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STUDENTS' MATHEMATICS ATTITUDES AND METACOGNITIVE PROCESSES IN MATHEMATICAL PROBLEM SOLVING

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Abstract:
Mathematical problem solving is considered as one of the many endpoints in teaching Mathematics to students. This study looked into the performance in mathematics problem solving among fourth year students of Central Mindanao University Laboratory High School and their relationship with students' attitudes towards Mathematics. The attitudes measured were Attitude towards success in Math, Mother's mathematics attitude, Father's mathematics attitude, Motivation, Usefulness of Math, Teacher's mathematics attitude, Confidence in learning math, and mathematics anxiety. It also investigated the metacognitive processes of students considering varying levels of their mathematics anxiety. It used the responses of 127 students. Of the 127, (nine) 9 were selected according to their mathematics anxiety levels to determine and compare their metacognitive processes. Results showed that students consider Mathematics as useful and they have a positive attitude towards success in Mathematics. The students' fathers, mothers, and teachers also have positive attitudes towards their mathematics learning. However, overall, the students' performance in mathematics problem solving is considered poor. Among the eight (8) mathematics attitudes only confidence in learning Math and mathematics anxiety were correlated with performance in mathematics problem solving. Confidence in learning Math was positively correlated, while mathematics anxiety was negatively correlated with performance in mathematics problem solving. Students with high mathematics anxiety tend to confirm their solutions with their classmates. Students with moderate anxiety are test-anxious and those with low anxiety are distracted by external factors, but can readily shift their focus back to problem solving. The three (3) cases showed that students with low, moderate, and high mathematics anxiety employed mostly orientation and execution procedures. There were only few instances of verification and lesser instances of organization procedures. Self-questioning was the most observed metacognitive skill. Furthermore, students from the three (3) cases were unable to correctly answer two (2)

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problems, both of which are non-routine due to unfamiliarity and "experiential interference".

Keywords: mathematics attitudes, mathematics problem solving, metacognitive processes

1. Introduction

George Polya, the Father of Mathematics problem solving, once said "*the main point in mathematics teaching is to develop the tactics of problem solving"*. An essential aspect of Mathematics is problem solving. Problem solving helps students develop critical thinking skills and logical reasoning abilities that they may employ in real life situations. However, even the phrase "problem solving" elicits negative feelings from students.

According to the Trends in International Mathematics and Science Study (TIMSS), Filipino students could not work with items which deviated from the usual problem types given in textbooks. This implies that the "problem solving" aspect of mathematics learning does not meet expectations, or perhaps is not given emphasis. Such feedback simply means that in the classroom, students deal mainly with routine problems, with perhaps teachers or students, or both, more concerned of step-by-step procedure rather than meaningful learning. Unfortunately, an emphasis on providing students with multiple mathematics problem solving tools has not been embraced universally within the educational community (Douville & Pugalee, 2003). Kaur and Yap, as cited by Yeo (2004), said that even lower secondary students (grades 7 and 8) from Singapore, who consistently ranks first in the TIMSS, do not perform well in solving non-routine problems. Hence, mathematics problem solving in its real sense, is not only a national issue, but an international one as well.

The National Achievement Test (NAT) results for grades 4-6 and fourth year high school in 2006 showed a decline not only in Mathematics, but in Science and English as well (FactSheets, 2007). In the same report, schools in Region X ranked 9th overall for grade 6 and fourth year. For grade 6, Region X schools fell 16.7 points behind rank 1 CARAGA, and for fourth year, 16.9 points behind Eastern Visayas, which leads the entire country.

Despite earning laurels in various international mathematics competitions, the Philippines still lags behind over-all, consistently ranking in the bottom rungs in the TIMSS. In the local scene, a select few students of Central Mindanao University Laboratory High School recently came out on top in the Metrobank-MTAP-DepEd Math Challenge. This accomplishment, however, is but an isolated case, with students of the said school having negative attitudes towards mathematics problem solving, and Mathematics as a whole.

For over thirty years, researchers have been investigating students' attitudes and beliefs towards Mathematics. In 1976, Elizabeth Fennema and Julia Sherman developed the Fennema-Sherman Mathematics Attitudes Scale (FSMAS). It has been a widely used scale to measure the attitudes and beliefs of students across all levels. In using various adaptations of this questionnaire, researchers have found a strong relationship between positive attitudes and beliefs towards Mathematics and academic success in the subject (Kalder & Lesik, 2011).

Determining students' attitudes towards Mathematics would greatly help in improving the teaching-learning process in the classroom, anticipate the need for remedial teaching, and help students realize their full potential in mathematics learning.

This study investigated the students' attitudes towards Mathematics and their relationship with performance in mathematics problem solving. It also investigated the metacognitive processes of students considering varying levels of their mathematics anxiety.

2. Review of Related Literature and Studies

Mathematics attitudes are students' predisposition towards Mathematics. These are divided into these areas: Attitude towards success in Math, Mother's math attitude, Father's math attitude, Mathematics anxiety, Motivation, Usefulness of Math, Teacher's math attitude, and Confidence in learning Math. These were measured using the Fennema-Sherman Mathematics Attitudes Scale. Metacognitive processes are the thought processes employed by the students during mathematics problem solving. Mother's math attitude refers to the perceptions and feelings of the students' mothers towards their children's mathematics learning, as perceived by the respondents. Motivation is the students' intrinsic drive to do Mathematics. Performance in mathematics problem solving is the student's scores in Mathematics problem solving. Teacher's math attitude refers to the perceptions and feelings of the students' teachers towards mathematics as perceived by the respondents. These teachers include Mathematics teachers as well as those of the other subjects. Usefulness of Mathematics refers to the students' perception on how the mathematics concepts that they are currently learning can be used in daily life and their future careers.

