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AN EXAMINATION OF THE NUMBER SENSE PERFORMANCES OF PRESERVICE ELEMENTARY SCHOOL MATHEMATICS TEACHERS[i]

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Abstract:
The aim of this study is to examine the number sense skills of preservice elementary school mathematics teachers. This study was conducted by using the survey model among descriptive research methods. A total of 111 preservice teachers studying at second and third grades in the elementary school mathematics teaching program at a state university participated in the study. The data of the study were collected by using the number sense test consisting of 17 questions and developed by Kayhan-Altay (2010). The findings of the study indicated that the number sense performances of preservice elementary mathematics teachers were lower than expected and there was a significant difference in favour of third-grade students. It is considered that the "Special Training Methods I and II" courses and the "Mathematics Curriculum" course, which are taught in third grades and in which subjects such as estimation and making mental calculations, etc. superficially related to number sense are mentioned, may cause this difference. Thus, it was suggested that it was necessary to include courses consisting of number sense and how it can be developed in the curriculum.

Keywords: number sense, preservice elementary mathematics teachers

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1. Introduction

Since there is not a precise definition agreed, number sense was defined in different ways by researchers. For example, while Howden (1989, p.11) defined number sense as "*a good intuition about the numbers and their relationships*", Reys and Yang (1998, pp. 225-226) made a definition as "*a person's general understanding regarding numbers and operations*". The strategies used by individuals who have and who do not have number sense in the solution of problems vary. While individuals who have number sense can develop flexible strategies in the solution of problems, individuals who have low number sense can make operations by applying to the rules they have previously learned and by depending on paper-pencil (Yang, 2005).

In recent years, mathematics educators in many countries have emphasized that the development of the number sense of children is very important and suggested that it is necessary to integrate the learning and teaching of number sense into the mathematics curricula at primary and secondary school levels (Yang & Li, 2008; Yang, Reys, & Reys, 2009). It is also pointed out in many national reports (Australian Education Council, 1991; NCTM, 1989, 2000; National Research Council, 1989) that the number sense in mathematics education is important. While there is no achievement or activity in terms of creating number sense in the mathematics curriculum in Turkey, the importance of number sense is emphasized (Umay, Akkuş & Duatepe, 2006).

There are many studies conducted on number sense with students at various education levels (Harç, 2010; Kayhan Altay, 2010; Markovits & Pang, 2007; Menon, 2004; Mohamed & Johnny, 2010; Singh, 2009; Takır, 2016; Şengül & Gülbağcı, 2012; Şengül, Gülbağcı & Cantimer, 2012; Yang, 2005; Yang, Li & Lin, 2008). However, there is a limited number of studies conducted with teachers (Yaman, 2015b). Some of these studies conducted indicate that the number sense of preservice teachers was low (Kayhan-Altay & Umay, 2011; Şengül, 2013; Tsao, 2005; Yaman, 2012; Yang, Reys, & Reys, 2009). For example, Şengül (2013) stated in the study in which the strategies preferred by final-grade preservice elementary teachers that the number sense of preservice teachers was low and they used rule-based strategies instead of number sense strategies in their solutions. Kayhan-Altay and Umay (2011) concluded in their study in which the relationship between the calculation skills and number sense of preservice elementary school teachers was examined that preservice teachers preferred standard calculations to find precise results such as finding common denominator, performing routine multiplication and division, turning decimals into fractions instead of using their number sense.

The number sense can be learned and taught (Griffin, 2004). In this respect, Kaminski (2002) applied a number sense program in the learning environment in which preservice teachers were active and could discuss their ideas during 12 weeks as being 4 hours a week with 43 preservice teachers. As a result of the study, it was determined that preservice teachers could make better correlations between numbers, they could make further mental calculations, and they made more logical explanations for the results they obtained and for the operations they made. Yaman (2015b) stated in the study in which the effect of the Mathematics Teaching I and II courses on the development of the number sense of preservice teachers was examined that there was a significant increase in the number sense performances of preservice teachers and a decrease in their calculations.

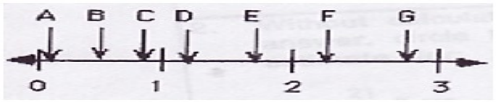
It is important to reveal the number sense skill performances of preservice teachers who will play the most important role in introducing these skills to their students. In accordance with the results to be obtained from this study, it will be determined whether it is required to structure teacher training programs in this respect and thus it will be possible to make a number of suggestions related to the planning to be made for teaching number sense. The aim of this study is to examine the number sense skills of preservice elementary school mathematics teachers. The following questions were attempted to be responded in this study based on this aim:

- Is there a significant difference between the number sense performances of second and third-grade preservice mathematics teachers?
- Is there a significant difference between the performances of preservice teachers in terms of the sub-dimensions of the number sense test?

