



STUDY AND RESEARCH IN TEACHING MATHEMATICS IN TWO HIGH SCHOOLS

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Abstract:

This work describes and analyzes the *dialectic study and research*, during implementation of Study and Research Path (SRP), in two high school courses. We analyzed over more than eight hundred protocols that generated the responses of (N = 75) students. The inquiry started with the question: “Which is the best savings plan to generate the greatest amount of income with a bank deposit that pays interest?” for which the students did not have an immediate response. That is to say, they did not know with what mathematical knowledge, or extra-mathematical, they could approach it. The analysis shows that implementing this type of teaching in an institution with previous experience in this paradigm works better than in one that lacks of prior experience.

Keywords: study, research, questioning, high school

1. Introduction

In general, the teaching of mathematics is reduced to “the study” of a set of “finished work”, meaningless and without *reason of being*. These works are studied in the teaching system as anything they were important “per se”. Students are invited to “visit”, admire and venerate these set of knowledge as they visit a “monument”. This “way” of considering the teaching of Mathematics defines what Chevallard has called “monumentalism” (2009; 2011a; 2011b; 2011c; 2012; 2013). The consequence of this teaching is reflected that mathematics is considered: incomprehensible, esoteric destined for a few and above all useless for life. The proposal of Chevallard is to study the mathematics imbued in the culture. Teach the mathematical knowledge linked to a

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*generating question*ⁱⁱ that comes from problems that arise in society, and that makes sense to the student. It is an epistemological and didactic revolution, from which emerges the *Paradigm of Research and Questioning the World* (PRQW). A didactic device related to it, are the Study and Research Path (SRP).

In a Study and Research Path (SRP), a group of students is supposed to study a question Q , with enough “generative power”, in the sense that the work done by the group is bound to produce a rich succession of problems that they will have to solve – at least partially – in order to reach a valuable answer A^\heartsuit to the question Q studied (Chevallard, 2009). In order to make a PRQW live, an SRP was implemented in two secondary schools. The SRP was based on question Q_0 : “Which is the best savings plan to generate the greatest amount of safe income?”. The question is linked to savings plans (Donvito, Sureda, Otero, 2013; Donvito, Otero, Sureda, 2014; 2017; Otero, Fanaro, Corica, Llanos, Sureda, Parra; 2013). The study of mathematics through this device allowed to re encounter some praxeologies related to exponential functions, sequences, series and logarithm.

This paper analyzes the characteristics of the *dialect of study and research* during the implementation of the SRP.

2. Literature Review

Within the framework of the ATD there are twenty-two works developed in the context of teaching by SRP, both in secondary and university. In Europe, the codisciplinary works to the economy made in secondary are those carried out by Ruiz Munzón (2010), Ruiz, Bosch and Gascón (2007). Regarding the codisciplinary works to the Sciences of Engineering, we locate the work of Byache, Beaubiat and Spaier (2016). While the monodisciplinary SRP in mathematics are those carried out by Barachet, Demichel and Noirfalise (2007), Gaud and Minet (2009) and Minet (2008).

In Argentina, Llanos and Otero (2015) implemented a monodisciplinary SRP in two fourth-year courses (14-15 years) of secondary school. The implementation was made during three consecutive years. The generating question corresponds to the multiplication of curves. The initial question, raised in the geometric, graphic and functional domains; it gives meaning to the remarkable points of the curves and to the general characteristics of the graphic representation of function mentioned before. A detailed analysis was realized according to the didactic functions (mesogenesis, topogenesis and chronogenesis).

On the other hand, Otero, Gazzola, Llanos and Arlego (2016) developed a co-disciplinary SRP to physics, in the last years of high school. The *generating question*: “Why did the Movediza Stone of Tandil fall?” Issues were studied related to the oscillating systems, simple harmonic motion, equations of motion, harmonic functions, damped and forced oscillations, resonance, notions of period, frequency, cycle in an oscillation,

ⁱⁱ A *generating question* is a question that must have the capacity to lead to the study of a set of praxeologies. At the same time, a general question and the derived questions must allow to “go over” the proposed study program in a course or at least a good part of it.

sine and cosine functions. The results were analyzed in terms of derived questions, mathematical and physical organizations, and built models.

