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EMPIRICAL APPROACHES TO PROBABILITY PROBLEMS: AN ACTION RESEARCH

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Abstract:
The purpose of this study is to explain the modelling and modelling process problems which require probability thinking ability by using simulations in an instructional way. For that purpose, action research was performed with 46 prospective mathematics teachers who were university students in Turkey. The simulation based activities executed in this study are based on Look, Think and Act cycle in Stringer's action research. Two open ended popular probability problems (Cereal Box and Birthday Problems) were asked to the prospective mathematics teachers. The responses about each problem are analyzed and presented in tables with percentage and frequency. When the data were investigated, it was seen that the prospective teachers gave the wrong answers or did not answer the questions. Since the aim of the study is to adopt experimental rather than theoretical approaches in the solutions of the problems, the focus was on creating simulation models for problems, doing experiments, visualisation of the results and calculation processes. Then, experimental and theoretical solutions were compared. Thus, the relations and models which help to understand the theoretical solutions of probability problems by using experimental data were proposed. Moreover, the study also sheds light on the questions of how to use the TinkerPlots in order to increase the comprehension of students and how to integrate the technology into probability teaching. In this case, it is thought that the study will be beneficial to researchers, teachers and students alike.

Keywords: teaching probability, simulation, experimental probability, theoretical probability, action research

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1. Introduction

Recently, statistics and probability have been placed in the curriculum of many countries more than ever [1; 2]. However, statistics and probability are some of the difficult subjects for the students [3; 2]. Students may face many unpredicted situations while solving probability problems to the intuitive nature of probability concepts. This case may be challenging for the students as well as the teachers who may be forced while analysing the problems during the lecture.

There is a need for alternative methods since the required visualisation for running experimental probability problems in traditional environments are not provided. The potential of the dynamic statistical software can be used while creating environments for teachers and students [4]. With the rise of importance of probability subjects in curriculums and of the access to technology in schools, the teachers should be promoted to use experiments with concrete materials or computer simulations in order to provide students with experience [5]. Students need experimental studies in order to understand the theoretical basis of probability. These experimental studies are the opportunities for the students because they develop their probability intuitions, helping them to create strong probability understanding and to motivate them [6]. The studies on probability teaching offered the use of computers as a way of understanding abstract or difficult concepts and developing the abilities of students [7; 8; 9; 10]. Batanero and Diaz emphasize the use of simulations which may help students to solve simple probability problems which cannot be done by the students alone [11]. Simulations are the most useful strategies for focusing on the concepts and reducing technical calculations [6]. Simulations enhance the understanding of statistical ideas [12] and support the learning process of students while studying on probability experiments [13].

The modelling of mathematical situations becomes easier through some software after the development of technology. Thus, the students motivated by the experiments can research and discover theoretical solutions. Moreover, they can be satisfied with the theoretical solution model by comparing the theoretical and experimental solutions of a problem. Some researchers stated that modelling and mathematical thinking can be applied to real life; that is, learning mathematical concepts enhances probability intuitions and contributes to the understanding of mathematical concepts [14; 10; 13]. On the other hand, it is necessary to establish a relation between experimental and theoretical probability. Namely, modelling should provide the advantages of understanding how mathematical concepts affect the observed situations.

Simulation based approach requires special effort for learning for both teachers and students alike since it does not only require statistical perspective but also modelling abilities. However, many teachers have less experience about doing probability experiments and using simulation tools; thus, they may have difficulties in applying experimental approach. Therefore, different approaches for probability instruction are required. This study aimed at designing the learning environment for solving probability problems experimentally by using simulations. Learning environment also presents the necessary instruction tools for teaching probability and possible necessary samples. Moreover, this study can be considered as a guide to teachers in their attempt to integrate technology into probability teaching.

2. Methodology

In this study, action research was used. Action research is a process in which teachers investigate teaching and learning so as to improve their own and their students' learning. It is a systematic review of previously planned, regulated and collaborative systems to enhance the quality of life through critical reflection and interrogation [15; 16, 17]. Fraenkel and Wallen define action research as "*research conducted by one or more people or groups in order to solve a problem or to gather information to provide information about a local practice*" [18]. Cohen and Manion defined action research as "*methods developed to solve the problem that emerged at a specific moment in the education-training process in practice*" [19]. In a definition by Kemmis and McTagard, "*action research is defined as a participatory self-reflection study conducted by teachers to improve their understanding of their own practices, colleagues' practices, and situations in which practices are being finalized*" [20]. According to Loftus, the action research is the individual form of the learners, with the simplest definition of "*learning by doing*" [21]. Action research is similar to the problem solving approach [22]. In the action research, researchers do the following.

