European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.1218119 Volume 4 │ Issue 4 │ 2018

ANALYSIS OF THE CAUSES OF LOW ACHIEVEMENT LEVELS IN SOLVING PROBLEMS WITH PARAMETER

Dalibor Gonda[i]
Faculty of Humanities,
Universities of Žilina in Žilina, Slovakia

Abstract:
This article describes research into the causes of failures of students in solving tasks with parameters. As a research tool, a non-standardized test was used, which was fulfilled by 124 respondents at the age of 18. Following analysis of the works of the respondents revealed basic misconceptions of the term parameter. Another reason, which was revealed by research, is formal setting of the conditions to solve the tasks. On one side the shortage is linked to the way students learn Mathematics. It is largely preferred learning of the whole task procedures to understanding the particular steps. On the other hand, it is related to the lack of definition of the terms parameter and the unknown. At the same time the misconception is unveiled, consisting in considering the term "-a" for always negative. The article, along with a description of the research and analysis of the test results, offers the basic solutions of discovered reasons of why students' fail to solve the tasks with parameters.

Keywords: creative approach; investigative skills; parameter; the method of zero points; understanding

1. Introduction

Teaching and learning can be broadly classified into two types. The names of months, days, people, animals, objects, physical quantities were created by agreements. They were not logically derived. It is sufficient to learn them without understanding. However, principles, laws, theories arose on the basis of logical operations. It is not enough to learn their wording. For their acquisition they need to be learned with understanding. Mathematical knowledge arose as a result of logical reasoning and operations of solving tasks of practical life. Using mathematical knowledge to solve new challenges requires an understanding of the already acquired solving procedures of already solved tasks. To develop mathematical knowledge and thinking it is necessary to seek answers how to teach math so that the students can understand. At

[i] Correspondence: email barboragondova@gmail.com

the same time we must look for the root causes of misunderstanding mathematics (Sierpinska, 1994; Ma, 1999).

The students seem to have to go to great effort to acquire the ways of solving the mathematical tasks without understanding. This attitude to learning mathematics results from the way they learn other, mainly humanity based subjects. These allegations are based on several studies. For example Vankúš & Kubicová (2010) found out the following attitude of students towards learning mathematics: "*Ordinary students cannot understand the math; they can only memorize the rules.*" (p. 280). Teaching mathematics in the early stages of mathematical education contributes to this attitude in some way. The primary objective of the initial phase of the mathematical education is to create a quality calculating apparatus - quality mathematical basis (Hejný, 2014).

At this stage, teaching mathematics is focused on acquiring the basic concepts and procedures for solving the mathematical tasks. The teacher gives students the curriculum mostly in the definitive final form. The teacher's explanation of the procedure is followed by training the solving procedures based on similar tasks, as it was the prime one. The students deal with most tasks by imitating, repeating the solving process that is already familiar for them. In some time students will get familiar with the idea that to be able to solve the task is identical to the calculating algorithms. They (students) pay a little attention to understanding why the calculating algorithm was used for solving the task. Finally, they get the idea that the successful solving of mathematical tasks is based on adopting solving procedures without understanding. It is enough to match the task assignment with the correct calculating algorithm. But in fact, the successful investigators of mathematical tasks are those students who can already adopt solving procedures in other areas of mathematics. Learning mathematics should contribute to developing a more in-depth learning style. It lies in trying to understand the curriculum, capture its meaning, and involve acquired knowledge into existing knowledge structures. Students learn not only to meet the requirements, but they want to apply the acquired knowledge in practice (Rovňanová, 2012). However, there is a necessity of teaching and learning with understanding that is essential for success in mathematics. Teaching and learning with understanding requires a different approach of teachers and learners. Novelty lies primarily in the fact that there is a priority to focus on, as to why the problem is solved in this way (not how it is solved). Accordingly, we use logical procedures of learning and we create logical chains (Petty, 1996).

2. From the calculating algorithms to the methods of solving

The development of mathematical thinking can be divided into two stages. The first, essential step is the acquisition of elementary calculating algorithms. The term algorithm means a set of clearly defined rules governing the succession of implementing a finite number of elementary operations, which ensures each task of that type will be solved in finite time. The algorithm can be characterized by its basic characteristics:

1. Elementarism: a finite number of simple, easily achievable actions (the steps of algorithm).
2. Determinism: After each step, we can say that the algorithm has already ended and if not, what step should follow.
3. Finality: The process described by an algorithm will end after some final time.
4. Finiteness: an algorithm leads the calculation from the input data to the result.
5. Mass scale: an algorithm is defined to solve the big group of tasks of the same type (Znám et al, 1986).

