European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.1059415

Volume 3 │Issue 11│2017

A SURVEY OF THE ERRORS OF STUDENTS IN GRADES 10 AND 12 WHEN USING VIETA'S FORMULAS TO SOLVE THE ASSOCIATED MATHEMATICAL PROBLEMS

Duong Huu Tong[1i], Bui Phuong Uyen[1], Ha Hoang Quoc Thi[1], Nguyen Thi Lai[2]
[1]School of Education,
Can Tho University, Vietnam
[2]Master Student,
Can Tho University, Vietnam

Abstract:
Vieta's formulas have many applications in the general mathematics program in Vietnam. Indeed, they appear in many forms of mathematical problems, ranging from 9th, 10th and 12th grades. The complexity of the mathematical problems makes students commit many errors when using the Vieta's formulas to solve them. The sample consisted of 246 students in grades 10 and 12, in which they addressed four questionnaires related to Vieta's formulas. The results were quite unexpected because the errors of tenth graders also existed for 12th grade students for many different reasons.

Keywords: Vieta's formulas, error, reason, quadratic equation

1. Introduction

1.1 Problem solving in mathematics, its errors and reasons

Problem solving is a major activity in students' mathematics learning because it requires them to acquire knowledge and be able to analyze for the solution. The four important steps in solving a problem given by Polya (1973) are as follows: understanding the problem, planning the strategy, implementing the plan, and reviewing the answers. Using these four steps in addressing mathematical problems is

[i] Correspondence: email: dhtong@ctu.edu.vn

not easy for many students, resulting in them making undesirable errors. From here, the author, Newman (1977) proposed the error analysis model associated with the following four errors: reading, comprehension, transformation, processing skills, and encoding.

Often the errors of students are due to many different causes. Carelessness, limited knowledge, the complexity of mathematical language make it easy for students to make errors (Thuc, 2009). Meanwhile, Herholdt and Sapire (2014) emphasized that particular areas of difficulty are the causes of the common procedural and conceptual errors. In addition, in their research, they put forward a model of error analysis that teachers can use to analyze students' errors when solving mathematical problems.

Jupri and Drijvers (2016) divided the difficulties in initial algebra consists of four categories: applying arithmetical operations in numerical and algebraic expressions, understanding the notion of variable, understanding algebraic expressions and understanding the different meanings of the equal sign. Furthermore, in their study, they also revealed that students committed errors in formulating equations, schemas or diagrams when formulating a mathematical model.

Sawardi and Shahrill (2014) pointed out that the systematic errors of students were the result of misconceptions. Besides, these errors reflected students' understanding of a concept, problem or a procedure in mathematics and there were different causes relatet to them as well. One of the causes revealed in their study was maybe a result of inattentiveness on the part of classroom teachers.

1.2 Vieta's formulas in textbooks in Vietnam

The Vieta's formulas are taught in the mathematics textbook in Grade 10. However, they are very much used in equations converted into linear and quadratic equations. In particular, they are presented in the following section:

" If the quadractic equation $ax^2+bx+c=0\ (a\neq 0)$ *has two* x_1, x_2 *then* $x_1+x_2=-\frac{b}{a}$*;* $x_1x_2=\frac{c}{a}$
Conversely, if two quantities u and v have $u+v=S$ *and* $uv=P$*, then u and v are two solutions of the equation* $x^2-Sx+P=0$*."* (Hao, 2015).

The Vieta's formulas have many applications in the following mathematical problems:

- Calculating roots of a quadratic equation in one's head.
- Finding two numbers if their sum and their product are known.
- Creating quadratic equations that satisfy the given condition.
- Calculating the relation between the roots.
- Calculating the remaining root if a root is known.

- Analyzing a trinomial into a factor.
- Find a relation between the two roots of the quadratic equation independent of the parameter.
- Find the value of the parameter that satisfies the given solution expression.
- Argue the number of roots of the biquadratic equation.

In Grade 12, the theory of Vieta's formulas is not repeated, but they are used in many mathematical problems related to the monotonicity of functions. In addition, it also appears implicitly in an exercise when students learn complex numbers as follows: *"Given that* $a,b,c \in \mathbb{R}$ $_1, z_2$ *are two solutions of the equation* $az^2 + bz + c = 0$. *Compute* $z_1 + z_2$ *and* $z_1.z_2$ *according to coefficients a, b, c."* (Hao, 2015).

Besides, Vieta's formulas are also applied to solve and argue the system of equations and equations.

The use of Vieta's formulas is not easy for 10th grade students because of the variety of mathematical problems. This can also lead to unwanted errors by students when using the formulas to solve the equations. In addition, there are no Vieta's formulas involved in the mathematical program 11, which draws students to forget how to use them. At the same time, the formulas are used as a tool to solve a class of mathematical problems in the mathematical program 12, and most of them appear in program 10. Therefore, there is a question necessary to research: Are the errors of the tenth graders when using the Vieta's formulas still in existence in the 12th graders?