- They identify a problem that arises in their practice,
- They work together to solve it,
- They develop and apply a strategy to solve the problem,
- They evaluate whether it is successful or not,
- If they do not find the current situation positive, they develop it by implementing another strategy.

2.1 The Participants

The sample of the research consists of 14 men, 32 women; totally 46 volunteering prospective teachers who are taking probability and statistics II course in spring semester. They are third grade students who are trained in the mathematics teacher's program at a state university in Turkey. Researcher was coded as R and prospective teachers were coded as PT1, PT2,…, PT46.

3. Theoretical Framework

Stringer claims that action research is not a panacea for all ills and does not resolve all problems but provides a means for individuals to "get a handle" on their situations and formulate effective solutions to problems they face in their professional lives [23]. Stringer provides a basic action research routine that provides a simple powerful framework (Look, Think, Act) that enables people to commence their inquiries in a straightforward manner and build detail into procedures as the complexity of issues increases. Stages of the routine related to the traditional research practices are shown in Table 1.

Table 1: The Stages of the Routine Related to Traditional Research Practices

A Basis Action Research Routine
Look • Gather relevant information (Gather data) • Build a picture: Describe the situation (Define and describe)
Think • Explore and analyze: What is happening here? (Analyze) • Interpret and explain: How/why are things as they are? (Theorize)
Act • Plan (Report) • Implement • Evaluate

Source: [23]

3.1 Data Collection and Analysis

In the first stage of this study, two open ended popular probability problems (Cereal Box and Birthday Problems) were asked to the prospective mathematics teachers. The responses about each problem are analyzed and presented in tables under the title of results. The researcher is also a faculty member teaching the prospective teachers probability statistics. Prospective teachers already know about dynamic software but they do not know how to make top-level applications. In the second stage, it is aimed to strengthen the intuition of the prospective teachers and to facilitate their decision making. In accordance with this purpose, the simulations were developed by the researcher to the probability problems using TinkerPlots [24]. Prospective teachers used these simulations for observations. The activities of this study lasted 5 weeks. In the first week questions were asked. In the second week, they were analyzed. Simulations were prepared for the next two weeks. In the last week, the developed models were used by prospective teachers to solve problems. At the end of the five-weeks period, the prospective teachers were asked for their opinions about the use of simulation in probability problems. The ways of thinking of the prospective teachers about the problems were tried to put forward by qualitatively analysing and observations. The opinions about using simulation were also qualitatively analysed and the outstanding opinions were presented directly with the quotations.

4. Results

4.1 Results from Cereal Box Problem

Cereal box problem is a fascinating and compelling problem for the students of all levels. This problem provides opportunities for students to gather data, to make a guess and to create a mathematical model. Cereal box problem carries an introductory characteristic to the ideas related to uncertainty situations and expected value. Here is the problem:

Look:
Assume that your favourite cereal box contains one of the six prizes. This prize may be a pen, a plastic film character or a picture card. The possibility of selecting a prize is independent of others. How many cereal boxes should be bought in order to win all the prizes?

Table 2: Prospective Teachers' Responses to the Cereal Box Problem

Situation	Solution or Explanation	Frequency
1	No answer	25
2	Six times	7
3	$\frac{1}{6}\cdot\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{1}=\frac{1}{720}$	6
4	At least six	2
5	It may not have six boxes forever	2
6	Six times for each box. So total thirty-six times	2
7	$1+6+6+6+6+6=31$	1
8	$\frac{1}{6}$	1

As seen in Table 2, the results showed that the prospective teachers did not answer the question or they gave the wrong answers. Sections from some answers are presented below.