In mathematics the basic calculating algorithms are considered to be all the calculating operations with expressions and solving algebraic equations, inequalities and their sets. On the basis of the teacher's explanation, the students acquire calculating algorithms and practice them on the appropriate number of examples. We can talk about math "drill", without which it is impossible to be a successful solver of mathematical problems. Basically, the students are expected to automate the implementation of the basic calculating algorithms. At this stage information-receptive didactic method in combination with the reproductive method is used (Patlák, 2004). It is very important that repeated use of basic calculating algorithms will help the student to receive necessary skill in their use.

The teacher can, even at this stage, hope to achieve that pupils acquire these calculating algorithms at least at the level of understanding, not only at the level of memorization.

The second stage of mathematical education is teaching algorithmic rules. Algorithmic guidelines, in contrast to an algorithm, are not characterized by determination or formality. Turek (2008) stated that the individual operations take the content character and require mental activities, based on the understanding of their meaning. Therefore, it is necessary to put some stress on teaching them. At this stage, it is very important to monitor and control the understanding of supporting ideas of the method. It is recommended to choose the tasks to solve them using the algorithmic guidelines applied in different variations. And this prevents students from memorizing the algorithmic guidelines as the calculating algorithm.

Solving tasks by using algorithmic guidelines leads to application of acquired algorithms in different areas of mathematics and other disciplines, or in practical everyday life. At this stage, the mathematical "drill" is substituted by mathematical reasoning. This is the stage in which students learn to create the solving of the task. The essential feature of this phase of mathematical learning is to develop capabilities to deal with the task by applying already acquired knowledge and skills from different areas, not just mathematics. In addition to new knowledge, there is the place to apply already acquired skills. A student learns that the first step of task solving is not to count but to think. A priority for him/her is to create a solution and not to match task assignment with a calculating algorithm. In principle, there is a fundamental shift in thinking and approaching to dealing with (not only) mathematical problems. Shift is from “repeating” of what I learned to "wonder" how to use what I know. Therefore, it is necessary to remember each step of the algorithmic guidelines, but at the same time it is not a sufficient condition for using the algorithmic guidelines in solving problems. The teacher becomes a moderator of solving. Hence, a didactic heuristic method is considered the most appropriate.

2.1 Difficulties solving the tasks with a parameter

In terms of successful solving of mathematical problems, transition from the acquisition of basic calculating algorithms to the stage of algorithmic guidelines becomes critical. In most cases, the problem lies in students trying to understand the discussed methods of solving as algorithms, which are enough to learn and match them with the task assignment. In this context, a serious problem appears to be the indirect methods of solving mathematical problems as the most frequently used algorithmic guidelines. (Odvárko, 1990), although these methods have their internal structure with the same sequence of steps, we think that the perception of them as calculating algorithms is very limiting for further development of the mathematical thinking of students. When using the indirect methods, the task often needs to be divided into the sub-tasks that can already be solved by the relevant calculating algorithms. Indirect methods are largely universal methods and they can help to solve different types of problems. Their effective use is necessary, in addition to acquiring basic structure of the method, to understand the method. Given the increased level of acquiring algorithmic guidelines, there is a difference in the successful tasks being solving by individual students (Turek, 2008).

The tasks with parameters create a large group of tasks solved by indirect methods. The task that contains the parameter is a set of the same type of tasks. The particular task is obtained by replacing the parameter by a number. If we put a parameter to the task, the task does not change (quadratic equation remains quadratic). Therefore, the task procedures are basically the same as for the task without parameters up to the moment when the next step of solving is dependent on the value of parameter. Although parameter in task assignment does not change the type of task, that would require a new way of solving. However, serious problems in solving problems with parameters occur in educational practice.

2.2 Students´ perception of the term parameter

The parameter will be added as an unknown, as a new concept to solve the equation with similar content.

