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# INVESTIGATING THE SPATIAL REASONING SKILLS OF STUDENTS IN THE CONTEXT OF MATHEMATICAL THINKING PROFILES

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**Abstract:**
The aim of this study was to examine whether there is a relationship between the spatial reasoning skills and mathematical thinking profiles of elementary school students. The quantitative method of descriptive survey model was used in the study. The sample of the study comprised 103 fourth grade elementary school students in Turkey. The Mathematical Process Test and Spatial Reasoning Test, which are valid and reliable, were used as data collection tools. According to the answers given by the students to the Mathematical Process test, it was decided based on the upper and lower limits of each group determined beforehand that the students had a thinking profile. Scores were also collected from the Spatial Reasoning Test. MANOVA test was performed to examine whether the spatial reasoning skills differed from the mathematical thinking profiles of the students. The results revealed that students with a visual thinking profile were more successful in both the Mathematical Process Test and the Spatial Reasoning Test.

**Keywords:** mathematical thinking profiles, spatial reasoning, elementary school students

## 1. Introduction

In human cognition, it is suggested that there are five systems for representing geometry (space), number, objects, actions and social relations, and each system guides and shapes the mental lives of adults (Kinzler & Spelke, 2007; Spelke & Kinzler, 2007). It is generally thought that some cognitive abilities, including spatial and verbal ability, are related to the preference of individuals for visual or analytic processing during

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problem-solving. Thus, correlational studies investigating the role of visual and analytic processing in human cognition have gained importance (Kozhevnikov, Hegarty & Mayer, 2002). Since the late 1970s, the distinction of visual/verbal cognitive profiles in educational research has become popular in the field of mathematics education.

Leading studies by Krutetskii (1976) and Presmeg (1986a) have examined students' preferences for visual and non-visual (analytic) methods-styles-processes in the problem-solving phase. Krutestki (1976) stated that students with visual-pictorial or verbal-logical cognitive processes of the mind preferred different mathematical thinking styles (visual, harmonic or analytic thinkers) to succeed in mathematical activities. Visual thinkers prefer to use visual-pictorial elements (figure, diagram, table, graphic, picture, etc.). Analytical thinkers prefer to use visual-pictorial processes. They do not need to use visual representations while using only mathematical operations. Harmonic thinkers use both visual-pictorial and verbal-logical processes. However, which cognitive process has come forward indicates variety from individual to individual. Additionally, researchers (Presmeg, 1986a, 1986b; Suwarsono, 1982) have suggested that students may be able to improve their preference for solving mathematical problems with visual or non-visual processing.

As seen in Fig. 1, when analytic learners encounter a problem, this makes their reasoning a strength of verbal-logical processes instead of visual-pictorial processes. That is, analytical thinkers do not benefit from visual representations in problem-solving. When visualizers encounter a problem, they make their reasoning a strength of visual-pictorial processes instead of verbal-logical processes. Harmonic learners are able to use both processes equally. So, one of the objectives of this study to reach a distinction between the level or the type of mathematical abilities, determined by the strength of, and preference for, visual-pictorial or verbal-logical components of thinking.

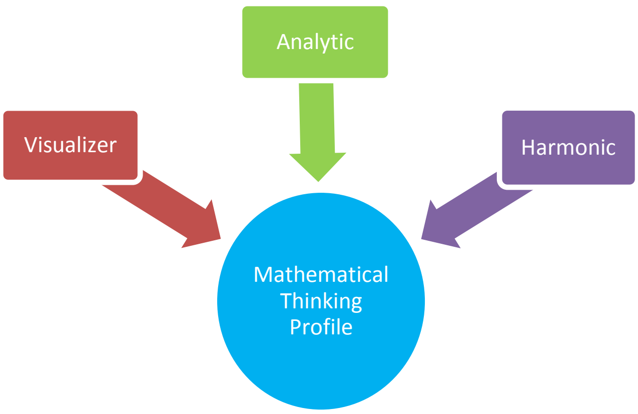



**Figure 1:** Mathematical Thinking Profiles

At the same time, it is believed that this study is important for students who prefer verbal-logical processes or visual-pictorial processes to examine the relationship between problem-solving performance and spatial reasoning abilities. When past studies are examined, Moses (1977), Suwarsono (1982), Hegarty and Kozhenikov (1999) and Samuels (2010) found that the mathematical thinking processes of students did not affect their problem-solving performances, spatial skills, calculus and nonverbal reasoning skills when they studied primary and middle school students. However, some researchers such as van Garderen and Montague (2003), Bremigan (2005) and Van Garderen (2006) revealed that students' preference for mathematical thinking processes affected their problem-solving performances, spatial abilities and calculus skills.