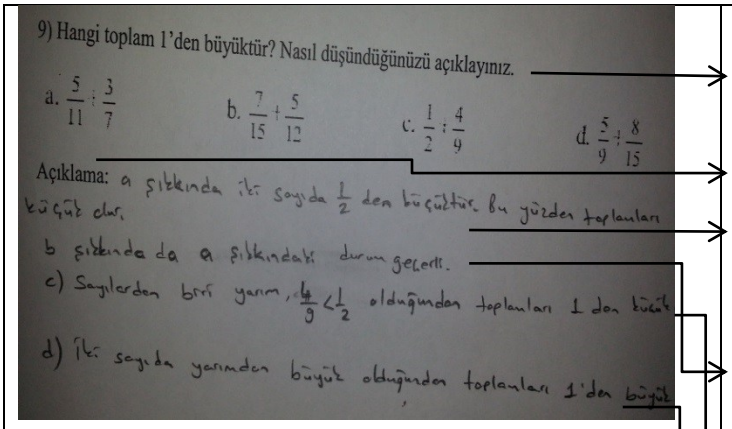
2. Method

Since the current state of the number sense performances of preservice teachers was described by using quantitative data collected in this study, the study is a descriptive survey study. The sample of this study consists of 52 (46.8%) second-grade and 59 (53.2%) third-grade students studying in the elementary school mathematics teaching program at the faculty of education of a state university. 79 (71.2%) of these preservice teachers are female, and 32 (28.8%) are male. The “Number Sense Test” consisting of 17 questions and three sub-dimensions and developed by Kayhan-Altay (2010) was used as a data collection tool in this study. In this test, there are eight questions in the sub-dimension of “Flexibility in Calculation”, four questions in the sub-dimension of “Conceptual Thinking in Fractions” and five questions in the sub-dimension of “Using a Reference Point”.

Table 1: Example questions of the Number Sense Test Components and explanations

Flexibility in Calculation	How can you perform the following operation in an easy way? Explain how you did it. 5 000 032 + 2 000 725 + 1 000 068 – 1 000 725.	The questions are related to the skills of using numbers in a flexible way, practical thinking, choosing the most effective and practical strategy. In the questions of this component, students are expected to solve the question in a practical way without making four operations in a long way.
Conceptual Thinking in Fractions	Which letter on the number line corresponds to the fraction of which numerator is slightly larger than the denominator? Explain how you found it. 	The questions consist of skills related to the concept of fraction. In the questions of this component, students are expected to show fractions on the number line and on the figure.
Using a Reference Point	Which sum is greater than 1? Explain how you think. a. $\frac{5}{11}+\frac{3}{7}$ b. $\frac{7}{15}+\frac{5}{12}$ c. $\frac{1}{2}+\frac{4}{9}$ d. $\frac{5}{9}+\frac{8}{15}$	The questions are related to the skill of using numbers such as 1, 1/2 as a reference point. It is a characteristic that usually helps in the decision-making process on greatness and in the facilitation of mental calculation.

It is expected that a practical way will be selected in simple operations encountered in the questions of the component of flexibility in calculation, fractions will be expressed in different forms such as number line, area model in the questions of the dimension of conceptual thinking in fractions, and the reference point will be determined and used in the questions of the dimension of using a reference point. Kayhan-Altay (2010) found the reliability coefficient for the test measurements to be 0.86. In this study, the KR-20 internal consistency coefficient was calculated to be .76 for the test measurements. This value indicates that the test measurements are reliable. The solutions made by using the number sense in the data analysis were assessed as 1 point, and the solutions based on calculation or made in a routine way were assessed as 0 points. Therefore, the highest score to be obtained from the test is 17, and the lowest score is 0. Below there is an example solution made by using the number sense of a question of the component of using a reference point and thus scored 1.



Which sum is greater than 1? Explain how you think.

Explanation:

a) Both numbers in "a" are smaller than 1/2. Thus, their sums are small.

