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MIDDLE SCHOOL MATHEMATICS TEACHERS' KNOWLEDGE OF MATHEMATICAL REASONING[i]

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Abstract:
Acknowledging the importance of mathematical reasoning in mathematics teaching, the present study analyzes middle school mathematics teachers' understanding of reasoning. In line with the purpose, the study was performed using a phenomenological design, which is one of the qualitative research methods. The study was carried out with a total of 16 middle school mathematics teachers serving in different state schools in the 2016-2017 school year. The study employed an interview form consisting of four open-ended questions to determine the teachers' theoretical and practical understanding of mathematical reasoning. Drawing on qualitative data, this study employed content analysis method for data analysis. As a result, it is understood that the middle school mathematics teachers do not have a comprehensive and adequate knowledge and view about mathematical reasoning when it is considered how they define, exemplify and support mathematical reasoning. Because it is appeared that mathematical reasoning means only making explanations, justifications and producing different solutions for a problem according to them. It is important that the teachers should broaden their view about mathematical reasoning to support their students' reasoning better.

Keywords: mathematical reasoning, mathematical thinking, middle school mathematics teachers

1. Introduction

Mathematics is one of the fields where many resort to reasoning. By its very nature, mathematics entails discovering patterns, reasoning, making predictions, motivated

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thinking, and reaching the conclusion. For instance, one cannot start solving a problem without thoroughly examining the problem and understanding what is being asked; as such, it is not possible to advocate mathematical ideas without providing a basis and a justification for them (Umay, 2003). In this process, mathematical reasoning is essential to formulate, to assess mathematical arguments, and to choose and to utilize a variety of representations (NCTM, 2000). Mathematical reasoning is similarly fundamental in solving a problem and questioning the validity of an argument (NCTM, 2000). Therefore, mathematical reasoning is considered as a constituent of doing mathematics (NCTM, 2000).

There are various definitions on mathematical reasoning in the literature. Some researchers described mathematical reasoning based on certain concepts such as *induction, deduction, abduction* (Holton, Stacey & FitzSimons, 2012) and *adaptive reasoning* (Kilpatrick, Swafford & Findell, 2001), while Carpenter, Franke and Levi (2003) reported that mathematical reasoning is "*to explain why a method works or a conjecture is true*." That is, reasoning is regarded as explaining, justifying and proving something. On the other hand, Brodie (2009) assumed that communication in *lines of thinking and argument* is prerequisite for mathematical reasoning. Based on this assumption, Brodie stated that communication may take the form of pictures, symbols, diagrams or models in the process of reasoning. With these definitions and given the function of mathematical reasoning, it can be argued that reasoning enables learners (students and teachers) to actively structure their mathematical ideas and to make sense of mathematics (Kilpatrick et al., 2001). It is due to this reason that the development of reasoning skills holds an important place in mathematics education (especially in basic education).

One of the most fundamental goals of mathematics education is to provide rational answers in response to why-questions, in other words, to develop reasoning skills (Altıparmak & Öziş, 2005). Since reasoning is a skill underlying mathematical thinking. Indeed, mathematical reasoning skills are also included in the mathematical process skills targeted in the course of mathematics at middle school level in Turkey. Considering the effectiveness of reasoning in facilitating school and non-school life as well, there has been a need to prepare settings to allow the development of these skills in the process of mathematics teaching (MEB, 2013). Regarding this, teachers play a significant role to improve these skills via classroom activities. They require classroom discussions enabling students to create alternative solutions, to share their solutions and ideas, and to communicate with one another so that they can improve their reasoning skills about a problem situation. Given the opportunity to discuss their ideas with peers and to develop their mathematical understanding through talk, students have a greater chance to develop reasoning competencies (Ayele, 2017). Therefore, it is paramount that teachers have knowledge of mathematical reasoning and are capable of creating rich reasoning environments in order to support the development of mathematical reasoning skills among students.