Regarding the codisciplinary works to the economy we can locate, besides the own ones proper, those of Parra, Otero and Fanaro (2015) and Parra and Otero (2017). These last, developed a SRP in the last year (16-17 years). The starting questions are linked to the market equilibrium of a supply and demand model. The results detail the most identified dialectics.

Finally, this work focuses on the characteristics, which had the *dialectic of study and research* in the implementation of the SRP.

3. The SRP in the Anthropological Theory of the Didactics (ATD)

The ATD defines the SRP as devices that allow the study of mathematics by means of questions. The SRP establish that the starting points of mathematical knowledge are questions called generative questions, in the framework of the ATD, because its study should generate new, derived, questions. Teaching by means of SRP is complex and demands rootle changes in the roles of the teacher and students.

The study of a question Q as starting point of knowledge supposes the emergence of a didactic system denoted by $S(X; Y; Q)$. In the case of a mathematics classroom, this means that a group of students (X) helped by one or more teachers (Y) will build an answer A to the question Q . The operation of this system responds to a scheme that Chevallard (2013b) calls Herbartian scheme. In its reduced form, this scheme is written as follows:

$$S(X; Y; Q) \rightarrow A^\heartsuit.$$

The symbol \heartsuit indicates that the answer to Q was produced under certain constraints, "working" as a response to that question under those limitations (Chevallard, 2009). The elaboration of A^\heartsuit from Q needs the "fabrication" of a didactic medium M . This is expressed by the semi-developed Herbartian scheme:

$$[S(X; Y; Q) \rightarrow M] \rightarrow A^\heartsuit$$

That is, the didactic system constructs and organizes (\rightarrow) the medium M with the aim to generate or produce (\rightarrow) an answer A^\heartsuit . This scheme indicates that the elaboration of M is articulated in a complex way with the elaboration of the response. This observation is applied in the developed Herbartian scheme (Chevallard, 2009), which is written as follows:

$$[S(X; Y; Q) \rightarrow \{A_1^\diamond, A_2^\diamond, A_3^\diamond, \dots, A_n^\diamond, Q_{n+1}, \dots, Q_m, O_{m+1}, \dots, O_p\}] \rightarrow A,$$

where $M = \{A_1^\diamond, A_2^\diamond, A_3^\diamond, \dots, A_n^\diamond, Q_{n+1}, \dots, Q_m, O_{m+1}, \dots, O_p\}$ is the didactic medium to study (Q). The available responses A_i^\diamond , the derived questions Q_j and other Works O_l are

potential instruments to study Q . These instruments have to be conveniently studied in "quality" and "quantity", to be used effectively and efficiently in the study of Q , that is in the construction and validation of A^\heartsuit (Chevallard, 2009). The objects noted by A_i^\diamond , with $i = 1, \dots, n$ are "already made" answers available, for example, a book, a web page, a teacher's course, etc. The entities Q_j with $j = n + 1, \dots, m$ are other works - for instance, theories, experimental setups, praxeologies, etc., considered useful to deconstruct A and extract what is necessary there to construct the response A^\heartsuit (Ibid). Introducing the developed Herbartian scheme, Chevallard (2012) specifies what can be described as a Study and Research Path (SRP) (Otero, Arlego, Llanos, 2017).

In this work, the dialectic that Chevallard calls "*of study and research*" is analyzed. This dialectic supposes a "good combination" of study (of answers A_i^\diamond , of questions Q_j and of other works W_k) and research to be able to formulate, from previous works, an answer A^\heartsuit . Answering a *generating question* generates a questioning of the works, notions and knowledge that are linked to that question. This questioning causes an investigation around these works and at the same time this research generates specific studies. This is how a dialectic becomes concrete: a research generates a study and a study, a research.

4. Material and Methods

This research is qualitative and ethnographic, with participant observation. It was carried out in two courses of the junior year (16-17 years) of the secondary level of two schools in the city of Tandil. These are courses of approximately 25 students, which are distributed in groups of 4 or 5 students each. The implementation of the SRP was carried out by the researcher in two weekly classes of 2 clock hours each. The generating issues of the SRP refer to issues related to savings through a system of compound interest. The SRP was implemented in the usual classes of each Institution. The two institutions (I1, I2) where the SRP were developed are the following:

- **I1** it is a state private management school that serves middle urban sectors, which allows access to computers in the classroom, but does not provide access to the internet. The 58 students had PRQW experience in their math classes. The researcher was not the owner of the class but was able to work freely.
- **I2** It is a private secondary school that serves the middle and upper-middle urban sectors. Here we worked with 22 students without experience in the PRQW. This school promotes the use of netbooks and Internet access during classes, requires quarterly integrative assessments, and encourages students to participate in interscholastic science competitions. The class was in charge of one of the researchers.