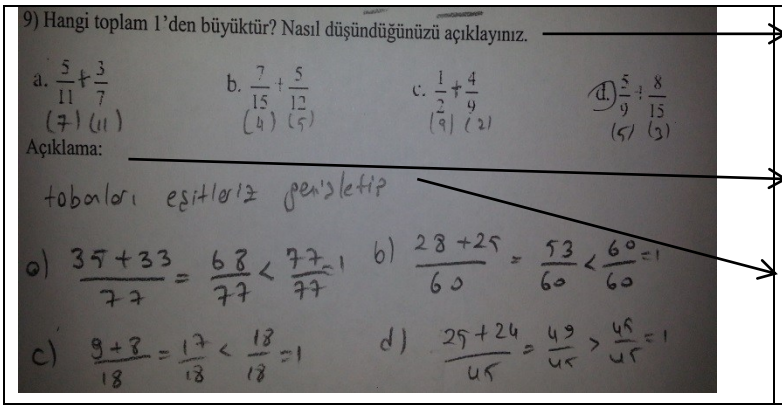
b) The condition in "a" is also valid in "b".

c) Since one of the numbers is half, 4/9<1/2, their sum is smaller than 1.

d) Since both numbers are greater than half, their sum is greater than 1.

Figure 1: An example solution made by using number sense

Below is an example solution of the same question based on the calculation or made in a routine way and thus scored 0.



Which sum is greater than 1? Explain how you think.

Explanation:

We make the denominators equal.

Figure 2: An example solution based on the calculation or made in a routine way

Since the data obtained did not show normal distribution, the Mann-Whitney U test among the nonparametric tests was used in the data analysis, and the data obtained were explained with the tables in the following section.

3. Findings

Firstly, the average scores and standard deviations of the number sense test of the students at each grade level were calculated to test the first sub-problem of the study, and they are presented in Table 2.

Table 2: Descriptive Statistics of the Number Sense Scores

Grade	n	X	ss
2	52	8.58	2.30
3	59	9.54	2.72

While the average of the scores obtained by second-grade preservice teachers from the number sense test was X=8.58, this value was calculated to be X=9.54 for third-grade preservice teachers. Considering the fact that the highest score that can be obtained from the test is 17, it draws attention that these values calculated are low. When the number sense scores of the preservice teachers are examined in terms of the number sense test sub-dimensions, Table 3 is obtained.

Table 3: The descriptive statistics of the number sense scores according to the sub-dimensions of the number sense test

	n	The Highest Score That Can Be Obtained	X	ss
Flexibility in Calculation	111	8	5.71	1.82
Conceptual Thinking in Fractions		4	1.99	.98
Using a Reference Point		5	1.38	.87

It is understood from Table 3 that while the average number sense scores of preservice teachers are the highest in the component of Flexibility in Calculation, they are the lowest in the component of Using a Reference Point. On the other hand, it was tested with the Mann-Whitney U test whether there was a significant difference in the number sense performances of preservice teachers according to the grade level, and the results are presented in Table 4.

Table 4: The U-Test Result of the Number Sense Performance According to the Grade Level

Grade	N	Mean Rank	Rank Sum	U	P
2	52	49.51	2574.5	1196.5	.044
3	59	61.72	3641.5		

A significant difference was found between the number sense performances of preservice teachers (U=1196.5, p<.05). Considering the mean ranks, it is understood that the performances of 3rd-grade students are higher than those of 2nd-grade students.

In accordance with the second sub-problem of the study, it was examined whether there was a significant difference between the performances of preservice teachers in terms of the sub-dimensions of the number sense test, and the findings obtained are presented in Table 5.

Table 5: The U-Test Result of the Sub-Dimensions' Scores of the Number Sense Test According to the Grade Level

Sub-Dimensions	**Grade**	**N**	**Mean Rank**	**Rank Sum**	**U**	**P**
Flexibility in Calculation	2	52	59.30	3083.50	1362.5	.302
	3	59	53.09	3132.50		
Conceptual Thinking in Fractions	2	52	51.73	2690	1312	.174
	3	59	59.76	3526		
Using a Reference Point	2	52	42.98	2235	857	.000
	3	59	67.47	3981		

Upon examining Table 5, it is understood that there is not a significant difference between the number sense performances of preservice teachers in terms of the sub-dimensions of Flexibility in Calculation and Conceptual Thinking in Fractions (U=1362.5, p>.05; U=1312.5, p>.05); there is a significant difference in favour of the third grades in terms of using a reference point (U=1196.5, p<.05).