2. Literature Review

2.1 Teachers' knowledge of mathematical reasoning

Most studies on mathematical reasoning have focused on the place and importance of reasoning in mathematics teaching (Brodie, 2009; Carpenter et al., 2003; Franke, Webb, Chan, Ing, Freund & Battey, 2009; Stylianides, 2010) or examined the mathematical reasoning skills of students (Caroll, 1998; Küchemann & Hoyles, 2006; Liu & Manouchehri, 2013; Stylianides, 2005). However, the studies on mathematics teachers' knowledge and understanding of mathematical reasoning are remarkably few in number (Clarke, Clarke & Sullivan, 2012; Loong, Vale, Bragg & Herbert, 2013; Simon & Blume, 1996; Stylianides, Stylianides & Philippou, 2007). Among them, the study by Clarke et al. (2013) aimed to find out what a total of 124 primary school teachers think and know about reasoning. Accordingly, the teachers in the study were asked what reasoning term they employed in classroom (*justifying, proving, reasoning, evaluating, analysing, generalising, inferring, deducing, adapting, transferring and contrasting)* and how regularly they used it. The researchers concluded that all teachers used the term '*explaining*' for reasoning and the term '*proving*' was also employed by some. Yet, the terms 'transferring', 'deducing' or 'contrasting' were less commonly associated with reasoning. The findings revealed that teachers integrated only certain aspects of reasoning in their teaching and the results on which aspects they integrated were not clear. In a similar way, the study by Herbert et al. (2015) with 24 primary school teachers aimed to probe their perceptions of mathematical reasoning by providing insights into both their way of teaching and student learning in their classrooms. Their findings demonstrated that teachers perceived reasoning as thinking, communicating thinking, problem solving, validating thinking, forming conjectures, validating conjectures, using logical arguments for validating conjectures and connecting aspects of mathematics. On the other hand, Loong et al. (2013) examined 7 primary school teachers' understanding of mathematical reasoning. The teachers were asked to explain how they define "*mathematical reasoning*" and to exemplify their classroom practices incorporating reasoning. Defining mathematical reasoning, two teachers used the terms "explaining" and "justifying"; other teachers failed to describe the concept of reasoning or provided inexplicit definitions. Also, the teachers reported that reasoning and working mathematically were similar. The examples of reasoning provided by the teachers from their classrooms merely offered student answers and explanations, and their problem-solving processes. Despite citing student answers to exemplify mathematical reasoning, teachers failed to define reasoning in the statement. The examples of problem solving processes offered by the teachers as an example for reasoning were trials and student errors. In conclusion, Loong et al. (2013), stated that the teachers may not have a clear understanding of reasoning by displaying insufficient knowledge regarding reasoning.

These being said, the available studies have revealed primary school teachers' understanding of reasoning; yet, there is no study examining their thinking about reasoning. Acknowledging the importance of mathematical reasoning in mathematics teaching, the present study analyzes middle school mathematics teachers' understanding of reasoning.

3. Material and Methods

This is a qualitative study with the purpose of finding out middle school mathematics teachers' understanding of mathematical reasoning. In line with the purpose, the study was performed using a phenomenological design, which is one of the qualitative research methods. A phenomenological design seeks to reveal common practices and to identify and describe the meanings assigned by participants (Annells, 2006).

3.1 Participants

The study was carried out with a total of 16 middle school mathematics teachers serving in different state schools in the 2016-2017 school year. The teachers were selected through random sampling and voluntarily participated in the study. They varied in teaching experiences, which ranges from two to 11 years, and represented different levels, including 5^{th}, 6^{th}, 7^{th} and 8^{th} grade. To ensure that the participants remained anonymous due to the research ethics, they were identified by assigned number as "T1", "T2", "T3",… "Ö16."

3.2 Data Collection Tool

The study employed an interview form consisting of four open-ended questions to determine the teachers' understanding of mathematical reasoning. Prior to administration, the form was reviewed by two field experts to ensure the clarity of the questions and to align them with the purpose of the study. The form was later revised in line with the feedbacks of the experts to establish the validity and reliability of the study. Table 1 presents the questions in the interview form.

Table 1: The Questions in the Interview Form

How do you define mathematical reasoning?
Could you provide examples from your classrooms regarding students' mathematical reasoning?
What kind of approach do you follow to support students' mathematical reasoning skills?
Below are the examples of some problem solutions provided by different students. What can you deduce from these solutions regarding the mathematical reasoning of the students? (Problem solutions are presented in the findings section.)