2.1 On Mathematics Attitudes

Ma and Kishor, as cited by Nicoladou and Philippou (2003), defines mathematics attitudes as "*an aggregated measure of a liking or disliking of Mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at Mathematics, and a belief that Mathematics is useful or useless*". Attitudes towards Mathematics then, may be either positive or negative.

Attitude towards Mathematics plays a crucial role in the teaching and learning processes of Mathematics (Farooq & Shah, 2008). Aiken, as cited by Jazdzewski (2011), discussed studies that conclude correlations between attitude and achievement, in which these correlations are stronger in Mathematics than in other subjects. A positive attitude towards Mathematics greatly helps in motivating a student to do Mathematics. Their beliefs and attitudes have the potential to either facilitate or inhibit learning.

Without interest and personal effort in learning Mathematics, students can hardly perform well in the subject (Yara, 2009). In addition, Robson, as cited by Farooq & Shah (2008), said that *"having a positive attitude towards Mathematics generally means enjoying working with Mathematics and having confidence in one's own ability to do it but it does not mean that a student will display this positive attitude towards the whole area of Mathematics all the time."* There will be questions that are difficult to solve and will affect the students' perception of Mathematics.

In Mindanao, Prado (1995) identified a positive relationship between mathematics attitudes and performance. Students from high schools of state universities and colleges have more positive attitudes towards Mathematics than their counterparts in the national high schools. The former also have better mathematics performance than the latter. In the same study, the students with less positive attitudes towards Mathematics tend to view Mathematics as a more difficult subject than those with more positive attitudes.

Children with high mathematics ability disengage from school if their competence or autonomy needs are unfulfilled (Miserandino, 1996). These students need to be challenged just enough. If given problems too easy for them, or those too difficult that they cannot answer, they lose interest. Also, these students have a method of solving problems, employing various shortcuts. If the teacher emphasizes procedure such as step-by-step solutions, they become frustrated and lose interest as well.

Studies of Milne (1992) and Norton and Rennie (1998) show that, among others, students have strong attitudes towards success in Mathematics.

2.2 Mother's and Father's mathematics attitude

How parents deal with their children and Mathematics have a profound effect on them. Tocci and Engelhard (1991) found that, among others, parental support was a significant predictor of attitude towards Math. As parents show strong support to their children involving Math, the more positive the latter are in mathematics learning. Involvement of parents in learning was positively perceived by eighth grade pupils, according to Alomar (2006). Brekalo (2012) found that students with low math anxiety mirror their mothers' positive attitudes towards math. Molleker (2000), on the other hand, determined that students' attitude towards Math is not related to those of their mothers and fathers. In the research conducted by Widmeyer Research & Polling for Carnegie-IAS in 2009, parents appreciate the value of Math and science in developing critical thinking skills more than the students. Furthermore, the study found a direct connection between parents' and children's beliefs about "nature versus nurture". In particular, if parents believe that math skills can be developed, their children believe so too. However, comments from the focus group discussion of the said research suggested that perceived importance of Math does not imply that the subject is personally important.

Dickens and Cornell, as cited by Martin (2002), discovered mothers' and fathers' expectations of their gifted girls had significant impact in the girls' belief about their abilities. Expectations and attitudes that parents have concerning their children's performance impact achievement. Tocci and Engelhard, as cited by Martin (2002), found that parents are among the most influential groups in attitude formation. Farooq and Shah (2008) discovered that some female students have less confidence in Math because their parents perceive that it is useless for them to do well in it.

Although an integral part of mathematics attitudes, motivation is often less emphasized in research involving attitudes. Yee (2010) determined that even if students have positive attitudes towards Mathematics, they lack intrinsic motivation. Students were motivated to learn Math due to extrinsic rewards or punishments. However, intrinsic motivation correlates positively with achievement. Regardless of the relationship between attitudes and achievement, teachers and Mathematics educators tend to believe that children learn more effectively when they are interested in what they learn and they achieve better if they like what they learn (Ma & Kishor, 1997). Various studies involving motivation contradict each other. Results of Norton and Rennie (1998) show that students across grades 8 to 12 are uncertain in their motivation to learn Mathematics. Milne (1992), on the other hand, found that students were highly motivated to learn Mathematics.

The results of Armstrong (1985) showed that math performance relates to its perceived usefulness. That is, students who do well in Mathematics know that they need Mathematics in their chosen disciplines. Murtonen, as cited by Orongan (2007), asserts that students who underestimate the value of skills for studies and future work are not committed to study.

According to Gray (2012), students are committed in learning Math because they perceive Math as a useful way to get a good job, or be able to go abroad. Such are the students' response despite having a "dull" curriculum and stereotyped teaching methods. Milne (1992) found that students perceive Math to be very useful.

The teachers' method of mathematics teaching and his personality greatly accounted for the students' positive attitude towards Mathematics. As the person who controls the classroom atmosphere, the teacher's attitude towards Math is easily caught by the students. Midgely, Feldlaufer, and Eccles, as cited by Kalder & Lesik (2011), determined that a significant relationship exists between teacher efficacy and students' confidence and beliefs in their ability to do Mathematics. Specifically, students in the classes of teachers with a positive sense of efficacy in teaching were more likely to believe that they were performing better in Mathematics than students in the class of teachers with a lower sense of efficacy in teaching Mathematics.