2. Research objectives

A survey was carried out to examine student's the wrong level of errors in solving a class of mathematical problems around Vieta's formulas.

Research questions are addressed:

- Does the student realize the errors in the incorrect solutions?
- Is there a difference between two students in Grade 10 and 12?
- What are the common errors students make when using the Vieta's formulas?
- What causes them to go wrong?

3. Methodology

3.1 Participants

The sample consisted of 137 students in Grade 10 and 109 students in Grade 12 at two high schools: Le Quy Don High School in Hau Giang province and Tap Son High

School, Tra Vinh province, in Vietnam. They already had the knowledge of Vieta's formulas and mathematical problems associated with them.

3.2 Instrument and procedure

The questionnaire was based on the problems we had anticipated containing the errors predicted. The problems were designed with the solutions containing the errors and asked the students to choose the correct one or the wrong one and point out the wrong place if there is. In addition to the problems with wrong solutions supposed to solve, an exercise offered to ask students to present their solution.

The questionnaire consisted of two parts: Part 1 was a problem that asked students to present the solution. Part 2 was the 3 questions that were designed with the wrong solutions. Subsequently, the questionnaire was distributed to students and collected for analysis.

Pre- Analysis of 4 items

Item 1:

Grade 10: Given the equation of the form $x^2-2x+m+2=0$. Find m such that the equation has two roots x_1, x_2 that satisfy $x_1^2+x_2^2=10$.

Grade 12: Find m such that the function $f\left(x\right)=\frac{1}{3}x^3-x^2+\left(m+2\right)x-1$ has two extrema x_1, x_2 that satisfy $x_1^2+x_2^2=10$.

Examine to identify the errors students make when addressing a problem related to the Vieta's formulas. In particular, the aim is to find out students' errors in using knowledge of the Vieta's formulas, determining the conditions such that the equation has solutions, and computing.

Item 2:

Grade 10: Let x_1 and x_2 be the roots of the equation $x^2-3x-7=0$. Find $x_1^2+x_2^2$.

Solution

Note that from our Vieta's formulas we have $x_1+x_2=\frac{b}{a}=-3$ and $x_1x_2=-\frac{c}{a}=7$. *Therefore*

$x_1^2+x_2^2=\left(x_1+x_2\right)^2-2x_1x_2=\left(-3\right)^2-2.7=-5$

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

Grade 12: Let z_1 and z_2 be the roots of the equation $z^2-3z-7=0$. Find $z_1^2+z_2^2$.

Solution

Note that from our Vieta's formulas we have $z_1+z_2=\frac{b}{a}=-3$ and $z_1z_2=-\frac{c}{a}=7$. *Therefore*

$z_1^2 + z_2^2 = (z_1 + z_2)^2 - 2z_1z_2 = (-3)^2 - 2.7 = -5$

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

Examine to find out students' errors in using the Vieta's formulas, in particular, are they confused about the sign in the formulas?

Items 3:

Grade 10: Given the quadratic equation of the form $x^2 + (2m-3)x + m^2 - 2m = 0$. Determine m such that the equation has two roots x_1, x_2 that satisfy $x_1.x_2 = 8$.

Solution

Note that from our Vieta's formulas we have

$$m^2 - 2m = 8 \Leftrightarrow m^2 - 2m - 8 = 0 \Leftrightarrow \left[\begin{array}{l} m = -2 \\ m = 4 \end{array}\right.$$

Conclusion. the given equation has two roots x_1, x_2 *that satisfy* $x_1x_2 = 8$ *if and only if* $m = -2$ *or* $m = 4$.

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

Grade 12: Find m such that the function $f(x) = \frac{1}{3}x^3 + \left(m - \frac{3}{2}\right)x^2 + (m^2 - 2m)x + 1$ has two extrema x_1, x_2 that satisfy $x_1.x_2 = 8$.

Solution

$$f'(x) = x^2 + (2m-3)x + m^2 - 2m$$

$$f'(x) = 0 \Leftrightarrow x^2 + (2m-3)x + m^2 - 2m = 0$$

Note that from our Vieta's formulas we have

$$x_1.x_2 = m^2 - 2m = 8 \Leftrightarrow m^2 - 2m - 8 = 0 \Leftrightarrow \left[\begin{array}{l} m = -2 \\ m = 4 \end{array}\right.$$

Conclusion. the given function has two extrema x_1, x_2 *that satisfy* $x_1.x_2 = 8$ *if and only if* $m = -2$ *or* $m = 4$.

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

In order for an equation to have solutions to satisfy certain conditions, it is necessary to have conditions for the equation to have solutions. This item is raised to check whether students have forgotten the conditions for the equation to have solutions.

Item 4:

Grade 10: Given the quadratic equation of the form $x^2-3x+2+m=0$. Determine m such that the equation has two roots $x_1,\ x_2$ that satisfy $x_1^2+x_2^2=\frac{9}{2}$.