As seen in Table 1, the questions directed to the teachers were intended to reveal their theoretical and practical understanding of mathematical reasoning. The first and last questions were designed to determine teachers' theoretical knowledge of reasoning while the second and third questions were aimed to explore teachers' thinking of mathematical reasoning in practice based on the examples and student answers they provided. The interview form was individually administered to the teachers over a period of 30 to 40 minutes. The teachers completed the form without being helped or interrupted by the researchers in any way.

3.3 Data Analysis

Drawing on qualitative data, this study employed content analysis method for data analysis. Content analysis strives to unveil the facts that may be embedded in the obtained data and to describe and organize these data in a way that readers can understand by bringing together similar data in the context of certain concepts and themes (Yıldırım & Şimşek, 2013).

In the first stage of data analysis, each question was analyzed separately; the researchers examined the data simultaneously and independently of each other and generated a code table with categories and sub-categories. In the second stage, the researchers coded all data. Subsequently, the researchers compared the data together to eliminate the difference between coding. In the third stage, they recoded the data to ensure the compatibility between the coders. Following the final revision on the code table, the data analysis was performed. The results of the analysis were displayed in the findings section. Also, in order to further establish the validity and reliability of the study, the data were elaborated and presented as direct quotations as much as possible.

4. Results

This section presents the findings from the analysis of the answers given by the teachers to the questions in the interview form. The researchers examined each question in the form through content analysis and identified different themes, which can be seen below in the relevant tables.

Table 2: Themes addressing the answers to the first question

Theme	Teacher
Regarding problem solving;	
Employing all the necessary thinking strategies	T1
Using mathematical knowledge to develop alternative solutions	T4
Justifying	T4, T11, T13, T16
Understanding the solution	T5, T7 (choosing the most appropriate solution),
Understanding the problem	T5, T6 (analyzing the question), T8 (identifying the relationship between the components of the problem)
Guessing	T8
Developing strategy	T9, T10
Having metacognitive skills	T10,
Performing problem-solving process	T12, T13,
Relating mathematical knowledge to the previous knowledge	T3, T12,
Creating new models through mathematical knowledge	T3
Reinterpreting mathematical knowledge	T3
Relating mathematical knowledge to real life situations	T14, T15

Firstly, the question of "How do you define mathematical reasoning?" was asked to the teachers. Table 2 presents the themes addressing the answers to the question directed to them, showing that there are four main themes. These themes indicated that the teachers related mathematical reasoning to different aspects of problem-solving. As for the theme of problem-solving, the teachers related mathematical reasoning to the aspect of justifying for the solution of the problem most. Thus, it can be argued that a large part of the teachers defined mathematical reasoning as justifying a solution presented for a problem. The statements of the two teachers defining mathematical reasoning as justifying are as follows:

> *"The ability to explain the reasons and the rationale." (T4)*

> *"It is being able to explain the accuracy or inaccuracy of the problem through justifications by asking why when solving a problem and to generate logical answers." (T16)*

This aspect was followed by the aspects of understanding the problem, developing strategy and performing problem-solving process. For example, emphasizing the importance of understanding the problem in relation to mathematical reasoning, the teacher numbered T5 articulated that "*One can understand the problem and anticipate the solution only through these skills*." Relating mathematical reasoning to developing strategy, another teacher described it as "*to contemplate on and to determine the ways to find the accurate solution of the problem in a logical way based on the limited data available and to know what one is doing in every step of the solution and to act consciously*" (T10).

Further, there were some teachers defining mathematical reasoning as relating mathematical knowledge to the previous knowledge and relating mathematical knowledge to real life situations. One of them, associating mathematical reasoning with generating new models through mathematical knowledge and reinterpreting such knowledge, described mathematical reasoning as "*the effort to relate numbers, algebraic expressions, mathematical thinking to one's previous knowledge, to generate new models, to reinterpret rather than to memorize and to use a formula*" (T3)

Table 3: Themes addressing the answers to the second question

Theme	Teacher
No answer/ Irrelevant answer	T7, T10, T12, T14
Deducing	T2, T13
Developing strategy	T3,
Generating alternative solutions	T4, T5, T8, T9, T15
Examples for the cases involving the association of different mathematical knowledge with each other	T6, T15
Generalizing	T11
Justifying	T16