Hacket (1985) found out that confidence in learning Mathematics predicts math-related performance. Pajares and Graham, as cited by Nicolaidou and Philippou (2003), found that boys and girls have the same confidence levels in their mathematics ability in the elementary level, but in the high school, boys are more confident than girls. Sometimes female students show less confidence in Mathematics partly due to their parents' perception that it is useless for them to do well in Mathematics, even if they find pleasure in doing Mathematics (Farooq & Shah, 2008). Furthermore, Stankov, Morony, and Ping (2011) found that confidence in Mathematics is a much better predictor of students' achievements than any other non-cognitive measure. He claims that if a student strongly endorses a wrong answer, then it is indicative that something went wrong in the teaching-learning process.

Mathematics anxiety is a component of the students' attitudes towards Mathematics. It is a state of distress and/or physiological arousal in reaction to stimuli including novel situations and the potential for undesirable outcomes (Brooks & Schweitzer, 2011). It is a discrete emotion characterized by high arousal, negative valence, uncertainty, and a low sense of control (Brooks, 2012). It includes fear, frustration, stress, tension, worry, apprehension, and nervousness. Anxiety is generally experienced in response to situations where the person is uncertain about an impending outcome of a personally relevant event, especially when the outcome is potentially harmful and feels unable to alter the course of events (Raghunathan & Pham, 1999). This definition of anxiety may very well describe mathematics test anxiety, where students' grades rely heavily on their performance in the given test.

Mathematics anxiety has been studied since the middle of the 20th century. It was first noticed in the late 1950s when undergraduate students were observed to react emotionally to arithmetic and mathematics. Dreger and Aiken, as cited by Scarpello (2005), labeled this as number anxiety. Schoenfeld, as cited by Yeo (2004), declared that mathematics anxiety is a component of mathematics attitudes. Mathematics anxious students do not like preparing for, waiting for, or taking mathematics tests (Scarpello, 2005).

Cemen, as cited by Yeo (2004), described three antecedents that interacted to produce an anxious reaction with its physiological manifestations such as perspiring and increased heartbeat. These antecedents are (1) environmental antecedents – negative mathematics experiences and lack of parental encouragement, (2) dispositional antecedents – negative attitudes and lack of confidence, and (3) situational antecedents – classroom factors and instructional format. Also, Buxton (1984) identified 3 emotional states fundamental to mathematics anxiety: (1) irritation, (2) bewilderment, and (3) frustration. Irritation occurs when common sense is not satisfied when doing mathematics. Students are bewildered when they cannot understand the problem or could not make connections. Frustration occurs when no amount of time spent, or number of strategies employed seem to provide a solution.

Richardson and Suinn, as cited by Yeo (2004), declared that "*mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematics problems in a wide variety of ordinary life and academic situations*". It is not only inside the classroom that students experience mathematics anxiety. Yeo (2004) added that such anxiety usually arises from lack of self-confidence. Students may perceive their inability to handle mathematics problems as "threatening", giving rise to feelings of self-doubt, fear of failure and loss of regard by others.

Fennema and Sherman, as cited by Yeo (2004), defined mathematics anxiety as "*feelings of anxiety, dread, nervousness, and associated bodily symptoms related to doing Mathematics*". These bodily symptoms are treated as physical pain by the brains of the students. Even anticipating a stressful event, such as a math test, can cause actual pain (Lyons & Beilock, 2012).

Fennema and Sherman divided anxiety into 4 categories: Thinking about Math, Taking Math Test, Learning Math Lesson, and Solving Math Problems. Students may have varied mathematics anxiety levels in each category. For example, a student may have low anxiety levels in Thinking about Math, such as budgeting weekly allowance, yet have high anxiety levels in Taking Math Test, especially surprise quizzes.

Mathematics anxiety often occurs as a result of instrumental learning because an understanding of the material is not obtained – only memorization (Kidd, 2003). High ability secondary students do experience mathematics anxiety. These students may perform well during lessons but fall short during examinations (Yeo, 2004). His results indicate that even high ability students show high anxiety levels in taking math tests and solving math problems.

2.3 On Mathematics Problem Solving

Problem solving is an integral part of Mathematics. It is not only a goal in mathematics learning, but a means of doing so. Problem solving means engaging in a task for which the solution is not known in advance, according to the National Council of Teachers in Mathematics (NCTM). Majority of mathematical concepts may be introduced by a problem familiar to the students. The use of problems where students can relate give meaningful learning experiences, and further reinforce retention. Also, according to Polya, solving a routine problem does not contribute to the mental development of the student. Non-routine problems provide an opportunity for students to develop higher-order thinking in the process of understanding, analysis, exploration, and application of mathematical concepts (Yeo, 2004).

The National Council of Teachers in Mathematics (NCTM) require that instructional programs from prekindergarten through grade 12 should enable students to: build new mathematical knowledge through problem solving; solve problems that arise in Mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; and monitor and reflect on the process of mathematics problem solving. Monitoring and reflection on mathematics problem solving are part of metacognition. Artz and Armour-Thomas, as cited by Ozsoy and Ataman (2009), claimed that poor metacognitive skills result in poor performance in mathematics problem solving.

Lester and Kehle, as cited by Yeo (2004), said that "*Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve the tension or ambiguity, such as lack of meaningful representations and supportive inferential moves, that prompted the original problem solving activity.*" Students then, should be continually trained to think analytically, look back to previously learned concepts, and connect prior knowledge to the task at hand.

George Polya outlined 4 stages of problem solving: (1) understand the problem, (2) devise a plan, (3) carry out the plan, and (4) review. Krulik and Rudnik, on the other hand, identifies 5 stages in problem solving: (1) read and think, (2) explore and plan, (3) select a strategy, (4) find an answer, and (5) reflect and extend. Krulik and Rudnik dub the problem solving process as heuristic process, or simply heuristics.