Solution

The given equation has two roots $x_1,\ x_2$ *if and only if* $\Delta>0$

We have $\Delta=(-3)^2-4(2+m)=1-4m$

$\Delta>0\Leftrightarrow 1-4m>0\Leftrightarrow m<\frac{1}{4}$

Note that from the problem's statement we have

$x_1^2+x_2^2=\frac{9}{2}\Leftrightarrow (x_1+x_2)^2-2x_1x_2=\frac{9}{2}$

$\Leftrightarrow 3^2-2(2+m)=\frac{9}{2}\Leftrightarrow m=\frac{1}{4}$ *(eliminated)*

Conclusion. There is no value of m satisfying the problem.

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

Grade 12: Given the quadratic equation of the form $z^2-3z+2+m=0$. Determine m such that the equation has two roots $z_1,\ \mathrm{z}_2$ that satisfy $z_1^2+z_2^2=\frac{9}{2}$.

Solution

The given equation has two roots $z_1,\ \mathrm{z}_2$ *if and only if* $\Delta>0$

We have $\Delta=(-3)^2-4(2+m)=1-4m$

$\Delta>0\Leftrightarrow 1-4m>0\Leftrightarrow m<\frac{1}{4}$

Note that from the problem's statement we have

$z_1^2+z_2^2=\frac{9}{2}\Leftrightarrow (z_1+z_2)^2-2z_1z_2=\frac{9}{2}$

$\Leftrightarrow 3^2-2(2+m)=\frac{9}{2}\Leftrightarrow m=\frac{1}{4}$ *(eliminated)*

Conclusion. There is no value of m satisfying the problem.

According to you, the above solution is true or false? □ Yes □ Wrong

Please point out the wrong point if any: ....

This item is aimed at identifying students' errors when differentiating conditions so that equations have two solutions and two discriminative solutions. When the equation requires two solutions, students will think that they are two distinct solutions or two solutions.

4. Results and Discussion

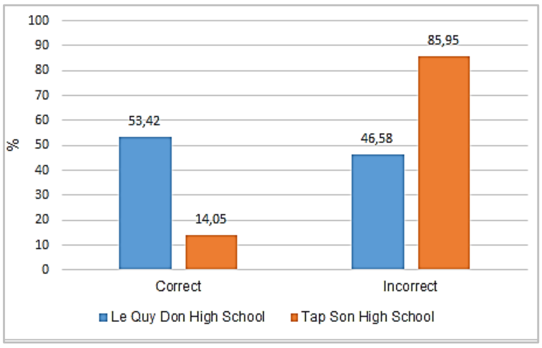
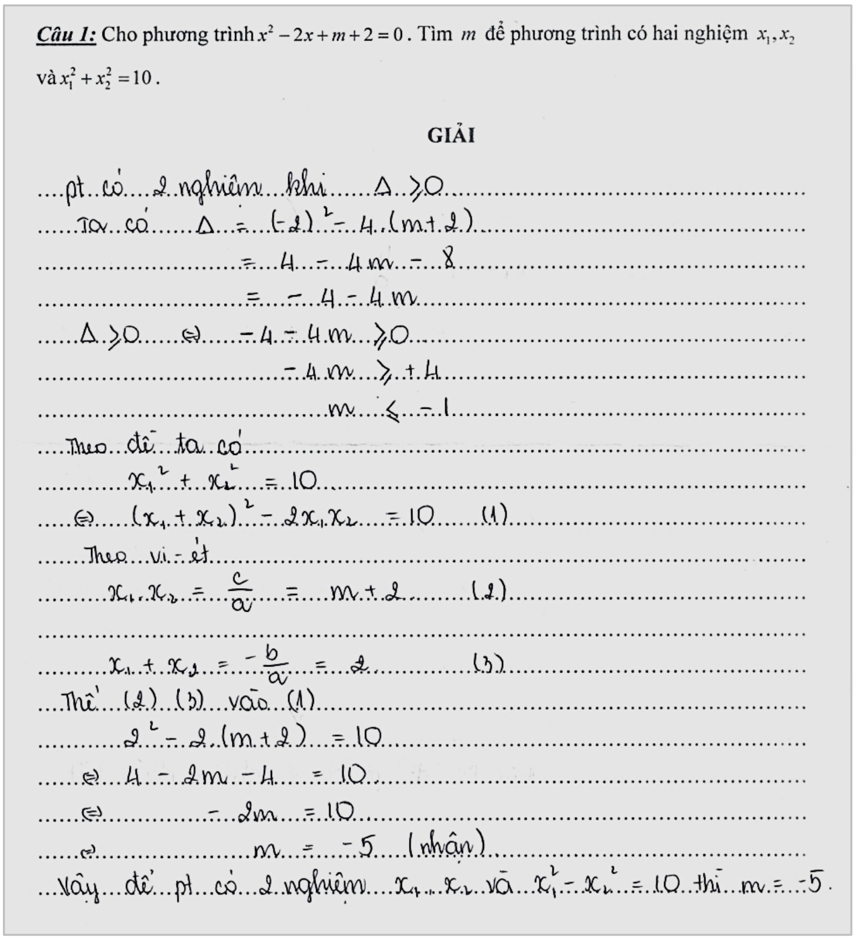