Additionally, the relationship between spatial reasoning and problem-solving performance was examined and a significant relationship was found (Lowrie, Logan & Ramful, 2016). Moreover, some studies have shown positive correlations between visual representation, which is an element of spatial thinking, and problem-solving. Spatial thinking is positively related to mathematical thinking (Battista, 1990); that is, students who perform better on spatial tests perform well on mathematics tests (Rasmussen & Bisanz, 2005).

According to MoNE [Turkish Ministry of National Education] (2015), in the problem-solving process, elementary school students are expected to be able to reach the level where they can express their own thoughts and reasoning easily, see the inadequacies or gaps in others' mathematical reasoning, use mathematical terminology and language to explain and share mathematical ideas in logical way, and express concepts with different forms of representation. For students to better understand the problem, to enrich the methods and strategies they use in problem-solving, it is necessary to first determine which mathematical thinking processes they prefer. However, when the literature was examined, no study was to determine the mathematical thinking profiles of elementary school students and compared this to their spatial reasoning abilities.

It is thought that, in the 3rd and 4th grades where there is a transition period from concrete to abstract, there are significant differences in students' thinking styles (Pilten, 2008). In these grades, students usually represent and solve tasks arithmetically, but most of them have difficulties with mathematical operations. Therefore, it is important to examine the influence of visual representation on the activities of students in this period (Montenegro, Costa and Lopes, 2018). Students on the 4th-grade elementary school level were selected in this study because they encountered complex problems and began to use their own methods. The purpose of the study was to determine whether there is a relationship between elementary school students' spatial reasoning abilities, problem-solving performances and mathematical thinking profiles.

In this study, the following research questions were examined:

- Are students' mathematical thinking profiles differentiated by their sex?
- Are there differences in students' spatial reasoning abilities and problem-solving performance that can be based on their mathematical thinking profile?
- Do students' spatial reasoning abilities and their problem-solving performance change according to their sex?

## 2. Methodology

The study chose the quantitative research design of correlation analysis. A correlational model explores and observes relationships among variables. Observation is quite achievable while collecting data (Karasar, 1994).

### 2.1 Participants

The participants were 103 fourth-grade elementary school students. The study took place at a public school in southern Turkey. The participants came from families with moderate socio-economic status, from a culturally and ethnically diverse. Table 1 shows the descriptive characteristics of the students who participated in the research.

**Table 1:** Student Characteristics

| Sex | f | % |
|---|---|---|
| Male | 43 | 41.7 |
| Female | 60 | 58.3 |
| Total | 103 | 100.0 |

As seen in Table 1, among the 103 students, 41.7% were male and 58.3% were female.

### 2.2 Procedure

In this study, we administered the Mathematical Processing Test and the Spatial Reasoning Test. All instruments were applied to students in groups of 25 to 35 students in their classrooms. The implementation of the tests was carried out by the researchers. The practices lasted for one class. The tests were administered to understand spatial reasoning ability and preferred method of processing or mathematical thinking profiles. Table 2 shows the measurement tests that were used to examine the abilities.

**Table 2:** Abilities and tests

| Abilities | Data Collection |
|---|---|
| Mathematical Thinking Profiles<br>Problem-solving Performance | Mathematical Processing Test |
| Spatial Reasoning | Spatial Reasoning Test |

### 2.3 Data Collection

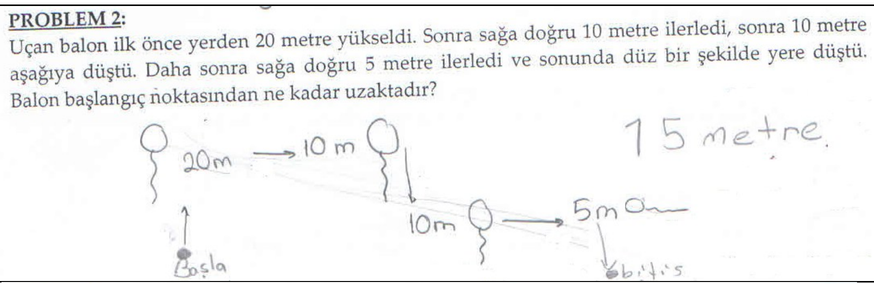
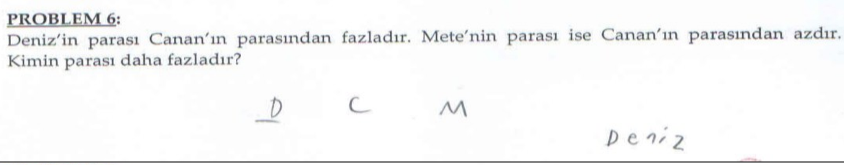
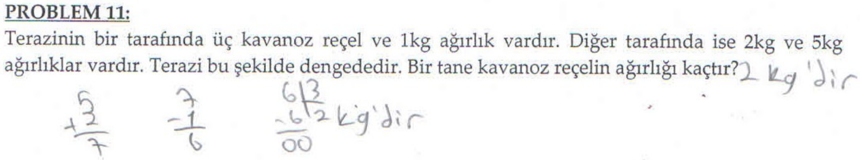
In this study, mathematical thinking profiles and spatial reasoning ability were measured by The Mathematical Processing Test and the Spatial Reasoning Test respectively.