Based on research dealing with how students understand the concept of a variable (MacGregor & Stacey, 1993; Bednarz & Lee, 1996; Trigueros & Ursini, 1999; Bardini, Radford & Sabena, 2005), the similar problems with the understanding of the term parameter are expected. These studies show that the variable is often viewed as a "potentially determined" number. The students see it as a temporarily unknown number that will be determined at some point. There are probably the roots of confusing the concept of a variable with the concept of unknown. The concept of unknown means the unknown number that is determined when solving equations or inequalities (Schoenfeld & Arcavi; 1988, Radford, 1996). We suppose that the reasons for a problem to solve tasks with a parameter are related to students' access to learning mathematics. According the current mathematic didactics, the teaching of mathematics is often based on the transmission of ready knowledge and their memorization, while it should be based on the creative learning process with the active participation of learners (Polák, 2014). This system teaches students to match a learned process and an algorithm, with the assigned task. Therefore, the task with a parameter is perceived as a new type of task, different from an analogous task with no parameter. The teacher is expected to teach them the new algorithm of for finding a solution. But the presence of a parameter does not change the strategy of calculating. Rather, it requires dividing the task into individual parts - to atomize. Atomization is the result of thinking when looking for solutions. It is not an automatic step of algorithm. We think that the main cause of little success in solving the tasks with a parameter is the students' effort to deal with the task using algorithms and matching calculations with the assigned task, without sufficient understanding of each step of the solution.

2.3 The research objective and the description of the research tool

We decided to do a pilot research on verifying the above-described reason. As a research sample, students - who are preparing future mathematics teachers, were chosen from the first year of the four pedagogical faculties of education. All respondents recently passed maturity in mathematics and they did not practice any mathematics examples before testing. The current research tool consisted of 4 test tasks (see Annex). The first two tasks are common inequalities without parameters. They were included to demonstrate the solving ability of respondents to deal with basic types of inequalities. The other two tasks were inequalities with parameter of the same type as the first two tasks. They were crucial for the pilot research, and therefore they were chosen very carefully. The following are the solutions of inequalities with parameters:

Example 1: Solve the inequalities with a parameter$a$: $|x| < \frac{a}{x}$.

Solution: It is clear from the assignment that $x \neq 0$ and at the same time for $a = 0$ the assigned inequalities has no solution.

1st way: The example is solved as inequalities with absolute value.

For $x > 0$ we solve the inequalities

$$x < \frac{a}{x}.$$

For $x < 0$ we solve the inequalities

$$-x < \frac{a}{x}.$$

Since the unknown x acquires only positive values, after the removal of a fraction in inequalities, we solve the simplified inequalities$x^2 < a$. It is obvious that the inequalities have the solution only for positive values of parameter$a$. Its solution on the set of positive numbers is $x \in (0; \sqrt{a})$.

Since the unknown x acquires only negative values, after the removal of a fraction in inequalities, we solve the simplified inequalities $x^2 < -a$. it is obvious that the inequalities have the solution only for negative values of parameter$a$. Its solution on the set of negative numbers is $x \in (-\sqrt{-a}; 0)$.

The acquired solution is put into the table below.

Table 1: Variable value depending on the parameter

a	x
0	$\emptyset$
$a > 0$	$(0; \sqrt{a})$
$a < 0$	$(-\sqrt{-a}; 0)$

2nd way: The task is solved as the inequalities with the unknown in the denominator. We annul the right side and create the fraction on the left side. After that we get the inequalities:

$$\frac{a - x|x|}{x} > 0.$$

One zero point is$x = 0$, the other zero points are acquired by solving the equation

$$a - x|x| = 0.$$

For $x > 0$ we solve the equation$a - x^2 = 0$. Solution of this equation is $x = \pm\sqrt{a}$ for positive values of parameter$a$. We set up Table 2 just for positive values of variable x

Table 2: The resulting sign of expressions on particular intervals

	$(\mathbf{0}; \sqrt{\mathbf{a}})$	$(\sqrt{\mathbf{a}}; \infty)$
$a - x\|x\|$	$+$	$-$
x	$+$	$+$
$\frac{a - x\|x\|}{x}$	$+$	$-$

Based on the table for > 0 , the inequalities solution is $x \in (0; \sqrt{a})$.
For $x < 0$ we solve the equation$a + x^2 = 0$. The solution is $x = \pm\sqrt{-a}$ for negative values of parameter$a$. We set up Table 3 just for negative values of variable x.

Table 3: The resulting sign of expressions on particular intervals

	$(-\infty; -\sqrt{-\mathbf{a}})$	$(-\sqrt{-\mathbf{a}}; \mathbf{0})$
$a - x\|x\|$	$+$	$-$
x	$-$	$-$
$\frac{a - x\|x\|}{x}$	$-$	$+$

Based on the table for $a < 0$ the solution of inequalities is $x \in (-\sqrt{-a}; 0)$.

Task 4: Solve quadratic inequalities with a parameter$a$: $x^2 + ax > 0$.