However, a heuristic is a guideline that may or may not yield success. Unlike an algorithm, it does not depend on knowledge of the problem to be successful. Heuristic is a method of thought that does not pertain to any specific problems or content (Carson, 2007). The heuristic differs from an algorithm because the latter applies to specific situations, for example, using the distance formula to identify missing coordinates of points given the distance, while the former requires critical thinking, being applied to unfamiliar situations.

Newman (1983) and Ransley (1979) devised a structure for heuristics, breaking them down into steps: (1) Reading, (2) Comprehension, (3) Strategy Know-How, (4) Transformation, (5) Process Skill, and (6) Solution.

One can read without understanding even a single word. Also, it is difficult to comprehend if one cannot read.

In Strategy Know-how, the student attempts to describe a method, a series of steps, on how he would go about solving the problem. He then goes to Transformation – where the problem is now expressed mathematically. After which, the student employs Process Skill, where he manipulates the mathematical symbols, or illustrations to reach a solution. The solution may either be correct or incorrect.

Students must go through each step to arrive at the Solution. Unfortunately, if a student encounters difficulties in a certain step, correct solutions are not arrived at. In the course of the problem solving, however, students may go back to previous steps, even if they have not yet arrived at a correct solution (Tan & Limjap, 2018). Yeo (2004) identified reasons why students do not arrive at a correct solution for each step, except in Reading. These were: (a) lack of comprehension of the problem posed, (b) lack of strategy knowledge, (c) inability to translate the problem into mathematical form, and (d) inability to use the correct mathematics. Incorrect solutions are obtained due to: (a) an inappropriate strategy used, (b) incorrect formulation of the mathematical form, (c) computational errors, (d) imperfect mathematical knowledge, and (e) misinterpretation of the problem.

In his study, Newman identified that students made the most error in process skills, such as errors in addition or multiplication. These are followed by comprehension, carelessness, reading, transformation, and encoding.

Wong identified 5 learning problems presented as heuristics which lead to errors in students' solutions: attach own meanings, incomplete of fuzzy thinking, mix up the rules, salient features, and a conformist attitude.

First, attach own meanings. Students confuse mathematical meanings of words with their own understanding. For example, in systems of linear equations, when told to "eliminate a variable", they may take it as "get rid of" or "disregard".

Second, incomplete or fuzzy thinking. As a result of boredom or distractions, students pay partial attention to a lesson, and use their own logic in reconstructing parts of explanations they miss, which may be faulty.

Third, Mix up the rules. With so many algebraic rules, students who have no understanding of the task at hand tend to use inappropriate rules.

Fourth, salient features. In an attempt to "make things easy for the students," some teachers give incomplete explanations by focusing on certain salient features that illustrate only some of the features of the concept. A trigonometry teacher may say that coordinates of angles that are multiples of will have an abscissa of. However, the angle does not have such abscissa.

Fifth, a conformist attitude. Since students are often trained to follow instructions meticulously, such as showing step-by-step solutions, students would be more concerned with form (solutions), rather than substance (understanding of the said solutions).

2.4 On Metacognition

Metacognition may simply be described as thinking about thinking. Metacognitive skills may be grouped into 2: metacognitive knowledge and metacognitive control. Metacognitive knowledge, according to Ozsoy (2009), is a person's mathematical processes and techniques students have and their ideas about the nature of Mathematics. Flavell, as cited by Ozsoy (2009), said it also describes a persons' cognitive skills and strategies, and the knowledge about what to do under which circumstances. Metacognitive control, on the other hand, is the person's ability to use his metacognitive knowledge. Artz and Armour-Thomas, as cited by Ozsoy and Ataman (2009), point out that the main reason underlying the failure of students in problem solving is that they cannot monitor their own mental processes during problem solving. This implies that poor problem solvers have poor metacognitive skills. Several researches, however, have implied that both thinking aloud and writing help in developing metacognitive skills.

Tan & Limjap (2018) identified the following metacognitive skills: self-instruction, self-questioning, self-monitoring, and self-evaluating. These skills are not linearly related, but are used throughout the problem solving process. In each step of Newman and Ransley's structure of heuristics, metacognitive processes are involved. However, as mentioned earlier, the steps are not always followed linearly. Students may go back to earlier steps in order to check or recheck their progress.

Garofalo and Lester, as cited by Pugalee (2004), identified a cognitive-metacognitive framework consisting of four phases of activities or behaviors involved in performing a mathematical task: orientation, organization, execution, and verification. Each phase includes several metacognitive behaviors related to problem solving. Orientation includes comprehension strategies, analysis of information and conditions, assessment of familiarity with a task, initial and subsequent representation, and assessment of problem difficulty and chance of success. Organization includes identification of goals and subgoals, global planning, and the local planning necessary to carry out global plans. Execution includes performance of local actions, monitoring progress of local and global plans, and trade-off decisions. Verification includes evaluation of decisions and results of executed plans. In his study, Pugalee (2004) identified that for the orientation, execution, and verification phases, there are significant differences between frequencies of demonstrated metacognitive behavior between students who thought aloud and those who wrote their thoughts. Think-aloud protocols are generally accepted as a means of assessing the mental processes of an individual (Pugalee, 2004). Ozsoy and Ataman (2009) cite: Asking appropriate questions activates the metacognitive skills of students (Hacker and Dunlosky, 2003). Questions asked by teachers, such as '*What about next?*', '*What do you think?*', '*Why do you think so?*' and '*How can you prove this?*' trigger the thinking and contribute to the development of metacognitive abilities (Yurdakul, 2004).

Rose (1989) view writing as "thinking aloud on paper". They provide information than input-output methods, yield rich data, and provide a means of observing important processes difficult to identify using other methods. It is posited as providing a level of reflection that promotes students' attending to their thinking about mathematical processes (Pugalee, 2004). In addition, writing creates and strengthens connections between acquired knowledge and new concepts being learned (Vygotsky, 1987). Meier and Rishel, as cited by Pugalee (2004), hold that writing, speaking and thinking are entwined such that narrative writing and speaking assist students in better understanding mathematical ideas.