We analyzed the students' productions (collected and scanned each class) and audio recordings. The purpose was, to analyze the type of questioning that led the investigation; and on the other hand, to determine the works, notions and knowledge that formed the study of each SRP.

5. Data analysis and results

5.1 Management of the SRP in the Institution 1 [I1]

To understand the question Q_0 : *Which is the best savings plan to generate the highest amount of income with a bank deposit that pays interest?* The students asked a total of eleven questions, for example: "How does a fixed term (FT) work?", "Are there different rates?", "What is a savings plan?", "How is the FT gain calculated?" Then, they searched for information on the Internet, outside of class hours, and formulated A^\diamond responses that were later disseminated in the class.

Most of the answers are transcriptions of partial responses existing on the internet. For example to answer the question, "What is a savings plan?" Group G5 transcribes the answer found on a page dedicated to the sale of automobiles (see Figure 1): "It is a system in which closed groups of people contribute monthly in a common fund. 70/30 savings plan: 70% in installments without interest. 30% upon delivery of the car "; that has nothing to do with the system of savings plan in compound interest. The students are limited to "compile" – that is, to copy without changing anything – an existing answer A , by way of answer A^\heartsuit , without question about the text where they have been found.

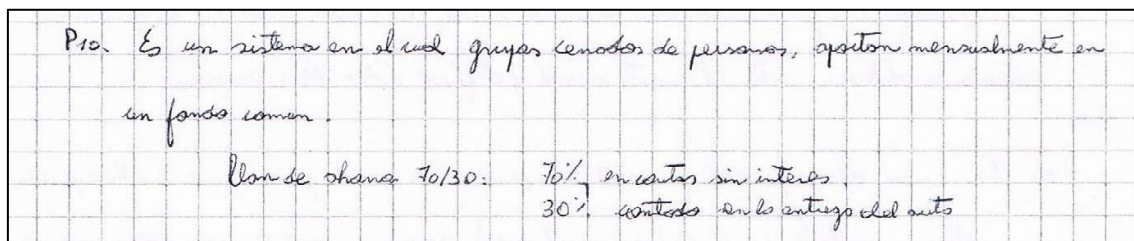


Figure 1: Answer of the G5 to the question, what is a savings plan?

There is, however, a small number of students of I1 who formulated answers that went beyond transcription, and that contributed an answer A to the study of the question Q_0 . An example is the response of the G2 group (see Figure 2): "A savings plan is a savings account, where the investor is depositing quotas at the pre-established value. It is not mandatory to deposit money, since this is obtained from the same interests. All these mechanisms are going to be built by banking entities. The types will depend on the type of bank and the fund deposited".

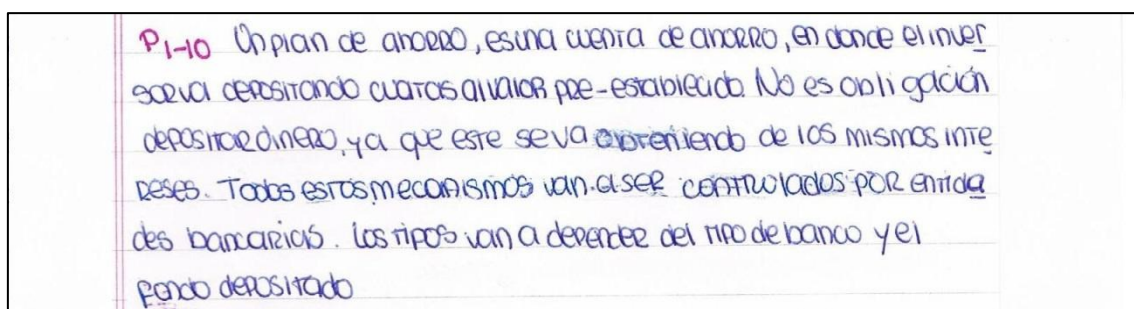


Figure 2: Answer of G2 to the question, What is a savings plan?

