and generalization of procedures (Ndalichako et al., 2013). Meantime, Nigerian authors, Ekwueme and Ali (2012) separated mathematical errors into four categories, such as arbitrary, structural, executive, and clerical errors.

In their study of students' errors in adding and subtracting fractions, Idris and Narayanan (2011) divided them into three kinds: careless errors, negligent errors, and accidental system errors. Meanwhile, Munasinghe (2013) stated that the students made errors because of wrong guidance from parents or family, teachers' unawareness of the students or the social and economic conditions of parents.

The two authors Brunei, Yusof and Malon (2003) did a specific study of students' errors in doing operations of fractions. One of the reasons drawn was the impact of the knowledge about natural numbers students learnt before, in particular, students often applied rules of solution in natural number problems for solving fraction problems.

1.2 The monotonicity of functions in textbooks in Vietnam

In the high school mathematics program, the monotonicity of functions is introduced at the beginning of the algebra textbook in Grade 10 and is repeated at the beginning of the calculus program in Grade 12 after the students learn about the derivative of the function in Grade 11. The monotony of functions is associated with many other mathematical knowledge and is widely applied in high school mathematics in general and 12th grade program in particular. Some popular applications consist of assessing and graphing functions, equation solving, equation system solving, inequalities solving, finding extreme of functions, and finding absolute maximum value and absolute minimum value of functions. (Hao, 2015)

Therefore, the teaching methods need to be effective so that students can master knowledge, apply mathematics and they contribute to the skill training and mathematical learning methods, and then create opportunities for intellectual development for students. However, the situation shows that when solving problems related to the monotonicity of functions, many students still do not master the knowledge, and then commit errors in the mathematical problem solving process. To better understand this, it is necessary to have a study of students' errors in solving mathematical problems associated with the monotonicity of functions.

2. Research objectives

Examine the students' ability to identify errors in solutions that contain errors. It is thought that if students do not recognize them in the solutions, they will probably commit the error in solving similar problems. At the same time, failing to find out errors also means that in terms of knowledge, students have errors.

Research questions are addressed:

- Do students realize errors in the wrong solutions?
- Do students have errors in judging wrong answers?
- What causes students to commit errors?

3. Methodology

3.1 Participants

The total number of students enrolled in the four high schools in Hau Giang province was 362. Since the students completed the chapter "Applications of the derivative in assessing and graphing functions" in the Calculus textbook in Grade 12, theoretically, at the time of the survey, students had sufficient knowledge and skills to solve mathematical problems related to the monotonicity of functions.

3.2 Instrument and procedure

Eight hypothetical problems built were derived from the problems we predict students may be committing errors. Among them, there are five problems that ask students to check whether the answer is true or false. If the answer is not correct, ask students to point out the wrong place, and in other three problems, ask the students to score, if the grade is less than 10 (10 points), ask students to tell the reasons.

3.2.1 Pre- Analysis of 8 items

Item 1: Examine the students' ability to recognize errors in the problem of the monotonicity of a function that is not continuous on a set. When considering the monotonicity of a function with the derivative tool (without definition), do the students commit an error?

Item 2: Examine the students' ability to discover errors in the problem of the monotonicity of a function on a closed interval. The task type that measures the monotonicity of the function on a closed interval is not explicitly stated by the textbook and at the ends of the [-1;1], the function has no derivative. When assessing monotonousness on a closed interval by derivative tool, do the students commit the errors?

Items 3 and 4: Survey of students' errors regarding the critical point of the function. Specifically, here we would like to know whether in item 3, $-\sqrt{2}$ is understood to be the critical point, and for item 4, do the students realize the critical point of the function $(x=0)$?

Item 5: Examine the students' ability to identify errors in mathematical problems related to the type of task for finding the parameter to let the function be monotonic over the given interval. In particular, investigate students' errors when applying the sufficient conditional theorem and the necessary conditional theorem on the monotonicity of the function in mathematics. Can the students distinguish when applying the sufficient conditional theorem and applying the necessary theorem on the monotonicity of the function in their solutions?

Item 6: Examine the students' ability to find out errors in mathematical problems using the necessary theorem on the monotonicity of the function. Specifically, when they find the parameters, students check the condition " $f'(x)=0$ at only a finite number of points" or not, if not check the conditions, which errors can students commit?