Secondly, the teachers were asked to provide examples from their classrooms regarding students' mathematical reasoning. Table 3 presents the themes, which were identified by means of the analysis of the examples provided by the teachers. The table demonstrates that the examples mostly focused on generating alternative solutions to the questions/problems. Some of the examples provided by the teachers related to generating alternative solutions are as follows:

"For instance, when I asked the question of "Is it possible to create different rectangles with an area of 36 cm^2?" to the students, most of them proposed alternative areas." (T5)

"When simplifying a square root, they seek to determine the perfect square by dividing the number by the smallest possible prime number, instead of using prime factorization." (T8)

The theme of generating alternative solutions was followed by the examples for the cases involving the association of different mathematical knowledge with each other and deduction. One teacher for each of the themes of generalizing and justifying provided cases as an example, which are as follows:

"When 5th grade students begin to learn four operations, they are told that they are going on a trip and asked to find out how many cars will be needed. Then, they identify the number of cars by means of grouping, which is actually an example of division. Or they are told that they are giving a small concert to their peers in the classroom and find out the number of those to attend the concert by counting the classroom. Then, they are asked to determine how much will be earnt in the concert and they perform repetitive addition by using cost per person, which is an example of multiplication. In this way, they could recognize the relationship between these cases." (T15)

"I performed an activity with 5th grade students to find the number of diagonals. We talked about how to do that by drawing triangles, quadrangles, pentagons. When finding the number of diagonals, sometimes they marked the same diagonal. I gave them homework to make an inference. Although they failed to reason as much as I wanted them to do, they obtained concrete data through their drawings." (T11)

Although there were various examples provided by the teachers for mathematical reasoning, four teachers offered irrelevant examples or failed to offer any example regarding the reasoning of the students in their classrooms, which is a remarkable finding implying that these teachers do not promote mathematical reasoning in their classrooms. Two of these teachers providing examples under these themes made the following statements:

"This happens when they solve complex problems. Sometimes they are not willing to think; unfortunately, they want everything fall into their lap. They think that the teacher should make everything ready for them; but, this is not the case for the course of mathematics, which is something they understand over time. When I incorporate two

subjects into a single question, my students usually ask me what they are going to do. Or when they encounter a new type of question, they ask similar things." (T7)

"I am trying to promote reasoning when appropriate, depending on the conditions of the classrooms. Yet, it is a disappointment experience for many students. Being accustomed to certain patterns, students want to multiply or add numbers randomly and without thinking and to find the solution. This is mostly common when they solve test questions." (T10)

Table 4: Themes addressing the answers to the third question

Theme	Teacher
Encouraging different solutions	T1, T4, T5, T8, T10, T14
Promoting communication between students	T1
Providing the opportunities to think for students	T1, T2,
Asking questions	
Questions intended to help finding the solution	T2
Questions to encourage justification	T3, T10, T15, T16
Questions intended to relate the previous knowledge to the new one	T6
Questions intended to make students realize their own mistakes	T8
Questions for problem solving	T9
Questions intended to understand problem situation	T12
Questions intended to reach the definition of a concept	T13
Concretizing	T3
Directing students to express with their own words	T4
Encouraging students to justify	T4
Using non-routine problem situations	T4, T7
Creating a model	T4
Forming a table	T4
Directing students to generate a problem	T4
Problem situations requiring students to relate to real life situations	T5, T15
Creating a classroom discussion	T6
Directing students towards a group work	T11
Creating an appropriate classroom environment	T15
Analyzing student solutions	T16

Thirdly, the question of "What kind of approach do you follow to support students' mathematical reasoning skills?" was asked to the teachers. A total of 16 themes were identified as a result of the analysis of the answers provided by the teachers. The answers demonstrated that a large part of the teachers asked their students various questions in order to support reasoning skills. These questions were mostly intended to encourage the students to justify their answers. The teachers also asked different types of questions to the students so that they can understand the problem situation, reach the definition of a concept, realize their own mistakes or relate their previous knowledge to the new one.