On the other hand, while the students were looking for information outside of class, the question "How is a fixed term calculated?" was studied in the classroom. For this, the professor introduced to a note on the functions and exponential equations that contained examples, definitions and tasks. Some tasks were linked to calculating the interest of the fixed terms. Then they returned to Question Q_0 : "What is the best savings plan to generate the greatest amount of income with a bank deposit that pays interest?" The students first analyzed the particular case of their own fixed term, through which they saved money for their graduation trip. Group G3, based on the idea that, the greater the capitalization period, the higher the interest rate, it designed a savings plan composed of twelve sub-plans that used, month by month, the best combination of fixed terms (Table1) .

Table 1: Savings plan composed of 12 different fixed terms

Month	Fixed-interest and kinds of capitalization	Final amounts
0	Fixed term to one year with the initial capital \$3877,65	\$4.449,60
1	\$ 660 posts 9 months to quarterly capitalization and then 1 time every two months	\$741,90
2	\$ 660 posts 9 months to quarterly capitalization and then 1 time every two months	\$734,03
3	\$ 660 posts 9 months to quarterly capitalization	\$726,46
4	\$ 660 positions 6 months to quarterly capitalization and then 1 time every two months	\$718,55
5	\$ 660 posts 6 months to quarterly capitalization and then 1 monthly time	\$710,93
6	\$ 660 posts 6 months to quarterly capitalization	\$703,60
7	\$ 660 posts 1 time to quarterly capitalization and then 1 time every two months	\$695,93
8	\$ 660 posts 1 time to quarterly capitalization and then 1 monthly time	\$688,55
9	\$ 660 posts once a quarterly capitalization	\$681,45
10	\$ 660 positions once a bi-monthly capitalization	\$674,03
11	\$ 660 posts 1 time to monthly capitalization	\$666,88
12	The last installment contributed by the group	\$660,00
Total		\$12.851,90

In month zero, they invested all the capital in an annual fixed term For month one, they started a new one with the accumulated weekly collection (\$ 660 in total each month) with quarterly capitalization during three periods and then capitalized that money in a fixed bimonthly term deposit; and so on, three quarters and then one month, three quarters, two quarters and a bimester, etc. The diffusion of this savings plan allowed us to reflect on the importance of studying the options, since this proposal allowed them to earn \$ 200 per year, with respect to the plan that the group already had.

Group G1 proposed a fixed-term deposit with monthly capitalization, to which \$ 660 was added every month. But because this involved recursively calculating the amount each month, they decided to formulate a mathematical expression (see Figure 3) that would allow them to calculate any month, independently of the previous one. For this, they modified the composite capitalization expression $M_f = (M_i) \cdot (1 + i)^t$ for $M_f = (M_i + 660 \cdot t) \cdot (1 + i)^t$.

$MF(t) = M_i \cdot (1+i)^t$
 - Nuestro $M_i = \$3877,28$
 $-(3877,28 + 660 \cdot t) \cdot (1+i)^t$
 $= (3877,28 + 660 \cdot t) \cdot (1 + 0,009860)^t$
 $11913,99$

33 alumnos con
 $\$20 / mes$
 \downarrow
 $\$660$
 \downarrow
 $M_i + 660 \cdot t$

Figure 3: Attempt of the G1 group to avoid recursion

The expression proposed was not useful for calculating the money of the savings system they were proposing (since they did not consider the accumulated interest of the previous months), the class formulated the question, “How is a fixed term calculated with an incorporation of periodic and constant capital?” which led to the study of works W_j : successions and series. Once this study was finished, the students were able to have a formula that would allow them to avoid the recursive calculation they had done previously, and which was accepted as A^\heartsuit .

So, in this institution (I1) it was possible to identify the characteristics of the *dialectic of study and research*. Since, to answer the generating question it was generated with the help of the teacher and the class in general, a questioning of the works, notions and knowledge linked to that Q_0 question that caused an investigation around these works. This research, also, generated the studies of exponential functions, linked to the calculation of compound interest; and that of successions and series.