Item 7: Examine the students' ability to uncover errors in the mathematical problems that apply monotonic function definitions to prove inequality. In particular, we want to know whether students understand the definition correctly, whether they know how to apply the definition to prove inequality.

Item 8: Examine the student's ability to detect errors in a problem that proves the equation has only one solution. Do the students think that: An equation has a side given by a monotonic function on the interval *K* and the other is a constant function, the equation has always given the unique solution on *K*?

4. Results and discussion

Table 1: Student survey results

Items	The number of students did not discover any errors	Percentages
Item 1	199	54.97
Item 2	211	58.29
Item 3	139	38.40
Item 4	123	33.98
Item 5	121	33.43
Item 6	213	58.84
Item 7	215	59.39
Item 8	198	54.70

The results are shown by the following graph:

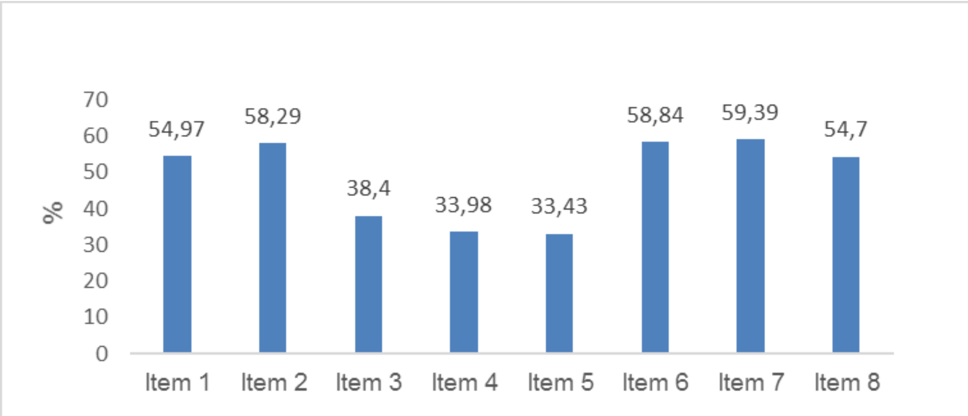
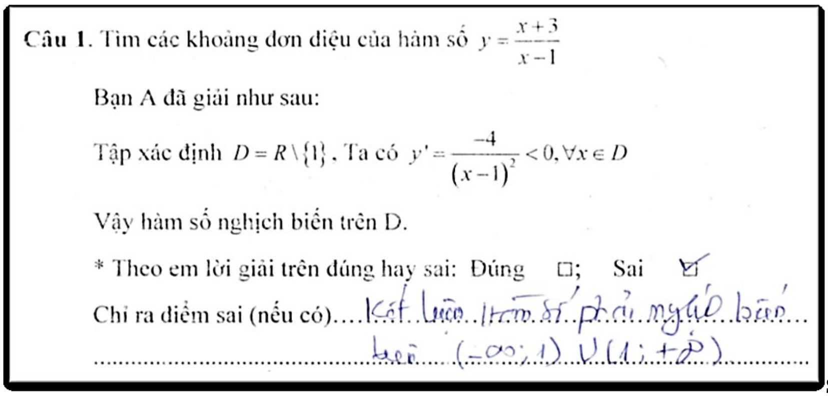


Figure 1: Comparison of the percentage of students who did not identify the errors

Based on the results of the survey, the percentage of students who did not find out the errors in the solutions was high. The following is the specific result of each item.

Item 1: Many students found that the derivative of a negative function on a set determines the decreasing function on that domain, but they were regardless of whether the function is continuous on the given set or not. That lead to an error in the solution. Here the correct conclusion of the problem must be: The function was decreasing on $(-\infty;1)$ and $(1;+\infty)$. Thus, it can be seen that when considering the monotonicity of a function, many students were not interested in the domain being defined as half-closed intervals, closed intervals, or open intervals. There were many students who thought that the function was not decreasing on its deterministic set. However, the students did not understand the reason, so they assume that the function was not decreasing on $D = R\setminus\{1\}$, but it was decreasing on $(-\infty;1)\cup(1;+\infty)$.