R: How many cereal boxes you think to buy in order to win all the prizes?
PT5: "*There is equal probability. All I know 1/6.*"
R: What is the minimum number of boxes we need to purchase?
PT23: "*We should buy six times.*"
PT12: "*At least six.*"
R: What is the maximum number of boxes we need to purchase?
PT37: "*We can never get the six prizes.*"
PT41: "*It's impossible to know since I can keep buying the same prizes*"
R: How many cereal boxes you think to buy in order to win all the prizes?
PT9: "*We go for each prize six times. The total is thirty-six times*"
PT16: "*It does not matter which one we get in the first prizes. Six times for each prize in the following. Total* $1+6+6+6+6+6=31$"
PT11: "*I think in the first 1/6 then 1/5 this way we multiply the possibilities. Result for me is* $\frac{1}{6}\cdot\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{1}=\frac{1}{720}$."

It has been seen that the answers are not often based on a logical basis or theoretical solutions are not related to the problem. This result reveals that prospective teachers cannot make consistent decisions in uncertain situations and that different teaching approaches are needed for the development of their perceptions.

Think:

Going to the store and buying all the cereal boxes all at once is not very practical. Assume that you came together with your friends and suddenly bought 8 boxes. You will be surprised when you see you don't have six prizes yet. What will happen if you don't have all of the prizes after you buy 10 or 15 or 20 boxes? How many boxes does it take to find all the prizes? [25]. At this point, attention can be drawn to important issues and motivation can be kept alive. Is there another way to model this problem without buying cereal boxes? How can a simulation model be created? What does it mean to acquire outputs on a trial? Which results will be considered for every trial?

To have a better understanding of the problem, this activity can be done. Assume that every number on a dice represents a prize. You can do 30 trials. A trial will be completed when all of the 6 prizes are collected. For example, in the first trial prize 1 was obtained in each 3 experience, prize 3 was obtained in each 5 experience. Now you can create your own table likewise. Tally sticks can be used on your table instead of numbers. If the trial continues like this, the total number of the products that were bought comes close to which value will?

Table 3: Table of the rolling a dice activity for Cereal Box Problem

Trial/Prize	Prize 1	Prize 2	Prize 3	Prize 4	Prize 5	Prize 6	Total number of products bought
Trial 1	3	2	5	1	3	2	16
Trial 2	1	2	2	2	2	2	11
Trial 3	3	2	1	2	2	2	12

After you complete your table, find the average of all the trials (For example, in the table above, the average of 3 trials is 39/3=13). This is the experimentally expected value. What will the expected value be if we do more trials? To answer this question, let us create a simulation. There are 6 prizes which the possibility of selecting each of them is 1/6. For this, the Mixer Device which can be seen Figure 1 or a Spinner with 6 equal regions can be used. A trial starts with buying the boxes and finishes when all of the six prizes are collected. Arithmetic mean of the numbers in the last column should be determined by adding the acquired results from all of the trials.

Act:

It was planned by the researcher to develop a simulation model for the Cereal Box Problem and prospective teachers were asked to use this simulation and evaluate the results they obtained. The images related to the creation and testing of the simulation model are shown below.