5.2 Management of the SRP in the Institution 2 [I2]

To understand the question Q_0 : *What is the best savings plan to generate the highest amount of income with a bank deposit that pays interest?* the students placed a question that seemed more concrete to them: “How to generate one million pesos with money placed at interest in a fixed term?” And they started looking for information on the internet. The search led them to ask questions such as: *How does a fixed term (FT) work? Are there different rates? What is a savings plan? How is the FT gain calculated?*

The A^\diamond responses that were then diffused to the entire class can be considered as “final”. For example, group G5 (see figure 4) first copied the definition of fixed term (FT) and its formula: “A fixed term is an investment constructed with renewable deposits and is calculated at the time of placement of the funds. Putting money at interest means that the money deposited delivers a small profit. The interest is calculated: $I = \frac{C \cdot R \cdot T}{100 \cdot 360}$ ”. Then he calculates the initial capital he would need if he wants to have 1,000,000 with a rate of 25% in 180 days.

UN PLAZO FIJO ES UNA INVERSIÓN CONSTRUIDA CON DEPOSITOS RENOVABLES Y SE CALCULA EN EL MOMENTO DE COLOCACIÓN DE LOS FONDOS. PONER DINERO A INTERÉS SIGNIFICA QUE EL DINERO DEPOSITADO ENTREGA UNA PEQUEÑA GANANCIA, EL INTERÉS SE CALCULA : $I = \frac{C \cdot R \cdot T}{100 \times 360}$

I = INTERES
 C = CAPITAL APORTADO
 R = TASA DE INTERES
 T = TIEMPO (MESES, DIAS)

360 = puede VARIAR segun el tiempo UTILIZADO (12 si son meses, 360 si son dias, etc)

100 = numero fijo.

PARA NOSOTROS ES posible llegar al millon de pesos se va a NECESITAR mucho CAPITAL inicial o mucho tiempo (que, en este caso, NO disponemos) ASIQUE VAMOS a optar por AUMENTAR el capital inicial.

$$1.000.000 = \frac{C \cdot 25\% \cdot 180 \text{ (DIAS)}}{100 \cdot 360 \text{ (DIAS)}}$$

Figure 4: Response of the G5 group to the question, How to generate one million pesos with money placed at interest in a fixed term?

The rate they place is invented, because at the time of implementation the nominal annual rate (NAR) offered by the banks were between 11% and 17%. The group realizes that the rate is small and explains that it is going to “*calculate the initial capital because if they do not need a lot of money, or a lot of time*”. The other groups had similar responses to this. In summary, the work of the students is reduced to copying the information and using it to give an answer A^\diamond that even though it is far from being the A^\heartsuit is considered as such. The answer to the real problem that was raised is school, decontextualized and fictitious. During the diffusion, the teacher questioned the answers by two questions: “*What happens if the interest calculated each year is added to the initial amount, before calculating the interest for the following year? Is there a formula that allows me to calculate the total money that I will have every month?*”

A- ¿Que pasa si los intereses de cada año le suman al capital inicial?

B- ¿Hay formula para calcular el capital inicial?

(A) Van a llegar más rapido a 1 millon porque:
 se dan 30.000

Por ej: se dan 30.000 el año de interes poniendo 500.000 el máximo año, en vez de sacar el interés neto 500.000, se saca el máximo 530.000. El máximo año, como el plazo es de 530.000 los intereses van a ser más, y así sucesivamente.

(B) $I = M \times ((1 + (\frac{i}{100})^n / 360) - 1)$

Figure 5: Response of group G2 to the question, What happens if the interest calculated each year is added to the initial amount, before calculating the interest for the following year?

These questions were intended to study the calculation of compound interest and the exponential function. The answers to these questions showed that there were two types of interest (see figure 5): "It is going to get faster to 1 million because 30,000 are given. For example: You get 30,000 a year interest by putting 500,000, if next year instead of taking interest out of 500,000 you take out over 530,000 the interest will be more, and so on".

Given that the students' answers did not generate the study of new works or knowledge, or new research, the professor requested a detailed analysis that would allow them to decide if it was convenient to place the money at simple or compound interest to obtain more quickly one million pesos. The students searched the information on the internet, and built graphs to show how the amount of money varied according to the interest rate. The straight line of simple interest suggested to them that this interest could be modeled by a linear function, and that of interest made up of a parable (a function they had studied the previous year). The teacher asked them to take the necessary actions to verify these statements.