Câu 1. Tìm các khoảng đơn điệu của hàm số $y = \frac{x+3}{x-1}$

Bạn A đã giải như sau:

Tập xác định $D = R \setminus \{1\}$. Ta có $y' = \frac{-4}{(x-1)^2} < 0, \forall x \in D$

Vậy hàm số nghịch biến trên D.

* Theo em lời giải trên đúng hay sai: Đúng ☐; Sai ☑

Chỉ ra điểm sai (nếu có).... Kết luận hàm số phải nghịch biến trên (-∞;1) ∪ (1;+∞)

s

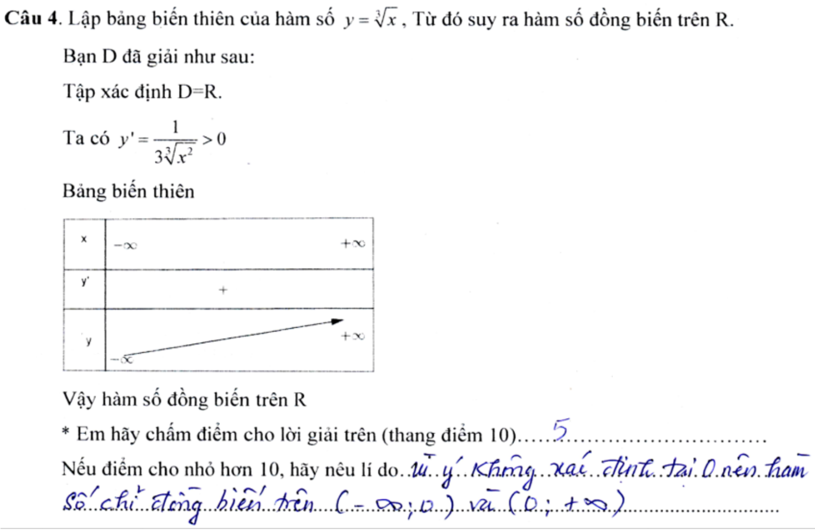
Figure 2: A student's work in item 1

Item 2: Although the requirement was to determine the monotonicity of the function on a closed interval, however, many students still accepted the false conclusion of the hypothetical solution (increasing on (-1, 0) and decreasing on (0; 1)). The cause was able to be that the monotonicity of the function in textbooks is often required in terms of the intervals. Therefore, students had the habit of concluding the monotonicity of the function on the intervals. For this item, the correct conclusion must be: The function was increasing on [-1; 0] and decreasing on [0; 1].

Item 3: The percentage of students who did not recognize errors was lower than the rest, but at a high level because they did not understand the critical point of the function. Specifically, in item 3, in the residual matrix of a critical point ($x = -\sqrt{2}$), it was possible that $x = -\sqrt{2}$ belonged to the domain being considered in the variance table to show points, which in turn lead to the index of the wrong derivative.

Item 4: In item 4, there was no any critical point ($x = 0$) that students should be aware that the function defined on *R* did not mean that its derivative was also determined on *R*. In other words, the derivative of the function was determined on *R*. It may also not be defined at some certain points (these are the critical points of the function), namely the function defined on *R* but its derivative was not defined at $x = 0$.

Therefore, excess or missing critical points could lead to wrong conclusions considered as the result of the problem. From the results of the survey, many students found that the function $y = \sqrt[3]{x}$ had a derivative not defined in $x = 0$, but they did not point out that this was the critical point of the function, but they concluded that the function only coincided with $(-\infty; 0)$ and $(0; +\infty)$.



Câu 4. Lập bảng biến thiên của hàm số $y = \sqrt[3]{x}$, Từ đó suy ra hàm số đồng biến trên R.

Bạn D đã giải như sau:

Tập xác định D=R.

Ta có $y' = \frac{1}{3\sqrt[3]{x^2}} > 0$

Bảng biến thiên

x	$-\infty$	$+\infty$
y'	+	
y	$-\infty$ ↗	$+\infty$

Vậy hàm số đồng biến trên R

* Em hãy chấm điểm cho lời giải trên (thang điểm 10)......5..........................

Nếu điểm cho nhỏ hơn 10, hãy nêu lí do..vì y' không xác định tại 0 nên hàm số chỉ đồng biến trên (−∞;0) và (0;+∞)..........................