4. Discussion and Conclusion

The findings of the study indicate that the number sense performances of preservice teachers are not as high as the level desired. This result is similar with the results of the previous studies (Şengül, 2013; Tsao, 2005; Yaman, 2012; Yaman, 2015a; Yang, Reys, & Reys, 2009) conducted with preservice teachers. Considering the average scores of the questions in the number sense test, it is understood that preservice teachers prefer making calculations or applying rules instead of applying number sense while making solutions. This finding refers to the same point with the results of many studies (İymen,

2012; Kayhan-Altay, 2010; Markovits & Pang, 2007; Mohamed & Johnny, 2010; Reys & Yang, 1998; Şengül & Gülbağcı, 2012; Şengül, Gülbağcı & Cantimer, 2012; Yang, 2005; Yang, 2007) conducted on samples at different levels. This situation is considered to be remarkable since it will be reflected on their teaching in the future professional lives brings to mind the need for including courses aimed at teaching number sense at the undergraduate level. This is because the literature indicates that the courses for developing the number sense contribute to the increase in the number sense performances of preservice teachers in a significant way (Nickerson & Whitacre, 2010; Whitacre, 2007; Whitacre & Nickerson, 2006; Yaman, 2015b).

In the study, it was also examined how the number sense performance varied by the number sense test components. The results of the previously conducted studies (Harç, 2010; Kayhan-Altay, 2010; Mohamed & Johnny, 2010; Yang, 2005; Yang & Li, 2008) are different since there is no association in the components suggested for the number sense in the literature. While the component in which preservice teachers succeeded at most in this study was the component of Flexibility in Calculation, the component in which they failed at most was the component of Using a Reference Point. This finding obtained matches up with the results of the studies of Takır (2016) and Cansız-Aktaş, Tuğrul-Özdemir and Yavuz-Mumcu (2017) who applied the same scale to secondary school students. On the other hand, it was determined that there was a significant difference in the Reference Point Component in favour of third grades in terms of grade levels.

Among the general purposes of mathematics in the mathematics curriculum applied in Turkey, there is a statement that "*The student will be able to use estimation and mental calculation skills effectively and develop problem-solving strategies and use these in daily life*". The number sense is not directly discussed in the content of the courses received by preservice teachers at the faculty, however, the subjects such as estimation, mental calculation; problem-solving strategies related to the number sense are mentioned in accordance with the general purpose mentioned. When the findings obtained from the study are examined, it comes to mind that the difference in favour of third grades may result from the Special Training Methods I and II courses taught at the 3rd-grade level and the Mathematics Curriculum course taught electively. Despite this difference, the number sense performances of preservice teachers are not at the desired level. Therefore, it is suggested to change the content of the abovementioned courses and to perform teaching about what kind of studies will be conducted for teaching the number sense to preservice teachers within the scope of the course. There may be an elective course in the program in which information about what the number sense is, its

components, how it will be developed, calculation, estimation, mental calculation skills can be provided in the content as also stated by Yaman (2015b).

References

1. Australian Education Council (1991). A national statement on mathematics for Australian schools. Melbourne: Curriculum Corporation.
2. Cansız-Aktaş, M. Tuğrul-Özdemir, E. & Yavuz-Mumcu, H. (2017). Ortaokul Öğrencilerinin Sayı Duyusu Becerilerinin İncelenmesi. Paper presented at 3. Türk Bilgisayar ve Matematik Eğitimi Sempozyumu, Afyon, Turkey.
3. Griffin, S. (2004), Teaching number sense. *Educational Leadership*, 61(5), 39-42.
4. Harç, S. (2010). *6. Sınıf öğrencilerinin sayı duygusu kavramı açısından mevcut durumlarının analizi* (Master's thesis, Marmara University, Istanbul, Turkey). Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/
5. Howden, H. (1989). Teaching number sense. Arithmetic Teacher, 36, 6-11.
6. İymen, E. (2012). *8. sınıf öğrencilerinin üslü ifadeler ile ilgili sayı duyularının sayı duyusu bileşenleri bakımından incelenmesi*. (Master's thesis, Pamukkale University, Denizli, Turkey). Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/
7. Kaminski, E. (2002). Promoting mathematical understanding: Number sense in action. *Mathematics Education Research Journal,* 14(2), 133-149.
8. Kayhan-Altay, M. (2010). *İlköğretim ikinci kademe öğrencilerinin sayı duyularının; sınıf düzeyine, cinsiyete ve sayı duyusu bileşenlerine göre incelenmesi* (Doctoral dissertation, Hacettepe University, Ankara, Turkey). Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/
9. Kayhan Altay, M., & Umay, A. (2011). Sınıf öğretmeni adaylarının hesaplama becerileri ve sayı duyuları arasındaki ilişkinin incelenmesi. e-Journal of New World Sciences Academy, 6 (1), 1277-1283.
10. Markovits, Z., & Pang, J. (2007). The ability of sixth grade students in Korea and Israel to cope with number sense tasks. InWoo, J. H., Lew, H. C., Park, K. S., & Seo, D. Y. (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 3, pp. 241–248), Seoul: PME.
11. Menon, R. (2004). Elementary school children's number sense. International Journal for Mathematics Teaching and Learning. Retrieved from http://www.cimt.plymouth.ac.uk/journal/ramamenon.pdf
12. Mohamed, M., & Johnny, J. (2010). Investigating number sense among students. Procedia Social and Behavioral Sciences, 8, 317-324.