"I do not focus on the answer while solving a question. I constantly ask why we conclude this or how we conclude if this happens. I try to encourage students to talk about their solutions. As they understand better while talking about it. Since I predict any potential misconceptions, I address them and prompt students to reason." (T16)

"As there is a problem about reading comprehension, I first ask guiding questions to learn to what extent that they understand what they have read. I direct them to find clues to enable them to link what they have read, to comprehend it as a story, and to relate. In short, sometimes I simply feel like I am in a Turkish language course. When I do this, they still may not solve the problem; but, at least I notice that they seem to be surprised to learn what the question actually tells." (Ö12)

"Rather than directly pointing out the mistakes they do while reasoning, I ask them distracting questions to make them notice their own mistakes." (T8).

Based on the opinions of the teachers, another way to support students' mathematical reasoning skills is to encourage them to engage in different ways to solve the problem. With the statements such as "*After solving the question on the board, I tell them there are different ways to solve the same problem, but I do not show these ways, instead, I give students an opportunity to articulate their own solutions on the board*" (T1), "*I present multiple solution suggestions for a problem situation and explain the reasons and justify them.*" (T4), "*I always encourage different solutions.*" (T8), a total of six teachers emphasized the importance of encouraging different solutions in the development of students' mathematical reasoning skills.

Besides, there were some teachers that provided different suggestions and found it significant to encourage students to relate their knowledge to real-life, to justify, to express the problem situation with their own words, to create a problem, to create a classroom discussion in supporting students' mathematical reasoning skills. Some of their statements are as follows:

"I encourage them to express the process that they suggest in order to solve the problem and the problems they solve with their own words."(T4)

"I enable them to relate the problem situation to real life and to engage in a reasoning process in a meaningful way. I suggest them to develop their own solution methods." (T5)

"What is important is ideas and thoughts. Given a free environment where they can articulate their ideas, they will begin to reason, of course, within certain flexible rules. I think this is the most fundamental point. In the following process, they may be provided with the situations they may encounter in everyday life or with the preliminary knowledge to enable them to reach the essential knowledge; then they may be asked to

utilize their own environment and to develop their own knowledge through materials/models." (T15).

Table 5: Themes addressing the answers to the first part of the fourth question

Theme	Teacher
Marking the solution as correct	
Explaining the operation being performed	T1, T2, T3, T5, T6, T7, T8, T9, T10, T11, T15
Determining that reasoning has been correctly performed	T2, T4, T11,
Preferring a way to solve the problem commonly used by students	T14
Elaborating the reasoning process	T16
Marking the solution as incorrect	T12*

In the last question, the teachers were provided with the solutions of different students and asked to evaluate them with regard to students' mathematical reasoning. Figure 1 shows the solution for the first problem:

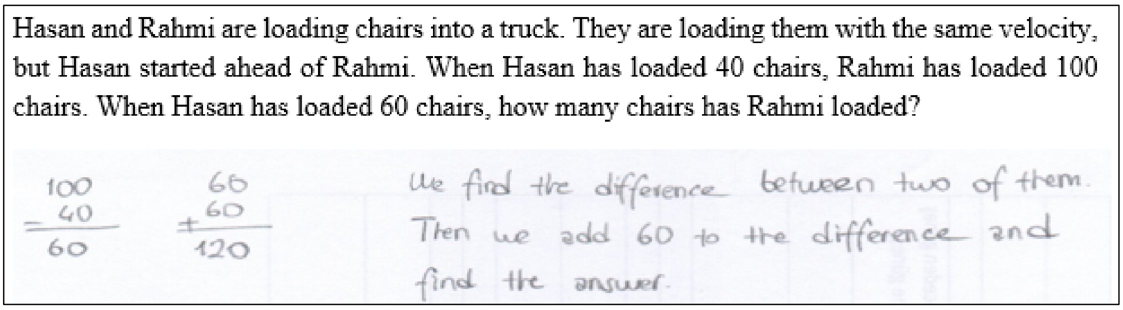

Figure 1: The first problem and the student solution

Table 5 presents the answers of the teachers, divided into two separate themes as marking the solution as correct and as incorrect. A large part of the teachers who marked the student solution as correct simply articulated the operation performed by the student, and failed to provide any explanation for the student's mathematical reasoning. Some of their statements are as follows:

"The student calculated the difference between them and found how many chairs Rahmi would load during this period." (T1).