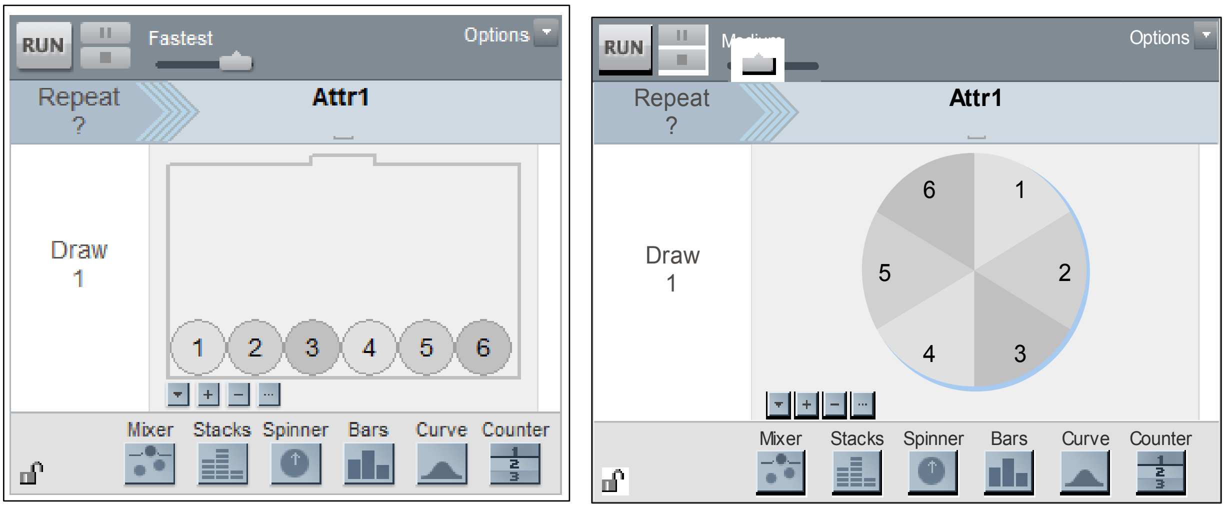


Figure 1: Samples for sampler selection

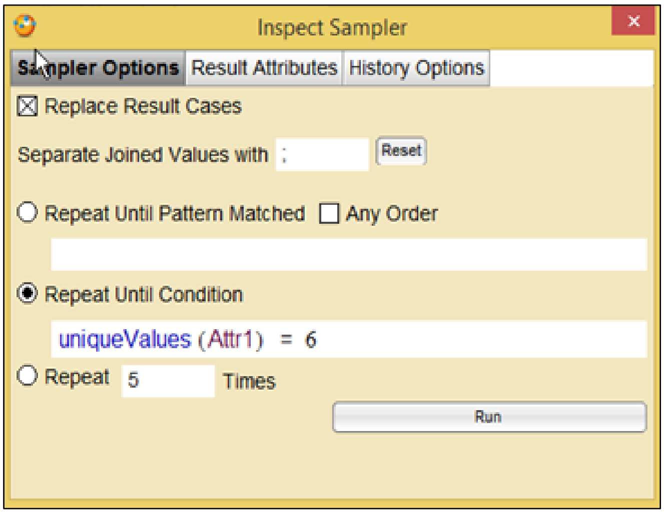


Figure 2: Creating sampler and sampler options for Cereal Box problem simulation

After the Sampler tool is determined, the steps below will be followed for sampler actions.

1) Click on the “Sampler” in TinkerPlots. Select “Sampler Options”
2) Click on the “Repeat until Condition”
3) Click on “Functions- Statistical- One Attribute- double click uniqueValues”
4) Write the attribute name inside the parentheses and set the statement equal to 6. (See Figure 2)