13. National Council of Teacher of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
14. National Council of Teacher of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
15. National Research Council. (1989). *Everybody counts* (A report to the nation on the future of mathematics education). Washington, DC: National Academy Press.
16. Nickerson, S. D., & Whitacre, I. (2010). A local instruction theory for the development of number sense. *Mathematical Thinking and Learning*, 12, 227-252.
17. Reys, R. E., & Yang, D. C. (1998). Relationship between computational performance and number sense among sixth- and eigth-grade students in Taiwan. *Journal for Research in Mathematics Education*, *29*, 225-237.
18. Şengül, S. (2013). Sınıf Öğretmeni Adaylarının Kullandıkları Sayı Duyusu Stratejilerinin Belirlenmesi. *Kuram ve Uygulamada Eğitim Bilimleri*, 13(3), 1951-1974.
19. Şengül, S., & Gülbağcı, H. (2012). Evaluation of number sense on the subject of decimal numbers of the secondary stage students in Turkey. *International Online Journal of Educational Sciences,* 4(2), 296-310.
20. Şengül, S., Gülbağcı, H., & Cantimer, G. G. (2012). 6. Sınıf öğrencilerinin yüzde kavramı ile ilgili sayı hissi stratejilerinin incelenmesi. *The Journal of Academic Social Science Studies,* 5(8), 1055-1070.
21. Singh, P. (2009). An assessment of number sense among secondary school students. International Journal for Mathematics Teaching and Learning. Retrieved from www.cimt.plymouth.ac.uk/journal/singh.pdf.
22. Takır, A. (2016) 6., 7. ve 8. Sınıf Öğrencilerinin Sayı Duyusu Becerilerinin Bazı Değişkenler Açısından İncelenmesi. *Dicle Üniversitesi Ziya Gökalp Eğitim Fakültesi Dergisi*, 29, 309-323.
23. Tsao, Y. L. (2005). The number sense of preservice elementary school teachers. *College Student Journal*, *39*, 647-679.
24. Umay, A., Akkuş, O., & Duatepe Paksu, A. (2006). Matematik dersi 1. – 5. sınıf öğretim programlarının NCTM prensip ve standartlarına göre incelenmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 31, 198–211.
25. Whitacre, I. (2007). Preservice teachers' number sensible mental computation strategies. Proceedings of the Tenth Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education. San Diego, CA. Retrieved from http://sigmaa.maa.org/rume/crume2007/papers/whitacre.pdf.

26. Whitacre, I., & Nickerson, S. D. (2006). Pedagogy that makes (number) sense: A classroom teaching experiment around mental math. In S. Alatorre, J. L. Cortina, M. Sáiz, & A. Méndez (Eds.), Proceedings of the twenty-eighth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 2., pp. 736-743). Mérida, México: Universidad Pedagógica Nacional.
27. Yaman, H. (2012, May). *Matematik öğretimi dersi alan sınıf öğretmeni adaylarının tahmin ve sayı duyusu becerileri.* Paper presented at 11. Ulusal Sınıf Öğretmenliği Eğitimi Sempozyumu, Çayeli-Rize, Turkey.
28. Yaman, H. (2015a). Sınıf Düzeylerine Göre Öğretmen Adaylarının Sayı Duyusu Performansları. *Kastamonu Üniversitesi Kastamonu Eğitim Dergisi*, *23(2)*, 739-754.
29. Yaman, H. (2015b). The Mathematics Education I and II Courses' Effect on Teacher Candidates' Development of Number Sense. *Kuram ve Uygulamada Eğitim Bilimleri*, *15(4)*, 1119-1135.
30. Yang, D. C. (2005). Number sense strategies used by 6th grade students in Taiwan. *Educational Studies*, 31, 317-333.
31. Yang, D. C., & Li, M. N. (2008). An investigation of 3rd grade Taiwanese students' performance in number sense. *Educational Studies*, 34(5), 443-455.
32. Yang, D. C., Reys, R. E., & Reys, B. J. (2009). Number sense strategies used by pre-service teachers in Taiwan, *International Journal of Science and Mathematics Education*, 7(2), 383-403.
33. Yang, D. C., Li, M. N., & Lin, C. I. (2008). A Study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. *International Journal of Science and Mathematics Education*, 6, 789–807.