"Here, the student reached the solution by calculating how far ahead Rahmi started in terms of the number of chairs. The student performed a correct reasoning." (T2)

"The difference between them is that Hasan started later; the student thought that they would continue to do the work in the same way." (T3).

In addition to articulating the operation being performed, three teachers expressed that the student performed the reasoning process in a correct way. Only one teacher elaborated the reasoning process in the student solution:

"The statement of "with the same velocity" is paramount in this question. As Hasan started ahead and they loaded at constant speed, there was a constant difference of 100-40=60 chairs between them. In other words, Rahmi would load 40 chairs more than Hasan would do under any circumstances. Therefore, when Hasan has loaded 60 chairs, Rahmi has loaded 60+60=120 chairs. The student achieved to establish the relationship between the two variables in the question. The student did not write down the equation as R=H+60, but conceived it in the mind." (T16).

Table 6: Themes addressing the answers to the second part of the fourth question

Theme	Teacher
Relating the solution to proportional reasoning	T1, T2, T3, T5, T7, T8, T9, T10, T12, T15, T16
Not engaging in mathematical reasoning/ Solving the problem by memorizing the rule	T4, T6
The solution is not sufficient to deduce something for mathematical reasoning	T11, T12, T13, T14

The teachers were provided with the solution of a different problem in the second part of the fourth question and asked to evaluate the solution with regard to the student's mathematical reasoning. Figure 2 shows the problem and the relevant solution:

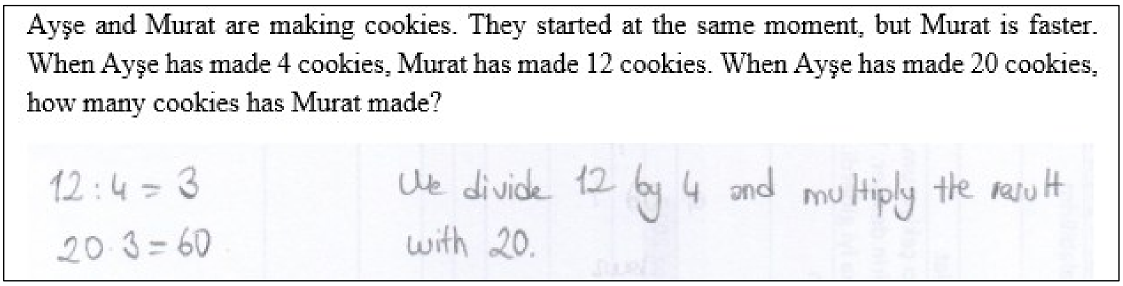
Ayşe and Murat are making cookies. They started at the same moment, but Murat is faster. When Ayşe has made 4 cookies, Murat has made 12 cookies. When Ayşe has made 20 cookies, how many cookies has Murat made?

Figure 2: The second problem and the student solution

The statements of the teachers for the solution were divided into three main themes. Accordingly, the teachers mostly related the solution to proportional reasoning. The statements of three teachers regarding proportional reasoning are as follows:

"The student found that Murat is three times faster than Ayşe. Then, the student calculated what it would be three times for Ayşe." (T9).

*"The student actually performed a proportional reasoning in the question. The student performed Murat/Ayşe and found a flat rate. Based on that, the student calculated how many cookies Murat has made when Ayşe has made 20 cookies by using the same rate and performing 20*3." (T15)*

"The student identified the relationship between the speeds of Ayşe and Murat by dividing 12 by 4. Thinking that such relationship is still valid in the following problem situation, the student multiplied 20 with 3." (T16)

Although there were some teachers relating the student solution to proportional reasoning, two teachers stated that the solution did not exhibit mathematical reasoning. These teachers believed that the student reached the solution by means of memorized information:

"This is particularly a problem situation that can be used in the subject of factors and multiplies in the 6th grade level.
Solution: Ayşe makes 4 cookies while Murat makes 12 cookies per unit time. Thus, Murat is three times faster than Ayşe. To determine how many times Murat is faster, one divides the number of cookies and then triples the number of the cookies made by Ayşe.
Although the student followed the necessary steps to reach the solution and provided the correct solution, the student's explanation is insufficient; therefore, I believe that the student solved the problem by using memorized information, rather than reasoning.
I think when the student cannot explain what he or she does and why, he or she does not offer a logical framework and performs random operations without reasoning, and provides the result in this way." (Ö4).