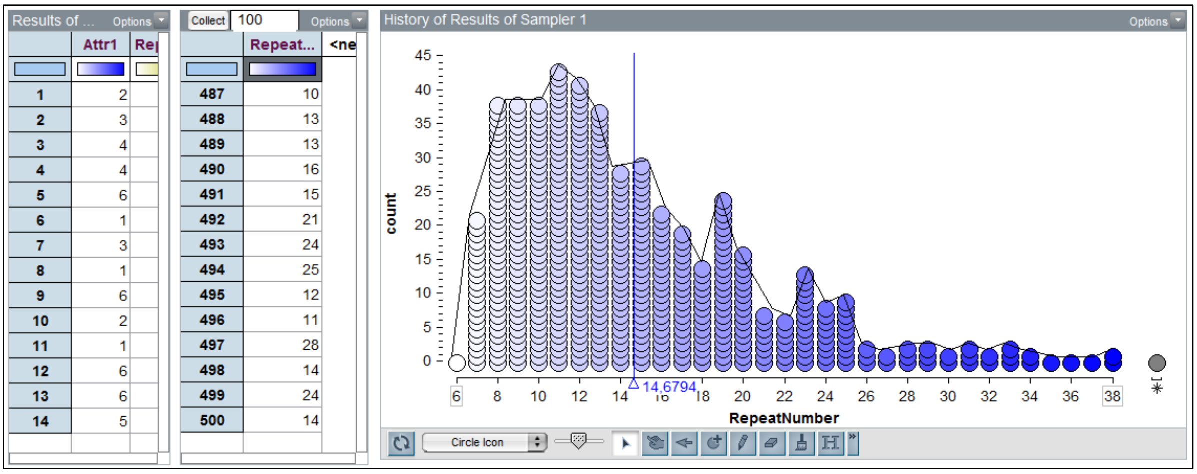


Figure 3: The trial results of the simulation of Cereal Box Problem

The results acquired on 500 trials in the simulations created for the cereal box problem can be seen in Figure 3. The arithmetic mean of the acquired results with the TinkerPlots software is 14,67. This value is the expected value. The expected value does not show how many times it takes to go for collecting all of the prizes. So, you may collect all of the prizes by going 7 times. Or you may not be able to collect all prizes even though you have tried 40 times. The expected value trials are the average of the acquired results of the trials you have made so many times. If the number of trials is increased more, the theoretically calculated value is approached gradually.

4.2 Theoretical solution of the expected value of Cereal Box problem

Simple probabilistic modeling shows that on average $n(1+\frac{1}{2}+\cdots+\frac{1}{n})$ boxes are required to complete a full set of n prizes. For example, it takes on average 14.7 boxes to complete a full set of six prizes. $\frac{6}{6}+\frac{6}{5}+\frac{6}{4}+\frac{6}{3}+\frac{6}{2}+\frac{6}{1}=14.7$

Assume that you need to get the prizes from cereal boxes all at once and you need to be 95% sure that you got all of the six prizes. Based on the simulation results it is easy to guess the number of the boxes you need to buy. According to Figure 4, result is 24. It is 95% possible that 6 prize will be collected.

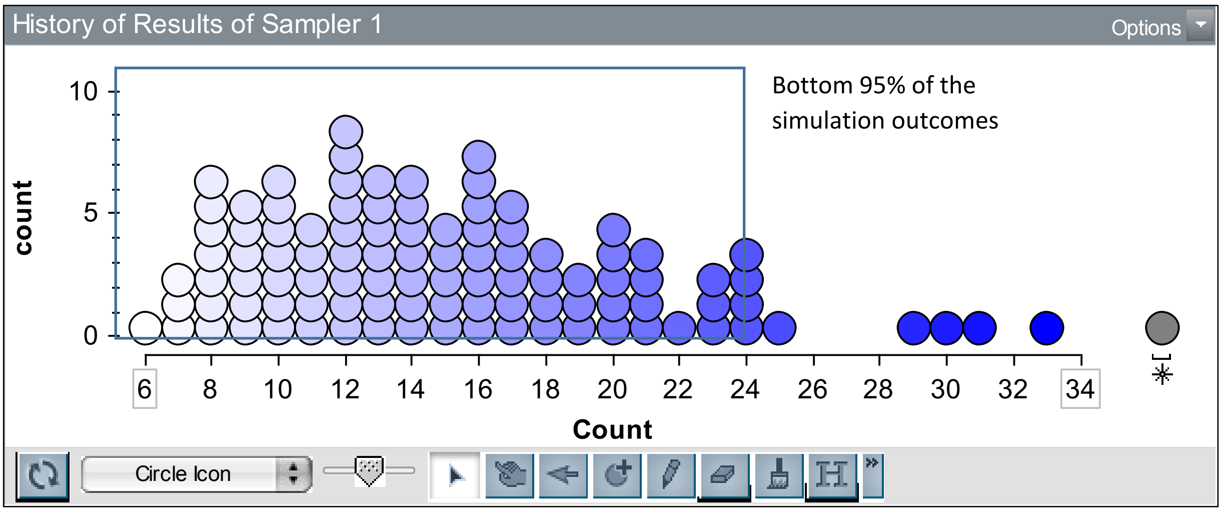


Figure 4: Cereal Box Problem simulation outputs (100 trials)

4.3 Results from Birthday Problem

"Birthday Problem" is a classic paradox example in the probability theory. It is one of the intuitive issues that confuse most students' minds.

Look:

What are the chances that at least two people share a birthday in a group of 23? Make a guess. What is your opinion on solving the problem?