Figure 3: A student's work in item 4

Item 5: In this item, quite a few students accepted the use of sufficient conditional theorem on the monotonicity of the function to solve the problem. Of course, such an interpretation could lead to an incorrect result. In particular, here students were hoped to apply the necessary and sufficient conditional theorem on the monotonicity of functions to solve, ie to increase on *R*, the function must satisfy the condition: $f'(x) \geq 0$. The cause was due to lack of the necessary and sufficient conditional theorem, not explicitly stated in the textbook, so the students did not understand the theoretic structure well enough and did not distinguish the difference between sufficient conditional theorem and the necessary and sufficient conditional theorem on the monotonicity of a function.

Item 6: Here the application of the necessary and sufficient conditional theorem to solve the problem was perfectly correct. However, many students found that the parameter *m* was the default value that did not check whether the parameter satisfied theorems. Specifically, students did not test the condition "$f'(x) = 0$ at only a finite number of points". Obviously, the function given was a constant function because there were countless solutions in *D*. It could be seen that when applying the necessary and sufficient conditional theorem to find the parameter as the problem above, a lot of students did not check with the parameters they found, which can lead to errors.

Item 7: It was easy to see $f'(x) \geq 0, \forall x \in \left(0; \frac{\pi}{2}\right)$. The conclusion $\forall x \in \left(0; \frac{\pi}{2}\right) \Rightarrow f(0) < f(x)$ was generally incorrect, because to have $f(0) < f(x)$ the students had to rely on the definition of the increasing function. Nonetheless, here,

$0 \notin \left(0; \frac{\pi}{2}\right)$, so it could not be inferred $f(0) < f(x)$. Unfortunately, the fact was that many students did not notice that.

Item 8: The survey showed that in this form of mathematical problem, many students committed the error in assuming the following statement in a correct way. "One side of the equation is a monotonic function on *K*, and the other is constant, then the equation has the unique solution". The error here was that students were not interested in the value domain of the function. Indeed, in this item the left side was an increasing function and has a value domain $\left[18\sqrt{7};+\infty\right)$ that corresponded to the domain $\left[3;+\infty\right)$. Accordingly, let the equation have the unique solution on $\left[3;+\infty\right)$ then $m \in \left[18\sqrt{7};+\infty\right)$.

4.1 Common errors students committed when dealing with the monotonicity of functions

From the survey results, it could be seen that when dealing with the monotonicity of the function, students were likely to commit the following common errors:

- Errors in assessing the monotonicity of a function on an interval, but the function is discontinuous over that interval.
- Errors in assessing the monotonicity of a function on a closed interval.
- Errors related to the critical points of functions.
- Errors in using the sufficient conditional theorem, the necessary and sufficient theorem of the monotonicity of a function on a solution.
- Errors in applying the monotonicity of functions to prove inequality.
- Errors in applying the monotonicity of the function to prove the equation have only one solution.

4.2 Reasons for students' errors in solving mathematical problems related to the monotonicity of functions

Based on the data collected from the actual survey of students, it was found that the common errors students commit when dealing with monotonicity of the function were due to several reasons:

A. First, students did not master basic knowledge.

A new knowledge is always built on the basis of old knowledge. Thus, the lack of basic knowledge, especially old knowledge directly related to new knowledge will make it difficult to acquire new knowledge, incomprehensive knowledge of new knowledge.

Understanding the basic knowledge unthoroughly also limits judgment, lack of logic, leading to errors in applying new knowledge into mathematics.

For example, to determine the monotonicity of a function on the interval *K* by definition, *K* is usually a closed interval, an open interval, or a half-closed interval. Hence, for the question of assessing the monotonicity of the function, students must first know the domain being considered. In case of the domain is not a closed interval, an interval or a half-closed interval, if students do not master the basic knowledge, they will not find out this, which will probably lead to errors in the mathematics.

For instance, when required to create the variation chart of a function $y = x - 1 + \sqrt{4 - x^2}$, to find the solutions of the equation $y' = 0 \Leftrightarrow \sqrt{4 - x^2} = x$, many students suggested that the equation had two solutions that lead to the wrong sign of its derivative. The reason may be due to lack of knowledge about solving the equations containing the radical. If students had a basic knowledge of solving equations, they would find that extraneous solution, which are not critical points of the function.