Solution:

1st way: The task is solved as quadratic inequalities with a parameter. First, we determine the discriminant value, which determines the number of solutions of corresponding quadratic equations

$$x^2 + ax = 0 \ (1)$$

In our case, the discriminant equals the expression$D = a^2$.

If the expression a^2 is positive ($D > 0$), the corresponding quadratic equation (1) has two different real roots. Thus, we can write:

$$a^2 > 0 \Rightarrow a \in R - \{0\} \Rightarrow x_{1,2} = 0; -a.$$

In the next step, we solve the quadratic inequalities $x^2 + ax > 0$ with parameter $a \in R - \{0\}$. The inequalities are solved by a method of zero points. Quadratic equation $x_1 = 0, x_2 = -a$ roots are obtained by zero points. Based on these zero points, we divide the domain expression $x^2 + ax$ to the intervals, where this expression takes the same „signed" values (positive or negative). For zero point $x = -a$ negative. We set up the table below:

Table 4: The resulting sign of expressions on particular intervals

	$(-\infty; -\mathbf{a})$	$(-\mathbf{a}; \mathbf{0})$	$(\mathbf{0}; \infty)$
x	$-$	$-$	$+$
$x + a$	$-$	$+$	$+$
$x(x + a)$	$+$	$-$	$+$

Based on the table, for $a > 0$ the inequalities solution is$x \in (-\infty; -a) \cup (0; \infty)$.

For $a < 0$ the zero point is $x = -a$ positive. We set up Table 5:

Table 5: The resulting sign of expressions on particular intervals

	$(-\infty; 0)$	$(0; -a)$	$(-a; \infty)$
x	$-$	$+$	$+$
$x + a$	$-$	$-$	$+$
$x(x + a)$	$+$	$-$	$+$

Based on Table 5, for $a < 0$ the inequalities solution is$x \in (-\infty; 0) \cup (-a; \infty)$.

If the expression a^2 is zero ($D = 0$), the corresponding quadratic equation has the only solution$x_p = 0$. In this case, the solution of this quadratic inequality is$x \in R - \{0\}$.

The expression a^2 does not obtain negative values which is why we do not think about the case of a negative discriminant.

We write the obtained solution in Table 6:

Table 6: The variable value depending on a parameter

a	x
0	$R - \{0\}$
$a > 0$	$(-\infty; -a) \cup (0; \infty)$
$a < 0$	$(-\infty; 0) \cup (-a; \infty)$

2[nd] way: The task is solved as quadratic inequalities without an absolute element. We will use the method of zero points. The expression on the left side of the inequality is adjusted to the product. We get the inequality in the form

$$x(x + a) > 0.$$

The expression on the left side has a value of zero for $x = 0$ and $= -a$. Based on these zero points we divide the domain of expression $x^2 + ax$ into the intervals at which this expression takes the same "sign" values (positive or negative). For $a > 0$ is the zero point $x = -a$ negative. We set up the table below.

Table 7: The resulting sign of expressions on particular intervals

	$(-\infty; -a)$	$(-a; 0)$	$(0; \infty)$
x	$-$	$-$	$+$
$x + a$	$-$	$+$	$+$
$x(x + a)$	$+$	$-$	$+$

Based on Table 7, for $a > 0$ the inequalities solution is$x \in (-\infty; -a) \cup (0; \infty)$.

For $a < 0$ the zero point $x = -a$ is positive. We set up Table 8.

Table 8: The resulting sign of expressions on particular intervals

	$(-\infty; 0)$	$(0; -a)$	$(-a; \infty)$
x	$-$	$+$	$+$
$x + a$	$-$	$-$	$+$
$x(x + a)$	$+$	$-$	$+$

Based on Table 8, for $a < 0$ the inequalities solution is$x \in (-\infty; 0) \cup (-a; \infty)$.

For $a = 0$ we have only one zero point$x = 0$. In this case the inequalities solution is$x \in R - \{0\}$.

There are several possible points of view how to perceive the tasks and they extended the range of solution choices. In both tasks respondents had to deal with key element of solutions, with the expression$-a$.

2.4 Analysis of students´ works

Percentage of respondents in solving various tasks is presented in Table 9.

Table 9: Percentage of students in solving particular problems

Task	$\lvert 3x - 5\rvert - 2x \leq 10$	$x^2 + 4x > 0$	$\lvert x\rvert < \frac{a}{x}$	$x^2 + ax > 0$
Sts. who looked for a solution	124	124	78	95
Sts. who solved	117	124	4	27

The results above describe the testing which confirms that the respondents, who want to go on mathematics in their professional life, have serious problems with solving the tasks with a parameter (in this case the inequalities with a parameter). At the same time, the testing revealed several causes of them having little success in solving tasks with parameters. The two most serious of them occurred in the majority of false solutions of test tasks with parameters. They will be described in the following paragraph.