Figure 1: The chart compares the percentage of 10th graders with correct / incorrect answers

At Le Quy Don High School, the percentage of students with correct answers was higher than the percentage of students with incorrect answers, but the difference was not large. The percentage of students corrected was 46.58%, in which 13.7% of the calculation was wrong in the process and did not finish the solution. 32.88% of the students solved relatively correctly because they were confused between two roots and two different roots in their solution. In the question of finding *m* such that the equation had two roots rather than two distinct roots, but students thought that they were two different roots, so they wrote the condition with the discriminant $\Delta > 0$.

At Tap Son High School, the percentage of students scoring correctly was less than the error percentage, and the difference in the school was quite large, the rate of students with incorrect solutions was approximately 6 times the rate of students with correct solutions. Like Le Quy Don High School, the ratio of students confused between two roots and two different roots was very large, accounted for 46.88%. The percentage of students who did not finish the problem was 25% and 10.94% of tenth graders were wrong in the calculation. In particular, 3.13% of students did not check the condition of the equation.



Câu 1: Cho phương trình $x^2 - 2x + m + 2 = 0$. Tìm m để phương trình có hai nghiệm x_1, x_2 và $x_1^2 + x_2^2 = 10$.

GIẢI

pt có 2 nghiệm khi $\Delta \geq 0$

Ta có $\Delta = (-2)^2 - 4.(m+2)$

$= 4 - 4m - 8$

$= -4 - 4m$

$\Delta \geq 0 \Leftrightarrow -4 - 4m \geq 0$

$-4m \geq +4$

$m \leq -1$

Theo đề ta có

$x_1^2 + x_2^2 = 10$

$\Leftrightarrow (x_1 + x_2)^2 - 2x_1x_2 = 10$ (1)

Theo vi-ét

$x_1 . x_2 = \frac{c}{a} = m + 2$ (2)

$x_1 + x_2 = -\frac{b}{a} = 2$ (3)

Thế (2) (3) vào (1)

$2^2 - 2(m+2) = 10$

$\Leftrightarrow 4 - 2m - 4 = 10$

$\Leftrightarrow -2m = 10$

$\Leftrightarrow m = -5$ (nhận)

Vậy để pt có 2 nghiệm x_1, x_2 và $x_1^2 - x_2^2 = 10$ thì $m = -5$.

Figure 2: A tenth grade student's worksheet in item 1

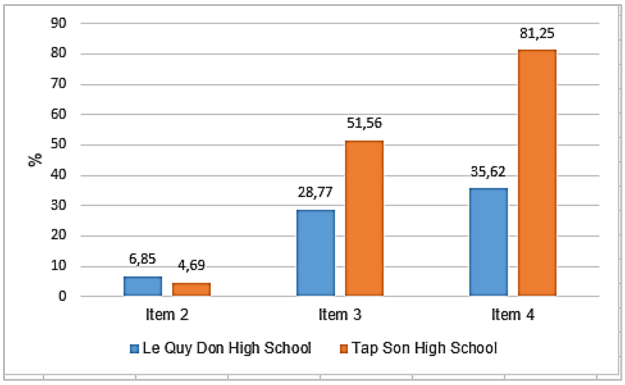


Figure 3: The chart compares the percentage of 10th grade students who did not find out the errors

According to the statistics, it was found that the rate of students in the Tap Son High School not detecting the errors was higher than the one of Le Quy Don High School. In item 2 few students did not uncover the errors (less than 10%). This rate in item 3 was higher than the one in item 2, but lower than the one in item 4. In item 3 and item 4, many students of Tap Son High School did not discover the errors, especially in item 4 there were 81.25% students of Tap Son High School did not recognize the wrong point of the problem.

At Le Quy Don High School, the percentage of students who did not recognize the mistakes of the problem was the highest in item 4 (36.62%) and the lowest in item 2 (6.85%). Meanwhile, in Tap Son High School, this percentage was similar to that of Le Quy Don High School. Specifically, item 2 had the lowest rate (4.69%) and the fourth item had the highest rate (81.25%).