For the problem of finding *m* for the equation $2x^2\sqrt{x^2 - 2} = m$ have a solution on $[3; +\infty)$, here new knowledge, monotonous knowledge of functions used to indicate the left side of the equation is a function, increasing on $[3; +\infty)$, and to prove the equation have only one solution, the students must mobilize and manipulate old knowledge. In order to solve this problem, students need to know:

- Since the left hand side is an increasing function, its graph is a continuous line going up from left to right on $[3; +\infty)$.
- The right side is the parameter *m*, so its graph is a straight line parallel to the horizontal axis and cut to the horizontal axis at *m*.
- Since the number of solutions given by the given equation is the number of intersections of the two graphs above, the *m* must be greater than the absolute minimum value on the left on $[3; +\infty)$, ie $m \geq 18\sqrt{7}$.

According to survey results, many students did not master the knowledge in the third plus sign, leading to an error in the solution. To get the right answer to this problem, students needed to figure things out geometrically: When the graph of an upward path (from left to right) was in the interval *K* and the line was parallel to the diagonal together. When they knew this knowledge, they would limit the errors occurred unfortunately.

B. Second, the students misunderstood the concepts.

The basic requirement, inevitably when learning the concept of mathematics is to understand the concept, to understand what is the connotation of the concept, what is the denotation of the concepts. If they do not understand them, they will understand the concept incompletely, even understand the nature of the concept wrongly. On the other hand, mathematical concepts are often interconnected, so understanding the concept incompletely can affect the cognition about knowledge. Consequently, one of the reasons for students' errors in solving mathematical problems is to misunderstand concepts.

For assessing the monotonicity of a function, the technology to solve problems given by the textbook is the derivative of the function, without mentioning the definition of the monotonicity of the function. Since then, students are no longer interested in defining the monotonicity of the function. This can also be the cause of students' errors.

For example, to assess the monotonicity of the function $y=\frac{x+3}{x-1}$ on its specific domain, students can use the definition or use the sufficient conditional theorem on the monotonicity of the function to solve the problem. If students use the definition, to decrease on *K*, then the function must satisfy the following two conditions:

- Function is determined on *K*.
- The function $y=f(x)$ is decreasing on *K*. If for any pair of points x_1, x_2 in *K* x_1 smaller than x_2 then $f(x_1)$ is greater than $f(x_2)$, namely $x_1<x_2 \Rightarrow f(x_1)<f(x_2)$.

If we use the sufficient condition theorem on the monotonicity of a function, to decrease on *K*, the function must also satisfy the following two conditions:

- The function has its derivative on *K*.
- $f'(x)<0$, $\forall x\in K$.

Note that, in the textbook, *K* is an open interval, a closed interval, or a half-closed interval.

The survey found that many students made the mistake of using the sufficient conditional theorem, but they did not care about *K*. In particular, *D* was not an open interval, a closed interval or a half-closed interval, it was a combination of two intervals $D=(-\infty;1)\cup(1;+\infty)$. By using the definition, students could easily point out that the conclusion was wrong, because if on *D*, students took $x_1=0, x_2=2$, then $x_1<x_2$ and they had so that the function could not be decreasing on *D*. Thus, by definition, the correct conclusion of the problem had to be: the function was decreasing on $(-\infty;1)$ and $(1;+\infty)$.

For the problem of assessing the monotonicity of the function $y=\sqrt{1-x^2}$ on [-1; 1], students could easily create the variation chart. However, when concluding the problem, many students mistakenly assumed that the function was increasing (-1; 0) and decreasing (0; 1). There were two reasons for this error. First, students did not master the definition thoroughly. In the definition, the domain could be considered as a closed interval, an open interval, or a half-closed interval, so students should think carefully when concluding the monotonic intervals of functions. Second, the requirement for the monotonicity of a function on a closed interval was not specifically presented by the textbook; it appeared only in the form of activity in finding the absolute maximum value and absolute minimum value of the function. Therefore, students often had habits to conclude the monotonous function on the intervals.

C. Third, students did not understand the logical structure of the theorem.

In order to ensure the efficient and correct application of theorems into the solution, an inevitable requirement is that the students must understand the logical structure of the theorem. Often mathematical theorems are expressed in the form $A \Rightarrow B$, where *A* is the supposition of the theorem, which indicates its scope of use. Therefore, if students do not understand the structure of the theorem, it is very easy to make the mistake of applying that theorem to the solution. The errors in applying a theorem to a solution are due to not understanding the supposition of the theorem, leading to the application of an incompatible theorem (in which case the theorem contains a different theorem), or applying the theorem when the supposition is not satisfied.