3. Main findings of the research

The most frequently occurred deficiency among our respondents was the perception of the term " $-a$". Many of the respondents regarded this term as always negative and the expression "a" always positive. A teacher often comes across this misconception in teaching equations with absolute values. Since the basic school students are already familiar with the concept of absolute value of numbers. On the basis of solved examples and tasks, students understand the concept of absolute value as follows: "the result of the absolute value always has a sign (+)." But when teaching the equations with absolute value, pupils get familiar with algorithmic rules. The key step of algorithmic rules is to remove the absolute value of the equation using the following definitions: Let a be a real number. The absolute value of the number a is marked $|a|$ and defined as follows:

If ≥ 0 , then $a = a$. If < 0 , then $a = -a$ (Polák, 2003).

The expression $-a$ in the definition was considered negative by many students. So, there is a contradiction with the concept of absolute values. At this stage of teaching, it is considered to be very important that the pupils learn to distinguish between numerical expression 3 and expression $-a$. The essential difference lies in the fact that -3 is a negative number, but $-a$ should be seen as a contrary expression to expression a. At the same time -3 is the opposite number to the number 3. The symbol "-" in the presented context can be interpreted in two senses: 1. negative, 2. contrary. The pupils get familiar with both meanings when studying about Integers in the curriculum. The fact is that they learn: opposite number to the number -3 is number 3 (- (- 3) is not used) and so the students usually keep the fact in their mind that symbol - (minus) before the number corresponds to the concept 'negative'. Based on this concept, they analogously perceive minus in front of the variable as a negative value of a variable. When students are taught in the curriculum "Equations with absolute value", it is necessary to relink minus = "contrary". It is so, because if the expression is negative in absolute value at a given interval, we replace it with the contrary expression but without absolute value. Pupils often avoid the resumption of the semantic links in the way they remember "practical": In the negative value of the expression in absolute value, it is then written without absolute value, and the marks are changed of each member. In this case, the expression a is positive and the expression $-a$ negative. However, we talk about a misconception that turns out to be the main cause of using incorrect tasks used to solve the problems as found in the testing.

The second major problem in solving test tasks can be called "formal conditions". During the task solving of the task, it is often necessary to determine the conditions that can basically be divided into two groups. The first group consists of the conditions under which it makes sense to deal with the task. Here, we include conditions based on the definitions of basic mathematical concepts. At the level of secondary schools, we talk mostly about conditions, at the set of real numbers, based on the following "definitions": division by zero is meaningless, extract the root of only non-negative numbers; there are logarithms of only positive numbers, or combinations of these "definitions". To set up the conditions of solvability, practically means the setting up and solving of an inequality. We can create an algorithm of these conditions, so that students do not have major problems with them. When analyzing the works of the respondents, we found out that most of them identified the conditions only after solving the task (they occurred behind the calculating part of the task). In several cases, they were not identified at all. This leads to the conclusion that the setting up conditions is not seen as a useful part of the solving.

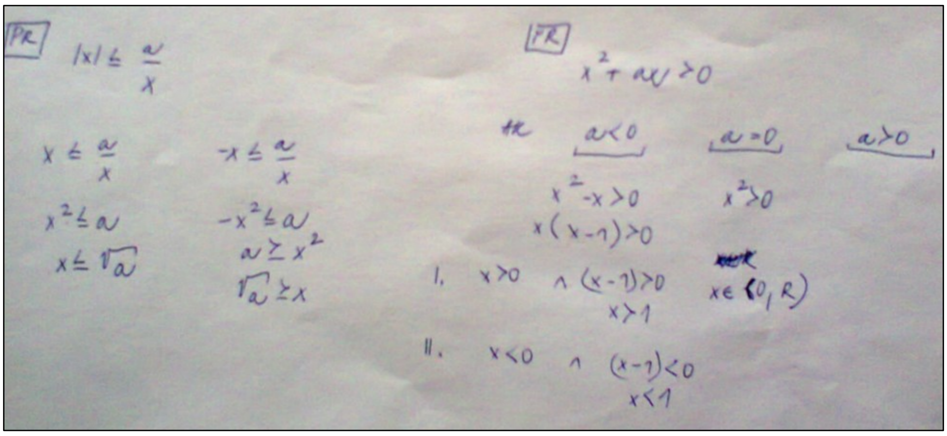
At the end of the task a significant minority of respondents unified the conditions and the results of the calculating part instead of making the intersection. That was basically the only mistake that occurred when solving the first two test tasks. This shows a lack of understanding of the importance of the conditions, in which it makes sense to deal with the task. Observed deficiencies revealed that students perceive the determination of the conditions of solvability as one of the steps of solving that type of task. For the development of mathematical thinking, formally written conditions to calculating part of the task should be replaced by the students understanding of them. The importance of these kinds of conditions is the fact that these conditions divided the set of real numbers into two disjunctive subsets. One subset includes numbers that can be the solution of the task and another subset contains numbers that cannot be the task solution. The application of this knowledge leads to the intersection of conditions with the result of the calculating part of the task solution. For practical reasons, it is recommended to help students to establish the conditions before calculating part of task solution, to enhance their relevance and applicability also within the calculations. In the case of an "algorithmic" task, students will also be lead to have to think about the task and only then begin with the solving of the tasks