Unlike others, four teachers stated that it is not possible to evaluate the solution process in terms of reasoning. These teachers needed more detailed explanations for the evaluation of mathematical reasoning. Some of their statements are as follows:

"Yes, we have explanations for operations. But, I realize that there is no explanation on the content. This makes me ask the following question: "Did the student reason about it, or does he or she do the same thing in this type of question by memorizing the relevant information? A student correctly multiplies or divides numbers but has he or she reasoned about it enough?" (T11)

"When I first looked at the solution, I did not understand what the student performed, but he or she correctly understood the question and linked the information. The student divided Murat by Ayşe, that is, he or she realized that 3 Ayşe equals to 1 Murat. Then, he or she reached the result. Here, I should not overlook the possibility that the student may have failed to link the information and thought that "I should perform this operation for this type of question" based on his or her memorization. The student's explanation merely includes operations. The student should be asked to clarify what 3 refers to or what 60 refers to in this question." (T12)

5. Discussion

The aim of this study is to investigate the understanding of middle school mathematics teachers about mathematical reasoning. With this aim, the definitions of the teachers about mathematical reasoning are examined at first. According to the explanations, it is seen that the teachers define mathematical reasoning as "*making justifications in the process of solving a problem*". A few teachers, on the other hand, explain that mathematical reasoning is related to problem solving process or linking new and old information. It is appeared in the study of Clarke, Clarke & Sullivan (2012) that many of primary school teachers define mathematical reasoning as "making explanations". In addition to making explanations, some teachers link reasoning with "transferring" and "deducing". Similarly, in another study conducted with primary school students, it is found that reasoning is defined as "making explanations" and "justifying" (Loong et al., 2013). It can be said that these results are parallel to the definition of mathematical reasoning by Carpenter, Franke and Levi (2003), since they define the concept as "*to explain why a method works or a conjecture is true*". On the other hand, if it is taken into consideration that mathematical reasoning includes different abilities as proving, reasoning, evaluating, analyzing, generalization, inferring, deducing, adapting, transferring, contrasting (Clarke et al., 2012), problem solving, verifying, making assumptions, using logical arguments for verifying and associating different aspects of mathematics (Herbert et al., 2015), it can be said that the definitions of the teachers about mathematical reasoning is not comprehensive enough. Therefore, it is though that the teachers who define mathematical reasoning as merely "reasoning" or "explanation" will probably evaluate or support mathematical reasoning of students only through these skills. This situation will cause mathematical reasoning skills of students to develop incompletely.

After the teachers define mathematical reasoning, they are asked to give examples related to mathematical reasoning of their students. It is found that the answers are mostly about producing different/alternative solutions for a problem. According to this result, the teachers think that a student who can solve a problem in different ways has the ability of mathematical reasoning. However, while defining mathematical reasoning, only one teacher makes explanations related to alternative solution methods. That's why, it is surprising that the teachers who do not define mathematical reasoning in terms of producing alternative solution ways, point out that it is a sign of mathematical reasoning to produce alternative solutions. Moreover, Loong et al. (2013) want teachers to sample the mathematical reasoning process and the teachers present examples related to explanations and justifications as opposed to the results of this study. But this result of the study suggests that although the teachers are closely associated justifying with mathematical reasoning, they are focused on producing alternative solution ways more and offer situations related to this to improve their student's skills in their lessons. Indeed, when the teachers are asked to explain what they are doing to support mathematical reasoning of their students, they indicate that they encouraged their students to produce different solutions for a problem at most. It is possible to say that producing different solutions for a problem is not only important for the development of mathematical reasoning, but also for the development of creativity (Haylock, 1997; Mann, 2006; Silver, 1997). When the studies on creativity are examines, it is understood that presenting more than one proposal to a situation is related to "fluency", one of the components of creativity (Leikin & Lev, 2007; Torrance, 1988), but it is required that these proposals are original (Silver, 1997; Sriraman, 2008). Therefore, it is important that the teachers support their students to produce alternative solutions to a problem. But it is not possible to say that this is enough for the development mathematical reasoning. This result is followed by asking questions to students that give them the opportunity to explain and justify what they do as parallel to the definition of the teachers on mathematical reasoning. It is stated that the questions that teachers ask their students allows the students to evaluate their thinking process, make arrangements, clarify the process in various ways and communicate with others (Chin, 2007). Moreover, it is known that asking questions are efficient ways of deepening the conceptual knowledge of students (Sullivan, Clarke, Spandel & Wallbridge, 1992; Perry, Vanderstoep, & Yu, 1993). Thus, although a question posed to students seems to serve one reason only, it supports the development of more than one mathematical skills. That's why, the result of this study that the teachers asked their students various questions to support their mathematical reasoning is important.