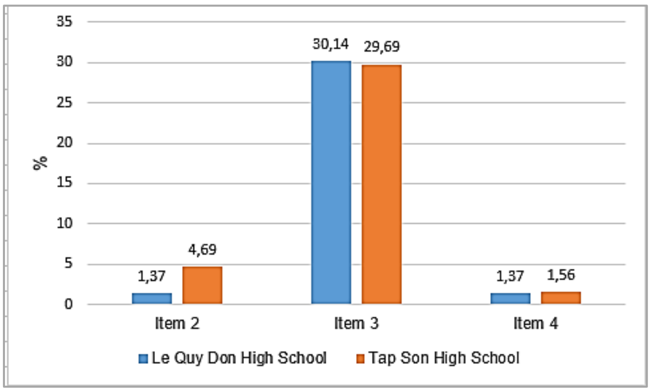
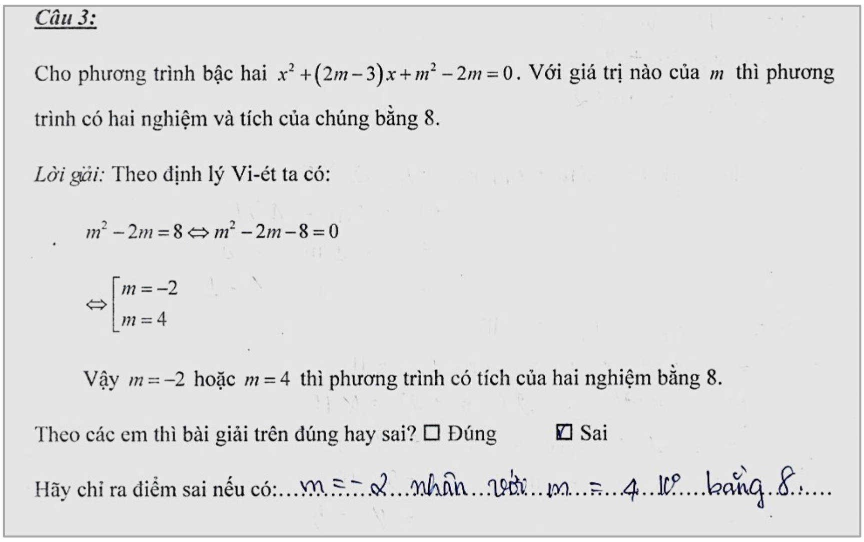


Figure 4: The chart compares the percentage of 10th graders not finding the reason of the errors

In addition to the students who did not find out the errors in the mathematical problem, there were also students claiming that the problem was wrong, but did not point out the wrong place. In this section, the ratio of the two schools was almost the same, with the exception of items 2 and 4, which were very low (less than 5%), item 3 had the highest rate and the rate of the two schools was approximately 30%.



Câu 3:

Cho phương trình bậc hai $x^2+(2m-3)x+m^2-2m=0$. Với giá trị nào của m thì phương trình có hai nghiệm và tích của chúng bằng 8.

Lời giải: Theo định lý Vi-ét ta có:

$m^2-2m=8 \Leftrightarrow m^2-2m-8=0$

$\Leftrightarrow \left[\begin{array}{l} m=-2 \\ m=4 \end{array}\right.$

Vậy $m=-2$ hoặc $m=4$ thì phương trình có tích của hai nghiệm bằng 8.

Theo các em thì bài giải trên đúng hay sai? ☐ Đúng ☑ Sai

Hãy chỉ ra điểm sai nếu có: m = -2 nhân với m = 4 là bằng 8

Figure 5: A 10th grade student's worksheet in item 3

In item 3, Tap Son High School had some students who misunderstood the requirement the problem, *"find m such that the equation has two solutions and their product is equal to 8"*, but when calculating the values of *m*, the students multiplied these values together. In addition, a few students thought that the right place was wrong. For example, the transformation was seen to be incorrect. In this item, Le Quy Don high school students pointed out the lack of the condition of the equation to have solutions, but instead of the condition $\Delta \geq 0$, students wrote that the condition was $\Delta > 0$. In particular, very few students thought the requirement of the problem was wrong, and some students misunderstood the requirement of the problem in the same way as the students in the Tap Son High School.