The determination of the conditions from the second group allows them to make some steps towards the calculation. It is possible to meet these conditions if it is necessary in a further step of solving by dividing both sides of the equation by the expression containing the unknown. The condition is defined under which the expression is different from zero, and then we can make a "wanted" adjustment of the equation. After solving this equation, it is necessary to solve the equation in case the expression that we used for division equals zero. These conditions are already the result of investigational considerations and students often have problems using them when solving the task. In the context of the described research, we meet these kinds of conditions when removing the absolute value. At the same time, they are key conditions to successful task solving with a parameter of any kind.

All respondents used the method of zero points when they solved the inequalities with absolute value. They knew when they solve inequalities; they should use this method consisting of two parts. In the first part, they defined the intervals at which the expression in the absolute value has non-negative and negative values. The second part consists of the replacement of the expression with the absolute value by the expression without absolute value and subsequent calculation of the newly emerged inequality. The tasks with parameters, however, revealed a misunderstanding of the first part. Research results confirmed that the students are not always sufficiently aware that the determination of the conditions is crucial to be able to replace the expression with the absolute value with the expression without the absolute value and thus, they continue with task calculation. Particular intervals are the conditions permitting to continue with the solving inequalities.

Based on the analysis of respondents' answers, the perception of conditions can be summarized as follows. They found setting up conditions as an act that should be done under certain circumstances resulting from the assignment. Broadly speaking, the conditions are determined by students because they remember their determination as one of the steps in finding a solution. They work with them a little but do not use them to make the calculating part of the task more effective during calculating. For tasks with the parameters, an attempt appears to determine the conditions also on the basis of remembering. Just the formality of setting conditions (determination of conditions to fulfil the rule of their need to be determined) is a manifestation of misunderstanding their function at solving the problem. And the tasks with the parameters require active work with conditions for either a parameter or variable. The formal definition of the conditions as the part of the algorithmic solution of the tasks is not sufficient.

To illustrate our claims, we include at least one, but not isolated, from the students´ work in our research.



The first example solving (on the left) contains only calculating part, the conditions are completely absent. Parameter a is perceived as nonnegative. In the second example (on the right) the conditions for parameter a are set totally formally.

They do not result from the assigned inequality and had no effect on the steps of solving mentioned in the example. Apparently, the investigator remembered that there is a necessity to set conditions for a parameter in this type of problems. Three conditions were set for a parameter (we point out that, as for the discriminant of quadratic equation), but subsequent solution of the task was not divided into three parts, depending on the value of the parameter.

3.1 Recommendations for practice

An in-depth analysis of the causes of failure of the respondents to solve problems with a parameter revealed a deeper cause. It can be identified as the root cause of the problems of students with tasks with a parameter. It involves an erroneous perception of the concept of parameter and its relation to the numbers and variables. In testing, the respondents perceived a parameter as an additional variable in the task. It is related to the fact that they meet the term parameter in teaching equations for the first time. They simply get familiar with the fact that "letters" in the equation (task) are variables whose values should be calculated. Basically, students miss the difference between parameter and variable. The students set the parameter values due to circumstances arising from task; and then, the values of variable are solved for predetermined value of a parameter. This is closely related to the problem with setting and using conditions during the task solving. The only limitation of our research is that we only looked at respondents' cognitive abilities. It is possible that students who experience low achievement in Mathematics might have reasons different from their own cognitive characteristics (Olkun et al, 2016). For example, according to Boaler (2016), up to 40% of children are afraid of mathematics due to frequent mathematical failures.