#### 2.3.1 Mathematical Processing Test (MPT)

The Mathematical Processing Test which was developed by Suwarsono (1982) and translated into Turkish by Hacıömeroğlu and Hacıömeroğlu (2013) was used in this study. MPT classifies students according to their choices of visual or analytic thinking while solving problems. Some techniques were considered to insure the validity and reliability of the test. For example, to insure validity, the Lawshe content validity ratio was computed. For this analysis, opinions were collected from two form teachers and three specialists. Thus, 15 items were selected from among 30 items. Moreover, pilot tests of MPT were conducted with fourth-grade students. 0 points or 1 point values were assigned for each question, depending on whether or not the questions could be answered. As a result of the item analysis, the questions below the item difficulty index value of .30 and below the item discrimination index value of .20 were removed. 11 items remained. Later, a Kuder-Richardson Formula 20 (K-R20) value of .78 was obtained, indicating that the test was reliable.

To represent the students' mathematical thinking profiles, 0 point was posted for each analytic solution and 2 points were posted for each visual solution, regardless of whether the answer was correct or incorrect. The highest score that could be obtained from this was 22 points. Therefore, a high score shows that the student has a visual preference, and a low score shows that the student has an analytical preference.
The students were also classified based on whether their answers were correct or incorrect. The score of 0 was assigned for each incorrect answer, and 1 point was assigned for each correct answer. The highest possible score in the test was 11 points. Table 3 shows the different answers given by the students in the fourth item and how these answers were classified.

**Table 3:** Item and the students' preference for visual or analytic processing

| **Mathematical Thinking Profiles** | **Item and Solutions** |
|---|---|
| Visual thinking profiles (Mert, 4B) | <br>**PROBLEM 2:**<br>Uçan balon ilk önce yerden 20 metre yükseldi. Sonra sağa doğru 10 metre ilerledi, sonra 10 metre aşağıya düştü. Daha sonra sağa doğru 5 metre ilerledi ve sonunda düz bir şekilde yere düştü. Balon başlangıç noktasından ne kadar uzaktadır?<br>15 metre. |
| Visual thinking profiles (Seçkin, 4B) | <br>**PROBLEM 6:**<br>Deniz'in parası Canan'ın parasından fazladır. Mete'nin parası ise Canan'ın parasından azdır. Kimin parası daha fazladır?<br>D C M<br>Deniz |
| Analytic thinking profiles (Damla, 4B) | <br>**PROBLEM 11:**<br>Terazinin bir tarafında üç kavanoz reçel ve 1kg ağırlık vardır. Diğer tarafında ise 2kg ve 5kg ağırlıklar vardır. Terazi bu şekilde dengededir. Bir tane kavanoz reçelin ağırlığı kaçtır? 2 kg 'dir |

As seen in Table 3, Mert was coded as a visual thinker (2 points) because he used visual representations in his solution. The item was coded as a visual thinker (2 points) also because he solved the question by imagining it in his mind. Damla was coded as an analytical thinker (0 point) because she solved the question by performing mathematical operations.

### 2.3.2 Spatial Reasoning Test (SRT)

The Spatial Reasoning Test, which developed by Ramful, Lowrie and Logan (2017), was used for measuring the students' spatial reasoning. The test consists of 3 categories: spatial visualization (10 questions), mental rotation (10 questions) and spatial orientation (10 questions). The test was prepared for students on a primary school level. Initially, the test consisting of 30 questions was reduced to a 20-question test as a result of item analysis (spatial visualization (6 questions), mental rotation (7questions) and spatial orientation (7 questions). Later, a Kuder-Richardson Formula 20 (K-R20) value of .71 was obtained, indicating that the test was reliable.

The students were also classified based on whether their answers were correct or incorrect. A score of 0 was assigned for each incorrect answer, and a score of 1 was assigned for each correct answer. The highest score that could be obtained was 20 points.