D. Fourth, students did not master the methods of solution. Knowing how to solve problems would significantly reduce errors in the mathematical solution, especially in mathematical problems with the given solutions, or at least the students could determine solutions. For example, for the problem of assessing the monotonicity of a function, the method of solution consisted of four steps:

- Find the domain of the function $y = f(x)$.
- Calculate the derivative of the function ($f'(x)$). Find the x_i (i = 1, 2, ...,n), where the derivative values are equal to 0 or undefined.
- Arrange the points x_i in ascending order and compile a sign chart.
- Draw conclusions on increasing, decreasing intervals of the function.

5. Conclusion

From the results of the survey, it was found that when dealing with the monotonicity of the function, students were still misled. Each student's error could be caused by a variety of reasons, which were inseparable and intertwined, and their role was constantly changing in the cognitive process. Hence, to prevent students' errors, teachers should apply various pedagogical measures at the same time. Nevertheless, at each stage, each students' cognitive process, teachers need to identify the main reason for the students' errors to take methods in correcting them.

For this reason, in the process of teaching and learning mathematics, the teachers should follow the students closely. When teachers find out that students have errors, they need to be repaired in time to prevent them from making errors in the long time systematically. Mathematical knowledge is often linked to one another and making a knowledge error in the long time most likely misleads others. When the scope of the errors is wider, the remedy for them will be more difficult, more time consuming.

When students commit errors, the teachers need to organize the follow-up activities so that students can take the initiative to spot errors and guide students to find the reasons for the errors. This will help students minimize errors in the near future. It also contributes to raising students' awareness in preventing errors in mathematical problem solving.

With these orientations, when designing and developing the teaching process, teachers must coordinate many pedagogical methods to achieve the following purposes:

- Help students understand the knowledge.
- Help students work correctly.
- Detect and help in time when students have difficulties.
- Preventing errors is necessary and if students commit errors, teachers should help students find them and repair in time.

References

1. Ekwueme, C.O. & Ali, A. (2012). Process Error and Students' Academic Achievement in Senior Secondary Certificate Examination in Mathematics in Nigeria. *Journal of Emerging Trends in Educational Research and Policy Studies* (JETERAPS) Vol. 3, No. 4, 600-603.
2. Hao, T. V. (2015). *Calculus 12 (Giải tích 12),* Vietnamese Publishing house of Education.
3. Idris, N. & Narayanan, L.M. (2011). Error Patterns in Addition and Subtraction of Fractions among Form Two Students. *Journal of Mathematics Education*, Vol. 4, No. 2, 35-54.

4. Loc, N. P. & Hoc, T.C.T. (2014). A Survey of 12th Grade Students' Errors in Solving Calculus Problems. *International Journal of Scientific & Technology Research,* Vol. 3, No. 6.
5. Loc, N.P. & Kha, N.T. (2015). Students' errors in solving problems on coordinate methods in space: Results from an investigation in Vietnam, *European Academic Research*, Vol. 1, No. 2.
6. Munasinghe, D. M. W. (2013). A study on error patterns in "Addition" in primary school children (7 years old children). *Merit Research Journal of Education and Review* Vol. 1, No. 7, 154-158.
7. Ndalichako, J. L. (2013). Analysis of Pupils' Difficulties in Solving Questions Related to Fractions: The Case of Primary School Leaving Examination in Tanzania. *Creative Education*, Vol. 4, No. 9, 69-73.
8. Newman, N. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute of Educational Research Bulletin*, Vol. 39, 31-43.
9. Thuc, P. D. (2009). *Methods of teaching mathematics in primary schools*, Hanoi: Publishing house of Education.
10. Yusof, J., & Malone, J. (2003). Mathematical errors in fractions: A case of Bruneian primary 5 pupils. In L.Bragg, C. Champbell, G. Herbert, & J. Mousley (Eds.), *Proceedings of the 26th Annual Conference of the Mathematics Education Research Group of Australia, 2*, 650-657. Geelong: Deakin University.

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