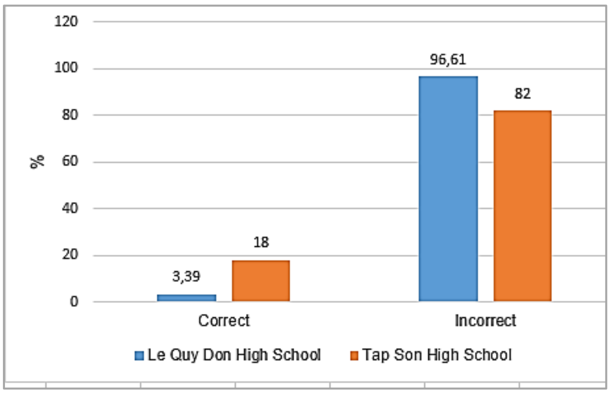
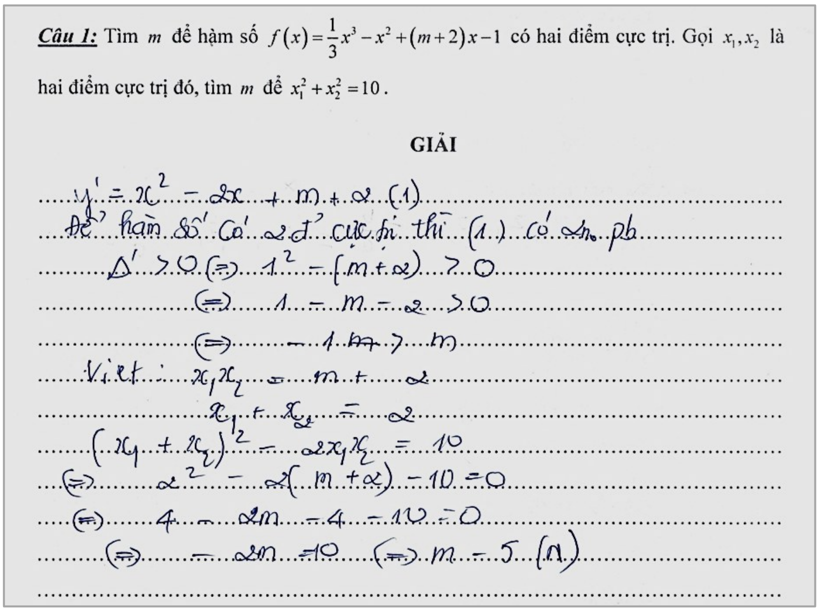


Figure 6: The chart compares the percentage of 12th graders with correct / incorrect answer

Based on the questionnaires, it was found that the percentage of 12th grade students who correctly solved this problem was very low, most of them made many errors. Le Quy Don High School had a very high rate (96.61%); however, most of them had wrong knowledge of extrema (44.07 %), in particular, they did not give $f^{'}(x) = 0$, instead they calculated the discriminant immediately. In addition, 10.17% of students did not complete the solution, and 30.5% of them made errors in the calculation process. 16.95% of them supposed that the condition for the function having two extrema. In particular, 5.08% did not use the Vieta's formulas to solve the problem.

The rate of students in the Tap Son High School with correct answers was higher than the one in Le Quy Don High School but the rate was still very low. The students also committed errors the same as Le Quy Don High School, 60% of them only missed $f^{'}(x) = 0$ when the rest of solution were correct. The percentage of students who made incorrect calculations during the problem solving process was rather high (22%). All of them used the Vieta's formulas to address this problem. Nevertheless, the highlight was that all students in the two schools tested the condition of the equation.



Câu 1: Tìm m để hàm số $f(x)=\frac{1}{3}x^3-x^2+(m+2)x-1$ có hai điểm cực trị. Gọi x_1, x_2 là hai điểm cực trị đó, tìm m để $x_1^2+x_2^2=10$.

GIẢI

$y' = x^2 - 2x + m + 2$ (1)

Để hàm số có 2 cực trị thì (1) có 2n pb

$\Delta' > 0 \Leftrightarrow 1^2 - (m+2) > 0$

$\Leftrightarrow 1 - m - 2 > 0$

$\Leftrightarrow -1 > m$

Viet: $x_1x_2 = m + 2$

$x_1 + x_2 = 2$

$(x_1 + x_2)^2 - 2x_1x_2 = 10$

$\Leftrightarrow 2^2 - 2(m+2) - 10 = 0$

$\Leftrightarrow 4 - 2m - 4 - 10 = 0$

$\Leftrightarrow -2m = 10 \Leftrightarrow m = -5$ (n)

Figure 7: A 12th grade student's worksheet in item 1

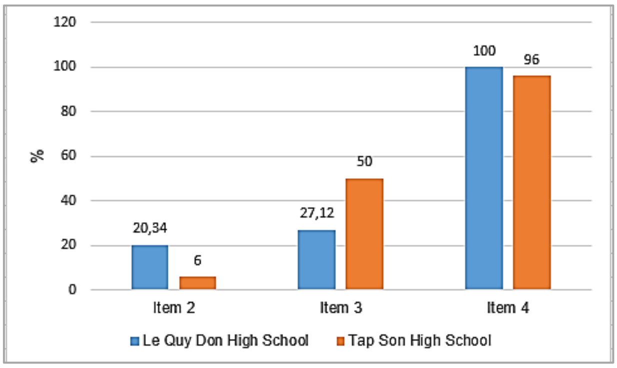


Figure 8: The chart compares the percentage of
12th grade students who did not find out the errors

In this section, many students made the errors. Especially, in item 4, 100% of Le Quy Don High School students did not recognize the error of the solution to this problem. In the two schools, the rates were increasing in items. At Le Quy Don High School, the percentage of students who did not recognize mistakes in the items 2 and 3 was just over 20%, but the rate in item 4 was 100%. Meanwhile, at Le Quy Don High School, the rate on the item 2 was very low, only 6%, but the one in the item 3 was up to 50%. The last item in the Tap Son High School had the lower rate than Le Quy Don High School, and it was still high (96%).