Table 4: Prospective Teachers' Responses to The Birthday Problem

Situation	Solution or Explanation	Frequency
1	No answer	30
2	$1-\frac{1}{365}=\frac{364}{365}$	3
3	$\frac{22}{365}.\frac{1}{365}=\frac{22}{133225}$	2
4	$\frac{1}{23}.\frac{1}{23}=\frac{1}{529}$	2
5	$\frac{\binom{23}{2}+\binom{23}{3}+\cdots+\binom{23}{23}}{\binom{365}{1}}$	2
6	$\frac{1}{365}.\frac{1}{23}=\frac{1}{8395}$	1
7	$\frac{2}{23}$	1
8	7^{23}	1
9	$\frac{23}{2}$	1
10	$\frac{23}{365}-\frac{2}{365}=\frac{21}{365}$	1
11	$\frac{23}{365}.\frac{22}{365}=\frac{506}{133225}$	1
12	$\frac{22}{23}$	1

As it can be seen from the Table 4, a large majority of prospective teachers did not respond. Other answers were based on incoherent or unrelated theoretical foundations.

Think:

Say your class has 23 students. We know that a year is 365 days long. This means everyone in the class was born on any day of the year. Our interest is, the probability of at least two people sharing a birthday in that group of 23. Many people think the possibilities are very low, almost zero. The actual probability is, of course, higher than the expected.

Birthday Problem can be solved by using a simulation. The plan in the modelling should be in this way: Between 1 and 365, arbitrary integers should be picked as many as wished (in this case n=23). Then, the focus should be on the repeated integers. The possible outcomes are: none of the numbers is the same, two numbers are same or more than two numbers are same. A calculation of probability based on the results of numerous trials gives the experimental probability number. In other words, we complete the calculation by dividing the number of trials in which one or more than one pair have the same integers to the total trial number.

Act:

It was planned by the researcher to develop a simulation model for birthday problem and prospective teachers were asked to use this simulation and evaluate the results they obtained. The images related to the creation and testing of the simulation model are shown below.

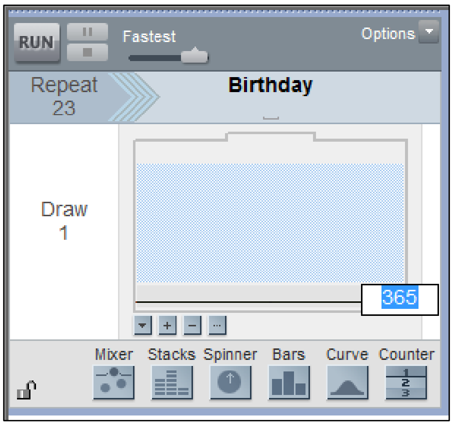


Figure 5: Sampler for birthday problem simulation

On Figure 5, the sampler device for the birthday problem simulation is seen. "Repeat 23" shows the number of people in the group. Stacks Device was chosen as the Sampler device. There are 365 items in this device which represent the days of a year. On Table 5, 20 trial results obtained from this device are seen.

Table 5: Birthday problem simulation results

Trial	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	233	265	250	57	228	81	260	146	355	110	344	36	253	5	289	19	171	245	56	188
2	132	241	107	259	69	320	291	354	192	54	318	136	214	44	249	152	111	325	353	261
3	55	234	210	126	357	44	271	145	20	365	65	326	96	201	349	305	334	301	85	286
4	305	22	341	23	73	238	146	271	212	183	81	179	351	27	93	322	38	281	128	43
5	98	299	364	293	320	122	303	238	259	363	331	33	160	239	351	328	161	294	116	349
6	299	345	268	156	301	36	122	124	235	241	217	218	87	243	95	65	75	84	192	12
7	349	296	289	272	305	235	281	61	329	179	139	218	162	212	174	88	304	340	364	290
8	317	317	197	84	66	346	1	265	261	163	317	131	319	115	316	297	215	232	350	183
9	169	60	221	188	287	330	299	133	144	245	334	33	292	354	207	155	288	128	81	7
10	68	7	39	211	101	331	210	112	91	303	334	237	329	132	296	314	297	336	187	340
11	162	219	88	158	26	361	201	264	176	80	362	9	117	256	6	24	286	119	302	306
12	323	154	190	37	304	240	160	228	225	98	58	219	39	228	202	201	186	174	140	88
13	92	160	319	347	175	345	232	333	49	347	277	85	266	248	49	74	208	137	152	140
14	306	365	137	12	198	268	305	196	303	331	341	179	260	207	206	174	23	244	208	201
15	61	265	211	98	279	77	48	72	160	268	219	365	61	29	297	181	233	166	140	25
16	323	145	135	125	16	141	14	338	171	349	131	167	147	168	273	65	273	3	346	113
17	56	253	364	281	267	6	224	66	252	258	174	279	78	297	211	126	250	107	263	248
18	245	303	199	35	10	164	56	340	56	25	97	58	331	217	239	357	207	205	201	240
19	208	78	58	353	209	268	76	128	8	163	277	157	248	328	142	206	333	234	211	71
20	245	115	77	109	147	308	94	120	90	21	14	171	6	296	39	313	343	329	349	343
21	201	170	229	299	220	242	66	62	337	136	97	268	240	285	348	304	297	332	119	138
22	252	50	140	318	183	355	280	297	325	282	158	51	244	144	179	351	113	174	202	51
23	304	47	244	300	239	348	147	251	300	245	343	295	143	66	339	319	261	313	183	174
R	**Y**	**Y**	**Y**	**N**	**N**	**Y**	**N**	**N**	**N**	**Y**	**Y**	**Y**	**N**	**N**	**N**	**Y**	**Y**	**Y**	**Y**	**N**