3. Conceptual Framework

This study is based on the idea that mathematics attitudes affect students' performance in mathematics problem solving, and mathematics anxiety levels affect students' problem solving heuristics and their metacognitive processes. Several researches have established that there is a relationship between mathematics attitudes and mathematics performance. Burstein, as cited by Yara (2009), determined that there is a direct link between students' attitudes towards Mathematics and their performance.

In her research, Brooks (2012) showed evidence that mathematics anxiety and performance are related following the Yerkes-Dodson law; where very low or high levels of anxiety are harmful, but moderate levels of anxiety may improve motivation. This describes an inverse parabolic relationship between anxiety and performance. Orongan (2007), on the other hand, determined that mathematics aptitude or performance is negatively correlated with anxiety. Other studies do not show any relationship between anxiety levels and performance (Preston, 1986).

Goos, Galbraith and Reenshaw, as cited by Ozsoy and Ataman (2009), stated that a failure in metacognitive skills ensures the corresponding failure in mathematics problem solving. However, the converse need not be true – that failure in mathematics problem solving is due to a failure in metacognitive skills.

4. Methodology

This section presents the research design, locale of the study, sampling procedures, respondents, research instruments, and data analysis.

The study employed the mixed methods design. Both qualitative and quantitative methods were used. The quantitative aspect of this study employed descriptive-correlational methods in determining the relationship between mathematics attitudes and performance in mathematics problem solving. The qualitative aspect of this study employed descriptive-case study methods to look deeper into the metacognitive processes of students, considering varying levels of mathematics anxiety. Three (3) data sources were used: interviews, observations and students' outputs in mathematics problem solving tasks for data triangulation.

The study was conducted at the Laboratory High School of Central Mindanao University. The said school has an estimated population of 500 students, with 3 sections per year level. Since its primary mandate is to serve as laboratory school for students of the College of Education, it is under the Commission on Higher Education (CHED) and not the Department of Education (DepEd). However, the K to 12 curriculum has been integrated as mandated by law. The study employed total enumeration. Among the 136 fourth year high school students, 127 were selected as respondents; the other nine (9) were unable to answer either the Fennema-Sherman Mathematics Attitudes Scale, or the problem solving tasks. Among all students, 9 were selected based on characteristics they possess: problem solving capability (poor, moderate, good), and according to their mathematical anxiety levels (low, moderate, high) who participated in the semi-structured interview.

5. Results and Discussions

This section presents the results and findings of the study.

5.1 Students' Attitudes toward Mathematics

Table 1 shows the summary of the responses of students' attitudes towards

Table 1: Summary of Students Attitudes towards Mathematics

	Mean	Qualitative Description
Attitude towards success in Mathematics (ASM)	4.13	Agree
Mother's mathematics attitude (MMA)	3.90	Agree
Father's mathematics attitude (FMA)	3.94	Agree

Motivation (MOT)	3.17	Uncertain
Usefulness of Mathematics (UM)	4.16	Agree
Teacher's mathematics attitude (TMA)	3.55	Agree
Confidence in learning Mathematics (CLM)	3.28	Uncertain
Mathematics anxiety (MAX)	2.84	Uncertain
OVER-ALL MEAN Interpretation	3.62	Agree Positive

Legend:

Rating	Scale	Descriptive Rating	Interpretation
5	4.50 – 5.00	Strongly Agree	Highly positive
4	3.50 – 4.49	Agree	Positive
3	2.50 – 3.49	Uncertain	Neutral
2	1.50 – 2.49	Disagree	Negative
1	1.00 – 1.49	Strongly Disagree	Highly negative

It summarizes the students' attitudes towards Mathematics per subscale. Students view Mathematics as very useful in daily life (UM), and have a positive attitude towards success in Mathematics (ASM). Also, the study reveals that students perceive their fathers, mothers, and teachers (FMA, MMA, and TMA) as having positive attitudes towards the students' mathematics learning, with mean values of 3.94, 3.90, 3.55, respectively. This implies that both parents and teachers of students encourage them to do better in mathematics, and are interested in their mathematics learning, regardless of how they feel towards mathematics. Also, overall, students are not motivated to learn mathematics (MOT), are not confident in learning Mathematics (CLM), and are not anxious about mathematics (MAX) with mean values of 3.17, 3.28, and 2.84, respectively. These results imply that the students are divided in these aspects – some are motivated to learn, while some are not; some students are confident in learning Mathematics, some are not; and some students are anxious about Mathematics, while some are not.

The results echo those of Visser (1987), where she found that both mothers and fathers have positive attitudes towards Mathematics, with the latter have more positive Students' Performance in Mathematics

5.2 Mathematics Problem Solving

Table 2 below shows that overall, students do not perform well in mathematics problem solving with a mean percentage score of 60.27%. Basing on Department of Education (DepEd) classification, the fourth year high school students in general are poor in mathematics problem solving (74% and below). This coincides with the results of Silva, as cited by Asparin (2013) that students achieve low in mathematics skills despite efforts done to improve their problem solving achievements.

This also reflects the poor over-all performance of students in the TIMSS. In 1999 and 2003 TIMSS items released by the National Center for Education Statistics of the United States of America, the Philippines consistently scored below average in 'Investigating and solving problems' and 'Solving routine problems' cognitive domains across the 5 content domains: Fractions and number sense, Algebra, Measurement, Geometry, and Data representation, analysis and probability. This implies that even with a higher standard compared to other secondary schools, the students have not yet mastered mathematics problem solving skills. This reflects the sad reality of mathematics learning in our country, and this means that the entire education system has much to do to improve mathematics problem solving in the nation.