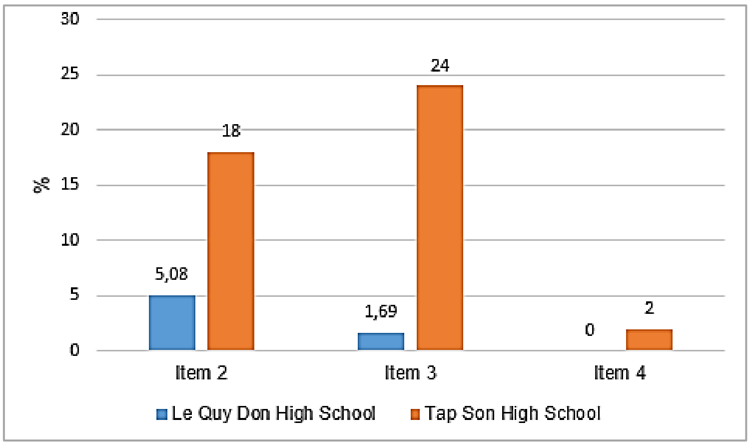
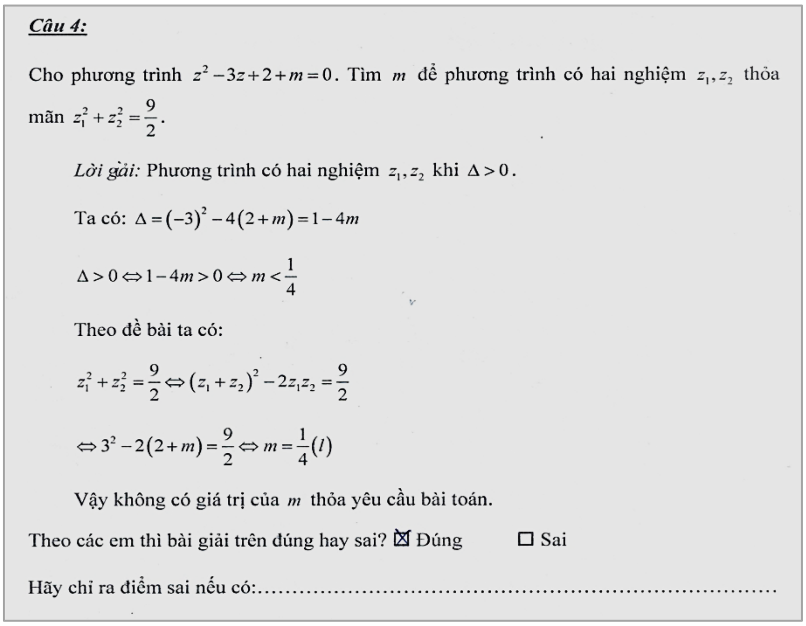


Figure 9: The chart compares the percentage of 12th graders not finding the reason of the errors

In this section, the percentage was very low for both schools, but especially for Le Quy Don High School. Tap Son High School still had students who recognized the wrong solution, but they did not indicate wrong place in the solution. Specifically, the rates on the items were 5.08%, 1.69% and 0% in turn.

Although these numbers were not large, but it was found that in which questions students of each school thought that the solution had a wrong place, but they did not point out the wrong place. In addition to the students who did not point out the error, some students said that the answer was wrong or that the result was wrong. In particular, some students still supposed that the product of two values m equaled 8 to be wrong, while the requirement of the problem was the product of two extrema rather than the product of values m.



Câu 4:

Cho phương trình $z^2-3z+2+m=0$. Tìm m để phương trình có hai nghiệm z_1, z_2 thỏa mãn $z_1^2+z_2^2=\frac{9}{2}$.

Lời giải: Phương trình có hai nghiệm z_1, z_2 khi $\Delta>0$.

Ta có: $\Delta=(-3)^2-4(2+m)=1-4m$

$\Delta>0 \Leftrightarrow 1-4m>0 \Leftrightarrow m<\frac{1}{4}$

Theo đề bài ta có:

$z_1^2+z_2^2=\frac{9}{2} \Leftrightarrow (z_1+z_2)^2-2z_1z_2=\frac{9}{2}$

$\Leftrightarrow 3^2-2(2+m)=\frac{9}{2} \Leftrightarrow m=\frac{1}{4}(l)$

Vậy không có giá trị của m thỏa yêu cầu bài toán.

Theo các em thì bài giải trên đúng hay sai? ☒ Đúng ☐ Sai

Hãy chỉ ra điểm sai nếu có:...

Figure 10: A 12th grade student's worksheet in item 4

4.1 Common errors students made when using the Vieta's formulas

From the survey results, there were two common errors students made as follows:

- The errors in calculating the value of the relation between the solutions of the quadratic equation.
- The errors in determining *m* such that the quadratic equation satisfies the given condition.