The teachers were finally given sample solutions for two different problems and asked to evaluate those solutions in the context of mathematical reasoning. The problems are related to proportional reasoning and includes situations that will be an example of additive relation and multiplicative relation, respectively (Fernandez, Llinares & Valls, 2013). The teachers are given the problems related to the proportional reasoning since it is essential for many concepts in the teaching program as algebra, measurement, numbers (Van de Walle, Karp & Bay-Williams, 2013). For the first problem, nearly all of the teachers could not have an explanation and only explained the operations given in the solution. Contrary to the first problem, almost all of the teachers related the solution to proportional reasoning in the second problem. Some of the teachers, on the other hand, stated that the explanation given in the solution part of the problem is not enough to make inference about mathematical reasoning. Because they indicated that it is not possible to determine whether student solve the problem by memorizing or with mathematical reasoning by only looking at the solution part. Since multiplicative solutions are most frequent in problems about proportional reasoning, it is thought that there exists such a difference between the answer given by the teachers to the solutions of two problems. Moreover, the teachers may identify better the reasoning process in the second problem because of the fact that the teachers are more likely to encounter problems in the second type or they are more likely to direct such problems to students. It is also possible to say that after students learn about proportionality, students give answers to problems by memorizing without questioning whether the relations in the given problems include direct or inverse proportion. Since the fact that students cannot define the concepts (Duatepe & Akkuş-Çıkla; 2002) or explain their solutions routely (Doğan & Çetin, 2009; Yenilmez & Kavuncu, 2017) although they can solve the problems about proportionality suggest that the conceptual knowledge of students about the topic is deficient. Thus, it is possible to say that the teachers suggesting that they cannot evaluate the given solution in terms of mathematical reasoning because of lack of knowledge in the solution part offer more accurate explanation than the teachers relating the solution with proportional reasoning. Because those who are associated with proportional reasoning may think that the problem can be solved by proportional reasoning after examining the problem and make explanations in that way. Indeed, in the solution part, only the operations of the student to solve problem exist and further explanation about the solution is lack.

6. Conclusion

As a result, it is understood that the middle school mathematics teachers do not have a comprehensive and adequate knowledge and view about mathematical reasoning when it is considered how they define, exemplify and support mathematical reasoning. Because it is appeared that mathematical reasoning means only making explanations, justifications and producing different solutions for a problem according to them. It is important that the teachers should broaden their view about mathematical reasoning to support their students' reasoning better. For this aim, it is necessary for them to do not limit themselves with only justification or different solutions to a problem and add other abilities to their existing cognitive structures about mathematical reasoning as transferring from one context to another, generalizing, proving (ACARA, 2012).

These skills should not only be limited to the definition of mathematical reasoning, but should be transferred into actions that the teachers will support these skills of students. Since another important result of this study is that the teachers cannot exemplify mathematical reasoning although they define what it is. This result may be related to the fact that that the professional experience of mathematics teachers involved in the study is less than twelve years. Therefore, it is educatory to investigate how the experience affects the perception and knowledge of teachers about mathematical reasoning by carrying out a similar study with more experienced teachers. What's more, it is thought that teachers need training that will enhance both their theoretical and practical knowledge. With this aim, the teachers can be presented with sections from different classrooms and their knowledge of mathematical reasoning can be supported by providing that the teachers reveal situations about mathematical reasoning existed in these sections and discuss about these situations. Such positive changes in the theoretical and practical knowledge of the teachers will undoubtedly positively support their students' mathematical reasoning process. Thus, the reasoning skills of the students can be supported in a comprehensive way and the students can have more advanced reasoning skills for both mathematics a lesson and everyday life.

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