Results obtained by Bulosan (2008) however, showed that pupils have "very satisfactory" ratings in the National Achievement Test.

Table 2: Students' performance in mathematics problem solving (PMPS)

Performance Percentage	Frequency	Percentage	Qualitative Description	Problem solving level
90 and above	0	0	Advanced	Good
85 – 90	0	0	Proficient	Good
80 – 84	1	0.79	Approaching Proficient	Moderate
75 – 79	1	0.79	Developing	Moderate
74 and below	125	98.43	Beginning	Poor
Mean Performance	60.27		Beginning	Poor

Table 3 below shows the relationship between performance in mathematics problem solving and the mathematics attitudes subscales. Over-all, there is no significant correlational relationship between performance in mathematics problem solving (PMPS) and mathematics attitudes. This contradicts the results of Schreiber, as cited by Jazdzewski (2011), where it was found that students with poor attitudes about Mathematics will score lower on Mathematics tests. The results also do not agree with the results of Prado (1995), where she found a positive relationship between mathematics attitudes and performance.

However, among the subscales, only confidence in learning Mathematics (CLM) and mathematics anxiety (MAX) are significantly correlated with performance in mathematics problem solving (PMPS). CLM is positively correlated with performance in Mathematics problem solving (PMPS) with a Pearson r coefficient of 0.199, this means the more confident students are in learning Mathematics, the better they perform in mathematics problem solving. This coincides with the results of Stankov, Morony, and Ping (2011) where they claim that confidence is a much better predictor of students' achievements than any other non-cognitive measure. Similar results were obtained by Kadijevich (2008) where it was found that self-confidence in learning mathematics mostly relates with mathematics achievement. His results, however, show that students' perception of the usefulness of Mathematics also relates with mathematics achievement, which this study does not confirm.

Table 3: Pearson r correlations of students' performance in mathematics problem solving (PMPS) and mathematics attitudes subscales

Mathematics attitudes subscale	Pearson r	p-value
Attitude towards success in Mathematics (ASM)	- 0.100	0.264
Mother's math attitudes (MMA)	0.036	0.686
Father's math attitudes (FMA)	- 0.037	0.683
Motivation (MOT)	0.172	0.054
Usefulness of mathematics (UM)	0.100	0.264
Teacher's math attitudes (TMA)	0.018	0.837
Confidence in learning Math (CLM)	0.199*	0.025
Mathematics anxiety (MAX)	- 0.187*	0.035
OVER-ALL	0.06	0.502

Mathematics anxiety (MAX), however, is negatively correlated with PMPS, with Pearson r value of – 0.187. This means that the more anxious students are at Mathematics, the poorer their performance is at solving word problems. This result supports the study of Hembree (1990), where he claimed that mathematics anxiety is related to poor performance in mathematics achievement tests. Ashcraft and Ridley, as cited by Lyons and Beilock (2012), revealed that mathematics anxiety is clearly an impediment to math achievement. Fennema and Sherman, as cited by Norton and Rennie (1998) reported that high anxiety was associated with lower achievement and self-esteem. The study of Birenbaum and Eylath, as cited by Orongan (2007) coincide with this result, where they found that low high school mathematics grades are connected with students' experience of anxiety.

The lack of a significant relationship between performance in mathematics problem solving (PMPS) and mothers' and fathers' attitudes coincide with the findings of Kleanthous and Williams (2010), and contradict other researches cited in the latter (Fan & Chen, 2001; Aunola et al., 2003; Ma, 2001). Results show that motivation is not significantly related to performance in mathematics problem solving (PMPS). This result concurs that of Reynolds & Walberg, as cited by Norton & Rennie (1998) with the idea that motivation may have a significant indirect effect in math achievement. However, this is contrary to recent research that indicates that success in mathematics is a powerful influence on motivation (Middleton & Spanias, 1999), and that motivation, coupled with study are keys to success in Math (Gray, 2012).

5.3 The Students' Metacognitive Processes during Mathematics Problem Solving

Nine (9) respondents were selected according to the following criteria: mathematics anxiety levels – low, moderate, high; and mathematics problem solving – good, moderate, poor. Good problem solvers had z-scores greater than 1.27, moderate problem solvers had z-scores close to 0, and poor problem solvers had z-scores less than – 1.68. The students were then assigned to three (3) cases according to their mathematics anxiety levels.

An inherent difficulty in research involving "thinking aloud" procedures is the fact that people can think faster than they can talk. Several thought processes may happen so naturally to some that it may be almost impossible to verbalize them. This was the researcher's concern throughout the data analysis phase. This may be the reason why there is little organization procedures in mathematics problem solving. A few other factors may have further influenced the results of this study. Despite the resolve to remain objective, the researcher is a novice. And with little experience in this type of research, it is most likely that personal bias may have been involved in the data analysis. The Hawthorne effect may also have affected the students' problem solving process. The students may have consciously or unconsciously modified their behavior by simply knowing they are being observed. Nevertheless, the results of the observations, interviews and outputs in the mathematics problem solving tasks were triangulated and the results follow. The summary of the results and findings of the cases is presented in Table 4.

In the orientation phase, the students exhibited comprehension strategies of translation, skipping of questions and visualization of the problem. They, in general, translated problems to understand them better. Also, they skipped questions whenever they could no longer continue, and returned to the skipped items later on. They claimed they checked their answers, however, the behavior was observed in three students only - Kat, David and Joshua. Furthermore, the students assessed familiarity of the problems. Although some students simply remembered they encountered the problem before, a few were able to recall their solutions. All students imagined problems, although a few did not draw. There was little evidence of students assessing problem difficulty.