4.2 Reasons for these errors

Based on the data collected from the survey, it was found that the common errors students made when solving problems using the Vieta's formulas was due to the following reasons.

- First, students did not master the knowledge of the Vieta's formulas.

In order to address a mathematical problem related to the Vieta's formulas, students had to master knowledge of the formulas, then they were able to be applied. For example, the problem required calculating the value of a related expression between the solutions of the quadratic equation, assuming that they converted the expression to the expression containing only the sum and product of the two roots, but when students applied Vieta's formulas wrongly, it seemed that the solution of the problem was wrong.

Therefore, the content of the Vieta's formulas was very important and students needed to master and remember correctly. Especially, if the content of the Vieta's formulas that students did not grasp correctly, the solutions to mathematical problems using the Vieta's formulas would be incorrect.

- Second, students did not know how to solve mathematical problems.

Understanding the solution method was also very important in addressing problems because when students mastered the method of resolution, they would limit mistakes. For example, when the problem required that the equation had two roots to the given condition, then it was necessary to find the equation for which the equation holds before finding *m* to satisfy the demand for the problem. If the student did not find the condition for the equation to have the roots, then sometimes the value *m* would not make the equation to have solution, so the value *m* did not satisfy the problem.

- Third, students did not clearly distinguish between two solutions and two different solutions.

There were many students confused between two solutions and two different solutions. Sometimes when a problem requires two solutions to satisfy the given condition, the student thought that the two solutions had to be two distinct solutions, thus setting the condition $\Delta > 0$ instead of $\Delta \geq 0$, then it would often lead to receive or eliminate incorrect values m.

- Fourth, the knowledge of the Vieta's formulas was interrupted during the study,

According to the general mathematics program, it was found that the 11th grade students did not have any basic mathematical problems related to the Vieta's formulas. Thus, the students did not gain knowledge. At that time, 12th grade students met quite a number of mathematical problems using the Vieta's formulas, so they would face many difficulties. Therefore, it was seen that 12th grade students discontinued the Vieta's formulas, and it was inevitable for students to use them incorrectly.

5. Conclusion

The remarkable result was that the 10th grade students made many errors and the 12th grade students were constantly committing the same errors as the 10th grade students. The errors made by students in the classes were similar, and they were caused because students encountered difficulties such as the complexity and variety of mathematical problems, and the undiscovered problems using the Vieta's formulas to address.

Therefore, in the process of teaching students, the teachers need to find out where students make errors and why they make errors. Identifying the difficulties that students encounter is very important, because from that the suitable measures to help students to correct errors are raised. If the place where they are wrong is not discovered, the students are not helped to recognize the errors, that lead to them continuing commit other errors.

In order to limit students' errors, it is important to prevent them. It is suggested that teachers must learn through their own experiences and with colleagues to find out the errors students often make, then teachers emphasize, draw attention to students so that they can deeply memorize knowledge.

When students solve a problem, if the students make errors, the teachers should not point out the wrong point, so guide the students to understand and identify the errors they are having, then help them to correct. So, if the teachers point out the errors, they just know that the place is wrong, but they do not really understand why it is wrong. Since they are not impressed with the errors, then they do not remember, they will continue to go wrong rather than to correct the errors.

References

1. Hao, T. V. (2015). *Algebra 10 (Đại Số 10),* Vietnamese Publishing house of Education.
2. Hao, T. V. (2015). *Calculus 12 (Giải tích 12),* Vietnamese Publishing house of Education.
3. Herholdt, R. & Sapire, I. (2014). An Error Anaysis in the Early Grades Mathematics – A Learning Opportinity. *South African Journal of Childhood Education*, Vol. 4, No. 1, 42-60.
4. Jupri, A. & Drijvers, P. (2016). Student Difficulties in Mathematizing Word Problems in Algebra. *Eurasia Journal of Mathematics, Science & Technology Education*, Vol. 12, No. 9, 2481-2502.
5. Newman, N. A. (1977). An Analysis of Sixth-grade Pupils' Errors on Written Mathematical Tasks. *Victorian Institute of Educational Research Bulletin*, Vol. 39, 31-43.
6. Polya, G. (1973). *How to Solve It: A New Aspect of Mathematical Method*. Princeton, N. J.: Princeton University Press. Prakitipong.
7. Sawardi, H, R, H. & Shahrill, M. (2014). Understanding Students' Mathematical Errors and Misconceptions: The Case of Year 11 Repeating Students. *Mathematics Education Trends and Research*, Vol. 2014, 1-10.
8. Thuc, P. D. (2009). *Methods of Teaching Mathematics in Primary Schools*, Hanoi: Publishing house of Education.

Received date	October 30, 2017
Accepted date	November 17, 2017
Publication date	November 22, 2017