### 2.4 Data Analysis

As a result of classification according to the scores obtained from MPT, it was determined that the ones with MPT scores between 0-7 had analytical, those with scores between 8-14 had harmonic, and those with scores between 15-22 had visual thinking profiles. The distribution of the students based on their mathematical thinking profiles is given in Table 4 below.

**Table 4:** Distribution of the students based on their mathematical thinking profiles

| Mathematical Thinking Profiles | f | % |
|---|---|---|
| Analytic | 30 | 29.1 |
| Harmonic | 54 | 52.4 |
| Visual | 19 | 18.4 |
| Total | 103 | 100.0 |

As seen in Table 4, approximately 29% of the sample were analytic, 52% were harmonic and 18% were visual thinkers. Then, the scores from SRT were calculated. All data were transferred to the SPSS 24 program.

## 3. Results

**Question 1:** Are students' mathematical thinking profiles differentiated by their sex?

In order to investigate the relationship between sex and mathematical thinking profiles, the students were divided into subgroups according to their thinking profiles. Pearson Chi-square analysis was then conducted to examine whether the students' mathematical thinking profiles changed based on their sex. Table 5 shows the results of the Pearson's Chi-squared test that was used to explore the proportion of analytic, harmonic, and visual processing for male and female students.

**Table 5:** Results of the Pearson's Chi-squared Test

| | | | Sex | |
|---|---|---|---|---|
| | | | **Male** | **Female** |
| **Mathematical Thinking Profiles** | Analytic | N | 16 | 14 |
| | | % | 15.5 | 13.5 |
| | Harmonic | N | 18 | 36 |
| | | % | 17.4 | 34.9 |
| | Visual | N | 9 | 10 |
| | | % | 8.7 | 9.7 |

*$X^2$=3.47 df=2 p=.176*

As seen in Table 5, the chi-squared test provided a "p" value of 0.17, indicating no significant relationship between sex and preference for visual or analytical processing ($X^2$ (df=2, N=103) =3.47, p>.05). In other words, the students' mathematical thinking profiles were independent of their sex.

**Question 2:** Are there differences in students' spatial reasoning abilities and problem-solving performance that can be based on their mathematical thinking profile?

A multivariate analysis of variance (MANOVA) was conducted to determine whether the students' spatial reasoning abilities and problem-solving performances produced a significant difference in terms of their mathematical thinking profiles. The considered factors were three levels of preference. The covariance matrices were homogeneous (p> .05). Then, the results of the Wilk's Lambda Test showed that linear combinations of the problem-solving and spatial reasoning ability scores of the students were significantly different in terms of their three levels of preference (Wilk's Λ= .856, F (4, 198)=4.010, p=.004). The results of the MANOVA test are shown in table 6.

**Table 6:** MANOVA Test results according to the students' mathematical thinking profiles

| **Dependent Variables** | **Mathematical Thinking Profiles** | **n** | **X** | **S** | **df** | **F** | **p** |
|---|---|---|---|---|---|---|---|
| Problem-Solving Performance | Analytic | 30 | 6.10 | 2.26 | 2-100 | 6.378 | .002 |
| | Harmonic | 54 | 7.72 | 2.11 | | | |
| | Visual | 19 | 7.94 | 2.24 | | | |
| Spatial Reasoning Ability | Analytic | 30 | 13.70 | 2.71 | 2-100 | 3.274 | .042 |
| | Harmonic | 54 | 14.44 | 2.61 | | | |
| | Visual | 19 | 15.68 | 2.62 | | | |

As seen in Table 6, there was a statically significant effect of the mathematical thinking profiles. It was seen that the students' problem-solving performance scores showed a significant difference in terms of their mathematical thinking profiles (F (2, 100) =6.378, p<.05). That is, on the problem-solving performance scores, the mathematical thinking profiles were significantly different from each other. This occurred in the favor of students with a visual thinking profile (X=7.94). It was understood that spatial reasoning ability scores also showed a significant difference in terms of mathematical thinking profiles (F (2, 100) =3.274, p<.05). In other words, on the spatial reasoning ability scores, the mathematical thinking profiles were significantly different from each other. This situation occurred thanks to the students with a visual thinking profile (X=15.68).

**Question 3:** Do students' spatial reasoning abilities and their problem-solving performance change according to their sex?