Self-questioning was the most observed metacognitive skill, which was exhibited in various phases of the problem solving process. Self-monitoring was also observed. Self-instruction and self-evaluation was less prevalent. These results echoed those of Montague, as cited by Oszoy and Ataman (2009), where it was found that self-instruction, self-questioning and self-monitoring were the three (3) most commonly used metacognitive skills in problem solving. According to Oszoy and Ataman, self-questioning enable students to systematically analyze the given information about the problem and manage appropriate cognitive skills.

It is noteworthy to mention that among the students; only Val claimed that he understands the problem once he gets the answer. The rest have responded with either instinct, or knowing the formula or procedure. These implied that they believed they understood the problem prior to execution. Val's response, on the other hand, implied a strong self-evaluation skill, which greatly helps in the Extending stage of Krulik and Rudnik's heuristics framework.

Most of the students avoided the hourglass problem because they have not encountered this before. They were not familiar with the problem. Even Val, who had low anxiety and high performance in mathematics problem solving (PMPS) did not answer this.

It is possible that another reason why this problem was not dealt with properly was because this problem asked for the procedure for finding the answer, and not a numeric answer the students are usually asked for. This is precisely the global planning behavior that was not exhibited by the students.

Another issue the researcher identified about mathematics problem solving was that students lacked relatable experience. Sometimes, students do not understand the problem because of the terms used, or they have not encountered the problem, or a similar one, in real life.

Table 4: Summary of Results and Findings of the Three (3) cases of Mathematics Level of Anxiety and Problem Solving Skills

Performance in Mathematics Problem Solving	**Anxiety Levels**		
	Low	**Moderate**	**High**
Good	Val - Loves math puzzles - Prefers formulas - Connects previously learned concepts to current problem - Sometimes solves without understanding the problem - Uses the same solution to check, and determine procedural errors	Jacob - When distracted, he can bring his focus back to solving - Does not use symbols to distinguish same-letter variables with different values - Starts solutions by estimation - Math anxiety comes from confusion in test items	Kat - Formulas are important - Prefers to use formulas or trial and error - Believes that if her answer is easily obtained, it is wrong - Relies on her classmates to confirm her answers - Easily distracted by music
Moderate	Carl - Translates questions to vernacular to understand better - Easily distracted, but can easily focus - Looks for formulas, also uses estimation and trial and error - Checks using the same solution	David - Studies hard before tests - Does not estimate answers - Looks for formulas - Solves without understanding the question	Maia - Finds math difficult, avoids it as much as possible - Dislikes multi-step problems - Relies on her classmates for solutions and answers - Prefers estimation and guess and test
Poor	Owen - Uses trial and error and estimation - Does not check, but if he does, he checks using a different solution - Enjoys math puzzles	Tania - Imaginative - Does not translate questions - Has a conformist attitude - Easily distracted without music - Distracting thoughts help in solving, keeping her "alive" - Prefers not to check her work - Instinct tells her if her answer is correct	Rose - Rarely translates questions - Relies on her classmates to confirm her answers - Distracting thoughts come when she becomes frustrated - Thinks she could do better in Math

This coincided with recent research that claims school mathematics rarely considers the social and cultural contexts of learning, with contexts differing significantly (Lowrie, 2004; Lowrie, & Clancy, 2003). In certain cases, their prior experience interferes with their understanding of the problem, which the researcher coins as "experiential interference". The varied responses of the interviewees below on the coins and the hourglass problem illustrate either experiential interference or lack of relatable experience.

Tania's explanation on the coins problem: "*Well, if we add the radius of the 2 identical coins, you're going to have twice as a single coin. If you merge them together and form a new coin, I think it's equal.*" The television documentary that she saw about coins being cast hindered her understanding of the coins problem. "*So I understood differently. I imagined the coins were melted, then were stacked one on top of the other. …the diameter remained unchanged, but its thickness increased. That's what stuck in my mind; if metal is melted, it is stacked*"; (experiential interference).

Rose's experience with hourglass timers that can be opened (experiential interference) led to this explanation: *"…no offense, but I think it's kinda dumb, because I think it's so easy to measure 6 minutes, just add sand from the 8 minute hourglass to the 5 minute hourglass to make the 5 minute hourglass 6 minutes, and to lessen the 8 minute hourglass, and it would be easier to measure the 6 minutes."*

Maia says "*It is difficult. It's annoying. Hourglass. What is that?*" (*lisod. Samok uy. Hourglass. Unsa man na?*)" (lack of relatable experience)

Kat said, *"Hourglass. I think that is sand. Sand is what is inside the hourglass, right? How do you count sand?"* (*Hourglass, mao ng sand sand. Sand sand ang hourglass di ba? How do you count sand?*) (lack of relatable experience).

David, who has moderate anxiety, said the following when asked about the hourglass problem: Nothing really. It's because in the previous exam, it was not given. The rest of the questions I have familiarized already. I answered those because it's those that I know how to solve. …I also do not know what that (hourglass timer) is sir. (*wala lang. kay katong niagi nga exam sir, wala man gud. Kaning uban, nafamiliarize na man gud. Mao ra akong giansweran kay mao ra akong nahibaw-an." "wala pud ko kaila ana sir.*); (lack of relatable experience).