The basis of improving student achievement in students´ dealing with the tasks with parameter is recommended to establish the correct concept of the term parameter in the minds of students. At the same time, it is necessary to distinguish the concept of parameter from that of the unknown. To achieve these objectives, it is appropriate to introduce the concept of parameter when teaching inequalities. The new concept of 'parameter' arises from the need to realize some part of the task. The proposed procedure is illustrated by particular example that students can solve without any problem.

Example: On the set R solve the inequality $2x - 3 > x + 6$.

After a simple adjustment, we obtain the inequality$x > 9$. The set of solutions of the inequality is written using interval: $K = (9; \infty)$. The need to introduce the concept of parameter can be justified as follows. Ask the question "how to do the correctness test? "We point out that even with the inequalities there is a need to verify the correctness of our solution even if the test is not a necessary part of the solution. The numbers that satisfy the inequality are written in two ways. In the form of inequality - $x > 9$ or of a set $K = (9; \infty)$. In both cases, it is the notation of all real numbers greater than 9. If the analogy was used with the test of correctness in the equations, we should gradually substitute all real numbers greater than 9. The problem is that there are infinitely many of real numbers greater than 9. We can remind students that, within the test of accuracy, we cannot substitute the variable for either inequality or a set; we can substitute only the number. Therefore, we need to find a new way how to write down all numbers greater than 9 and it must be a "number". Numbers greater than 9 can be written in the following ways: 9+1, 9+2, 9+3 ...etc. Any number greater than 9 can be written so that we add some (any) positive number to it. In mathematics, we can use a letter to write down any number. We add a set of numbers to it. Therefore, our wanted notation may be $9 + a; a \in (0; \infty)$. In our case, we used "a letter" to write down numbers with the same feature and "a letter" receives the name parameter. In general, we use a parameter in mathematics, if we want to write down (expression, equation ...) more objects with the same characteristics. The notation $9 + a$ is the set of numbers and can be substituted to an inequality. After substitution, we do the test of accuracy for infinitely, many inequality solutions and we do it with one calculation. We point out that the set, joined to the notation $9 + a$, can be seen as a condition that a parameter must meet, so that our expression represents our desired set of numbers. This creates a link between an expression containing a parameter and a condition that the parameter must meet so that the expression has the desired characteristics. The correctly created concept of the parameter and its dynamic integration into the world of students' thoughts forms the basis for students to master the work with conditions. At the same time, the suitable setting of the conditions and their effective use in solving problems can be seen as a sign of understanding of solving the problems with parameters.

4. Conclusion

The results of the above described research revealed the causes of the students' difficulties in solving problems with parameters. In general terms, when they learn mathematics there is an effort to remember the procedures for solving problems. They are satisfied they can calculate a problem but they do not try to understand the causes of using particular steps when solving problems. The aim of the mathematics teaching, however, is to develop creative thinking, and thus the overall human personality. A teacher can achieve this development of creativity by choosing the tasks that cannot be solved only by repeating the learned process. It is advisable to choose such tasks where it is necessary to create solutions, based on the proper acquirement of the concepts and the relationships between them. The problems with parameters fulfil this task. The assignment of these problems offers considerable variability and their solving require no new knowledge in mathematics. Their major benefit is that students learn to use already acquired knowledge and skills in different permutations and combinations and thus, develop creativity and enhance self-efficacy, thereby increasing the success rate of students in solving problems (Hoffman, 2010). You can develop pseudoabstraction with parameter tasks. According to Dumitrascu (2017), the pseudoabstraction consists in deriving new properties by transforming the initial data.

In the end, students learn „mathematical independence". This term refers to the awareness where the student is able to solve tasks based on his/her own knowledge and skills in his/her own way. The core in solving the problems is what he/she knows and not what he/she remembers. The approach of students´ is changing on the basis of their internal motivational factors. Understanding the new concepts and putting them into the world of thoughts becomes priority. For the reasons described above, it is recommended that the problems with parameters should be often used during mathematical education and not to be a separate chapter in teaching equations and inequalities. Based on our findings, it is recommended to focus on further research into the connection between using algorithms and mathematical student confidence.