After a new search on the internet, the students came to the conclusion that indeed the compound interest was calculated using the quadratic function, and that their graph corresponded to a parable.

The group G1 justified it in the following way (see figure 6): "The compound interest is a parable, it is the geometrical place of the points of a plane equidistant to a given line".

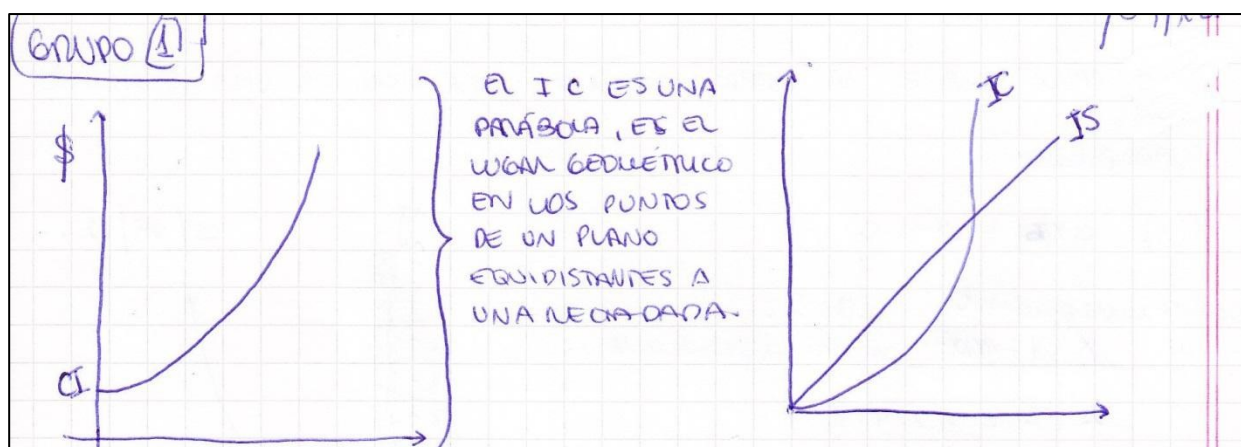


Figure 6: Justification of G1 that compound interest is a parable

The students transcribed the information from the internet, which did not serve to justify that the variation of compound interest was a parable, without questioning it. They did not ask for the terms that made up the definition, and were not able to explain them during the diffusion, nor to graph what they had just published. The teacher asked them, then, to evaluate the algebraic expression in the negative numbers. When doing so and completing the graphs for the negative values of the quadratic function and compound interest, the students recognized that both graphs were different, and that consequently compound interest could not be a quadratic function. This conclusion derived in the entrance to the study of the work W_1 : exponential functions.

For the study of this work the professor introduced a similar note to the one of the institution I1 on the functions and exponential equations that contained examples, definitions and tasks. Once they studied the type of variation linked to the increase in the amount of money placed at interest, the SRP was terminated. The students considered that knowing how the fixed terms worked at compound interest and their variation already had the answer A^n to Q_0 .

In this institution, an attempt made *Paradigm of Research and Questioning the World* (PRQW) live, but since the students limited themselves to transcribing the information, they found on the Internet without criticism or questioning, it was concluded that the students did not carry out a *research*. As a result, the questioning only came from the teacher, and the study of the mathematical work was made by a note, by its indication, and not due to the formulation of new questions (derived questions), or the generation of a questioning of the works, notions and knowledge that were linked to those questions. So in this institution there was no *research* that genuinely generated a *study*, or a *study* that generated a *research*, because it lacked the main thing, the attitude of generating new questions, which require going to seek new knowledge, because this is what keeps a teaching alive for research.

6. Conclusion

The evidence obtained in this paper shows that the *dialectic of study and research*, in the terms described by the ATD (see section 3), is very difficult to achieve. In the I1 institution, it was possible to identify some characteristics of the *dialectic of study and research*, which did not occur in the I2 institution. In the I1 institution, the questions asked by some students allowed the study of oeuvres. For example, the search for a formula to calculate the money that would be obtained when a monthly fee is added to the original fixed term, it derived in the study of series and sequences. In the I2 institution, the search for closed answers the lack of questioning of the students caused the interruption of the SRP. The difference can be attributed to the experience of I1 and the inexperience of I2. This reinforces the idea that the paradigm shift is very difficult to achieve.

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