Tania's solution is as follows: Initially, I imagined using technology, or by manual counting. If 5 minutes sir… (*una nako giimagine sir kay mag use ug technology, or by manually counting. Kung 5 minutes sir,*) you have to record 1 minute, and when you turn it around, you play that 1 minute. If not technology, then you count from 1 to 1 minute. Jacob explains: "*I think I have encountered this sir, but I left it alone. I do not solve sir.*" (*murag nakaencounter sir, pero ako ra ginapasagdaan. Dili ko gasolve sir.*) When asked if he imagined the hourglass being flipped, he replied "*No sir. the sand just falls, there's no flipping of the hourglass, so I get confused.*" (*wala sir.gakahulog lang siya, walay bali-bali. So malibog ko.*)

When asked, Val explains, "*This one sir, because I have not encountered this type of question. I really don't know (how to solve this);*" (lack of relatable experience).

Carl explains his solution: I thought of the hourglass as an old timer with sand inside. Its capacity will be emptied in 5 minutes. I should measure 6 minutes. Since 5 minutes is small, so you use it once, and after emptying it, estimate 20% and add it to 1 entire glass. That's 6 minutes. (*gihunahuna nako nga ang hour glass kay murag daan nga timer sa una nga naay balas sa sulod. Kana siya nga capacity, isa ka bali, mahurot in 5 minutes. Dapat i-measure kay 6 minutes. Gamay biya kayo so 5 minutes ra, so dapat kato gamiton to nimo ka-isa aron mahurot to tanan, estimate nimo nga ang mahulog nga sand kay igo-igo nga 20%. katong 20% nga kadaghanon sa sand, idungag to nimo sa tibuok usa ka glass. Mao dayon to ang 6 minutes.*)

Owen responds with a solution to a parallel problem which at first glance seems close to the correct one, but he was not able to execute. "*I had difficulty sir… I related it to 3 gallons and 6 gallons (problem), how do you get exactly 4 gallons. The first thing they did was when it reached 1 gallon, they filled up the 3 gallons, to get exactly 4 gallons. In the 8 minutes, if 1 minute passes, then after 5 minutes, we add the 1 minute. That is my solution, but I really had difficulty showing my solution.*" (*naglisud ko sir... Girelate nako sa 3 gallons ug 6 gallon s. unsaon nimo pagkuha ug exact nga 4 gallons. Ang pag-una, ang gibuhat nila nga pagabot ug 1 gallon, ipapuno ang 3 gallons. Para maexact ang 4 gallons. Sa 8 minutes kay pag 1 minute dayon, tapos macomplete ang 5 minutes, dri dayon sa 1 minute. Mao ra na akong gianswer, naglisud jud ko pakita sa solution.*

6. Conclusions and Recommendations

Based on the findings of the study, the following conclusions are arrived at:

The fourth year high school students of Central Mindanao University Laboratory High School consider Mathematics to be useful, and have a positive attitude towards success in Mathematics. Their fathers, mothers, and teachers have positive attitudes towards their mathematics learning as well. However, they have poor mathematics problem solving performance.

The students' confidence in learning Mathematics is positively correlated with their performance in mathematics problem solving. This means that the more confident the students are, the better they perform in problem solving tasks. Measures must be taken to increase their confidence.

Mathematics anxiety is negatively correlated with performance in mathematics problem solving. This means that the more anxious students are, the poorer they perform in problem solving. It is imperative then, that mathematics anxiety be addressed.

Students mostly involve orientation and execution procedures in problem solving. In addition, the students exhibited self-questioning skills more than the other metacognitive skills.

The students' inability to answer the coins and hourglass problems may be attributed to students' unfamiliarity of the problem, or "experiential interference".

The findings and conclusions of the study led to some recommendations which are believed to pave the way for further research, to improve the teaching and learning of Mathematics which would lead to improvement in mathematics problem solving performance.

Students must be encouraged further in learning Mathematics. They should be provided avenues and opportunities for independent and critical thinking in Mathematics problem solving. They need to be given strong support in mathematics learning. Parents and teachers must work as partners in the teaching-learning process. Both have significant influence in the development of the students' positive attitudes towards Mathematics. Teachers should be able to employ problem solving as a means to mathematics learning, and not simply teach it as a topic after conceptual understanding and procedural and computational knowledge has been attained.

Students need to be exposed to more problems in Mathematics, both routine and non-routine problems. With students assessing familiarity of problems, the more problems they encounter, and the more varied they are, the more likely they are to perform better. These problems, however, should be relatable to the students in the context which they understand to provide for meaningful learning. The levels of difficulty of the problems should match the levels of their abilities to reinforce their confidence. Difficulty level need to increase further, enough to challenge them, but not too difficult as to hinder their success and discourage them.

Students with mathematics anxiety, particularly those with moderate and high levels should be provided with strong support system in mathematics problem solving. Their anxieties must be acknowledged and addressed for them to perform better. Researches have determined one method to help students cope with mathematics anxiety which is to train them to treat their feelings of anxiety as excitement. Another method is letting students write their negative feelings about Mathematics before taking Mathematics tests, and reinforcing their self-worth. At home, parents should not pressure their children into doing very good in Mathematics, instead help them with difficult lessons using their children's learning styles.

Students should develop their metacognitive skills. Since they lack organization and verification procedures, teachers ought to provide avenues where these could be practiced. Self-instruction, self-monitoring, and self-evaluation should be encouraged in the classroom since these are keys to better problem solving.

Further studies involving students' metacognition and mathematics problem solving are recommended; for instance, determining how students use metacognitive processes in identifying errors in their solutions. Identifying students who have moderate and high levels of mathematics anxiety at an earlier age, as well as those with "experiential interference" and "suspension of sense-making" would help teachers, educators and administrators in providing remedial services to help them cope with the challenges they face in Mathematics.

Respondents and interviewees should be thoroughly trained to think aloud prior to observing their problem solving procedure. This will ensure that their thought processes would be more verbalized. A combination of thinking aloud and writing, also known as "thinking aloud in paper" would better capture cognitive and metacognitive processes.

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