A multivariate analysis of variance (MANOVA) was conducted to determine whether the students' spatial reasoning abilities and problem-solving performances produced a significantly difference in terms of sex. The factors considered were students' sex. Covariance matrices were homogeneous (p> .05). Then, Wilk's Lambda Test results showed that linear combinations of problem-solving and spatial reasoning ability scores of students were not significantly different in terms of their sex (Wilk's Λ= .989, F (2,100) =.564, p=.571). Table 7 shows MANOVA test results of students' problem-solving and spatial reasoning achievement scores in terms of their sexes.

**Table 7:** MANOVA Test results according to students' sex

| Dependent Variables | Sex | N | X | S | df | F | p |
|---|---|---|---|---|---|---|---|
| Problem-Solving Performance | Male | 43 | 7.44 | 2.32 | 1-101 | .316 | .575 |
| | Female | 60 | 7.18 | 2.28 | | | |
| Spatial Reasoning Ability | Male | 43 | 14.79 | 2.65 | 1-101 | 1.131 | .290 |
| | Female | 60 | 14.12 | 2.73 | | | |

As seen in Table 7, there were no significant differences within sexes. Once the results for the dependent variables were considered separately in follow-up MANOVA analyses, it was determined that the students' problem-solving performance scores showed no significant differences in terms of sex (F(1,101)=.316, p>.05). Spatial reasoning ability scores also showed no significant differences in terms of sex (F (1,101) =1.131, p>.05). That is, there were no significant differences between male and female students in their spatial reasoning ability and performance in problem-solving.

## 4. Discussion

In this study, it is important to note that the students' preferences for analytic (or verbal–logical) or visual (visual–pictorial) thinking was independent of their sex. Generally, these findings support the conclusion of Hacıömeroğlu, Chicken and Dixon (2013), who determined that discrepancy between spatial and analytic reasoning abilities revealed that preference and discrepancy were independent of sex.

Another result obtained from the study was that preference for visual or analytic processing appeared to be significantly correlated with spatial reasoning ability and problem-solving performance. The most noteworthy were the results that preference for visual processing was related to problem-solving performance and spatial reasoning ability. The results revealed that the significant correlation between ability and preference shows that the students' problem-solving and spatial reasoning abilities predicted their preference for visual or analytic processing. Moreover, it was seen that having a visual thinking profile had an important contribution to the students' spatial reasoning abilities and problem-solving performance. In parallel to this result, Bremigan (2005) reported that there is a significant relationship between visualization strategies (or visual processing preferences) and mathematics performance in the results of their study. For instance, visual learning is important for pupils to learn calculus, because many concepts are easier to understand when they are explained by visual representations (graphs, figures, etc.). Moreover, Hacıömeroğlu, Chicken and Dixon (2013) stated that preference of students or analytical thinkers might not work well for them due to their incomplete understanding of calculus and lack of usage of visual representations.

Hacıömeroğlu (2015) reported that visual processing students were better in terms of spatial visualization and verbal-logical reasoning performance. Garderen and Montague (2003) also showed that talented students had a visual thinking profile. Moses (1977) and Suwarsono (1982) reached the conclusion that students' problem-solving performance is not significantly related to their preference for visual or analytic processing, while Hacıömeroğlu and Chicken (2012) supported our study's results by reaching the conclusion that a strength preference for visual processing is related to higher mathematical performance.

It was seen that there were no significant differences within sexes on spatial reasoning abilities and problem-solving performance. In parallel this, Verdine Golinkof, Hirsh-Pasek, Newcombe, Filipowicz and Chang (2014), Olkun and Altun (2003), Olkun (2003), Lowrie, Logan and Ajay (2016) and Çilingir-Altıner (2018) found that the sex variable did not make a significant difference in spatial abilities. In fact, it turns out that there was no significant effect of sex on spatial and mathematical ability until puberty (McGee, 1979), so these differences are not observed in the age group. However, the meta-analysis studies by Maeda and Yoon (2013) and Reilly and Naumann's (2013) showed that male students performed better than female students in spatial ability tests. For this reason, according to McGee's hypothesis, the case may be that the participants in the collected articles were on undergraduate and high school levels.

## 5. Conclusion

Presmeg (1986a, 1986b) and Suwarsono (1982) suggested that students develop their own preferences for solving mathematical problems with visual or non-visual (verbal) approaches. Therefore, considering the results of this study, it is possible to make a positive contribution to the spatial reasoning ability and problem-solving performance of students by providing them with preference for visual processing.

This study examined some variables like sex and mathematical thinking profiles. Moreover, these variables are not enough to explain students' spatial reasoning ability and mathematical problem-solving performance. For example, variable such as socioeconomic status, math anxiety, attitudes, confidence, experiences and achievement in math may be important in determining factors of preference for analytic and visual processing.

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