References

1. Bardini, C., Radford, L. & Sabena, C. (2005). Struggling with Variables, Parameters, and Indeterminate Objects or How to Go Insane in Mathematics. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 129-136). Melbourne: PME.
2. Bednarz, N., Kieran, C., & Lee, L. (Eds.) (1996). *Approaches to Algebra, Perspectives for Research and Teaching*. Dordrecht /Boston/London: Kluwer. https://doi.org/10.1007/978-94-009-1732-3_1
3. Boaler, J. (2016) Mathematical mindsets. Jossey-Bass, A Wiley Brand, San Francisko.
4. Dumitrascu, G (2017) Understanding the Process of Generalization in Mathematics through Activity Theory. International Journal of Learning, Teaching and Educational Research. Vol 16, No 12 (2017). https://doi.org/10.26803/ijlter.16.12.4
5. Hejný, M. (2014). V*yučování Matematice Orientované na Budování Schémat: Aritmetika 1. stupně.* [Teaching building schemes oriented mathematics: Arithmetics of 1st grade] Praha: Univerzita Karlova v Praze, Pedagogická fakulta.
6. Hoffman, B. (2010). „I think I can, but I'm afraid to try": The role of self-efficacy beliefs and mathematics anxiety in mathematics problem-solving efficiety. *Learning and Individual Differences, 20(3),* 276-283. doi: 10.1016/j.lindif.2010.02.001.
7. Ma, L. (1999). Knowing and Teaching Elementary Mathematics: *Teachers´ Understanding of Fundamental Mathematics in China and the United States*. LEAP, Mahwah – New Jersey. https://doi.org/10.2307/749776
8. MacGregor, M., & Stacey, K. (1993). Seeing a pattern and writing a rule. In I. Hirabayashi, N. Nohda, K. Shigematsu & F. Lin (Eds), *Proceedings of the 17th International conference for the Psychology of Mathematics Education* (Vol. 1, pp. 181-188). University of Tsukuba, Japan. https://doi.org/10.2307/749345
9. Odvárko, O. a kol. (1990). *Metody Řešení Matematických Úloh* [Methods of solving the mathematical problems]. Praha: SPN.
10. Olkun, S., Altun, A., Sahin, S. G., Kaya, G.(2016). Psychometric Properties of a Screening Tool for Elementary School Students' Math Learning Disorder Risk.

International Journal of Learning, Teaching and Educational Research. Vol 15, No 12 (2016)

11. Patlák, E. (2004). *Všeobecná Didaktika* [General didactics]. IRIS, Bratislava. Petty, G. (1996). *Moderní Vyučování* [Modern teaching]. Potrál Praha.
12. Polák, J. (2003). *Přehled Středoškolské Matematiky* [Overwiev of high school mathematics]. Prometheus, Praha.
13. Polák, J. (2014). *Didaktika Matematiky* [Didactics of mathematics]. Fraus: Plzeň. Radford, L. (1996). The roles of Geometry and Arithmetic in the development of Elementary Algebra. In N. Bednarz et al. (Eds.), *Approaches to Algebra: Perspectives for research and Teaching* (p. 39-53). Dordrecht: Kluwer. https://doi.org/10.1007/978-94-009-1732-3_3.
14. Rovňanová, L. (2012). Učebné štýly žiakov - výzva pre didaktickú prácu pedagogických a odborných zamestnancov v školskom prostredí v kontexte novo vynárajúcich sa potrieb detí [Learning styles of pupils – a challenge for didactic work of pedagogical and professional staff in the school environment]. *Humanum Międzynarodowe Studia Społeczno-Humanistyczne.* Nr. 9(2)/2012, 327-338.
15. Schoenfeld, A. & Arcavi, A. (1988). On the Meaning of Variable. *Mathematics Teacher* Nr. 81(6), 420-427.
16. Sierpinska, A. (1994). *Understanding in Mathematics*. The Falmer Press, London-Bristol, 1994. https://doi.org/10.4324/9780203454183.
17. Trigueros M. & Ursini S. (1999). Does the understanding of variable evolve through schooling? In O. Zaslavsky (Ed), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 273-280). Haifa, Israel: PME.
18. Turek, I. (2008). *Didaktika* [Didactics], Iura Edition: Bratislava.
19. Vankúš, P. & Kubicová, E. (2010). Postoje žiakov 5. a 9. ročníka ZŠ k matematike [Pupils attitudes of the 5th and 9th year of elementary school to mathematics]. *Acta Mathematica* (vol 13, pp 277 – 283). Fakulta prírodných vied UKF v Nitre, Nitra.
20. Znám, Š., Bukovský, L., Hejný,M., Hvorecký, J. & Riečan, B. (1986). *Pohľad do dejín matematiky* [View into the History of Mathematics], Alfa: Bratislava. .