Table 5 shows trial results of the simulation obtained. As it is seen in the table, every column shows one trial results. These numbers represent the arbitrary birthdays created by simulation for everyone in the group. It is seen that some columns have one match, some have more than one and some does not have any. Every result on Table 5 is copied from the left table on Figure 6. The table on the left Figure 6 represents the last trial.

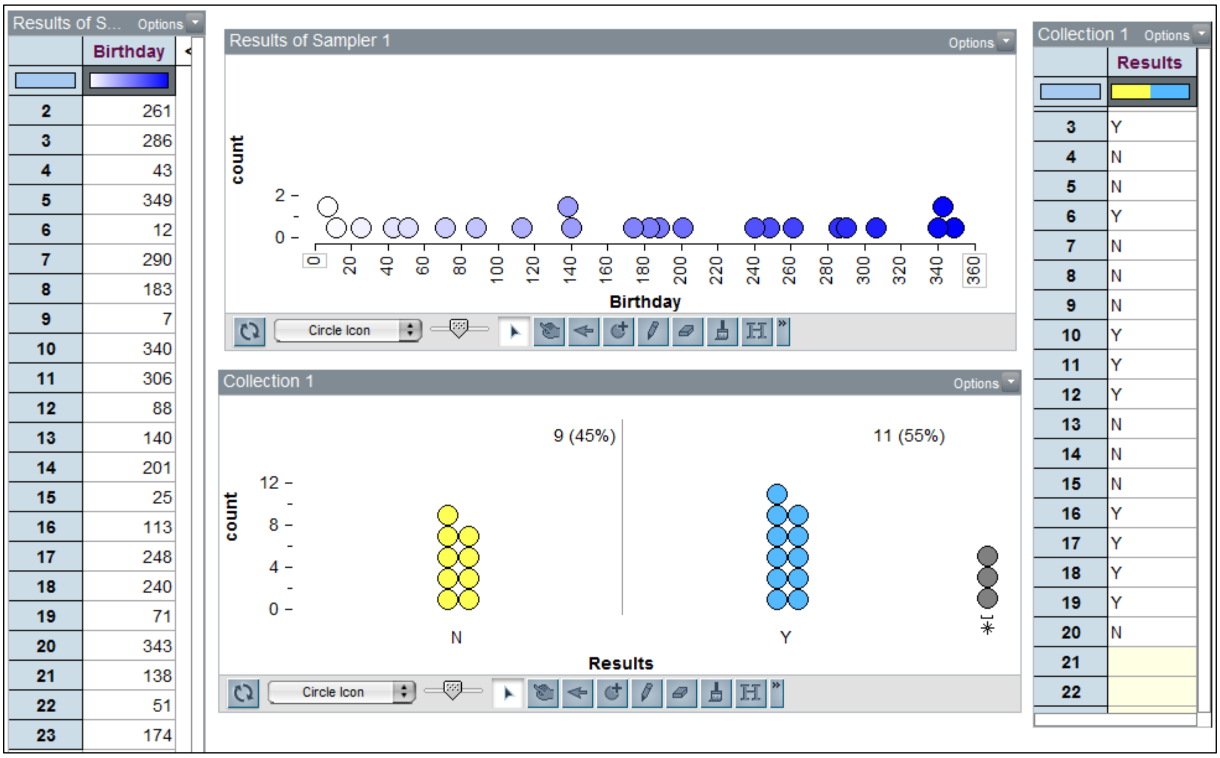


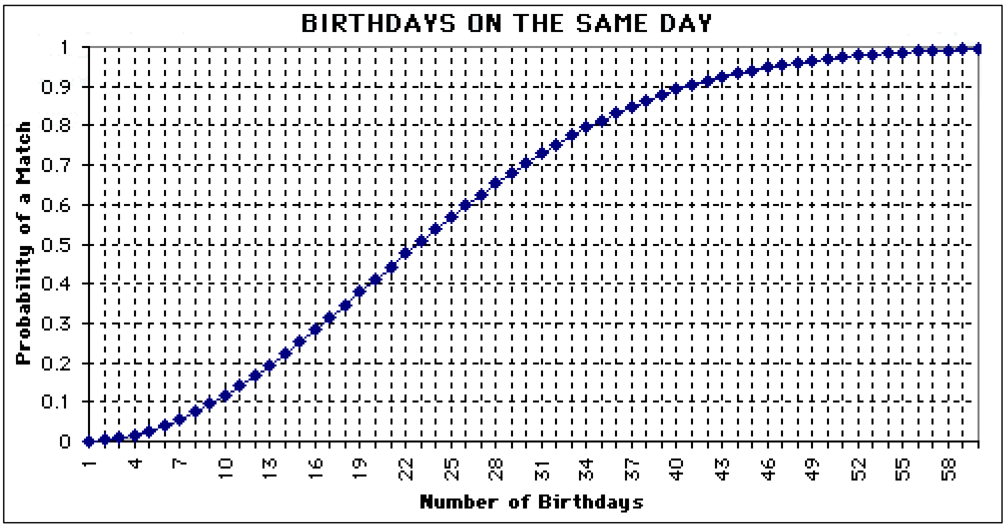
Figure 6: Birthday problem simulation results (23 people and 20 trials)

The chart on the up-centre in Figure 6 was created to determine if there are any matches. Two circles on top of one another means the numbers are same. The chart on the right on Figure 6 was created by writing the results obtained from Table 5. The chart on the down-centre in Figure 6 is the display of the chart on the right. It shows the quantile of Y and N numbers. That is, in 20 trials the results were obtained as: at least two people sharing a birthday Y (11), no people sharing a birthday N (9).

As it is seen in Figure 6, the probability of at least two people sharing a birthday in a group of 23 according to the results of 20 trials we obtained from the birthday problem simulation is:

$$\frac{\text{Number of 'successful' outcome}}{\text{Number of trials}} = \frac{11}{20} = 55\%$$

The theoretical probability will be approached gradually as the number of trials increases. A common false opinion is that students too often think that the sample distribution reflects whole distribution. Comparing 365 days to 23 people, students think that since there are too many days the probability of two people sharing a birthday in that group is very low. Arbitrary samples come in sets and possible results do not spread monotone or symmetrical. The use of simulations helps students to improve their understanding of this situation. (Alternatively, other simulations prepared for this problem have been examined. For example, http://mste.illinois.edu/activity/birthday/#Simulation)



Source: http://mste.illinois.edu/activity/birthday/explanation.php

Figure 7: Birthdays on the same day

As it can be seen from the Figure 7, a probability of over %50 is obtained after 23 people and the probability is above %90 before the number of people reaches 45.

4.4 Theoretical Solution to the Birthday Problem

Let's calculate the probability of students not sharing the same birthday in a class of 23. The event of different birthdays is A'. In this case;

$$P(A') = \frac{365}{365} . \frac{364}{365} \dots \frac{343}{365} = \frac{\left(\frac{365!}{342!}\right)}{365^{23}}$$

$$= \frac{42200819302092359872395663074908957253749760700776448000000}{85651679353150321236814267844395152689354622364044189453125}$$

$$= 0.4927027656$$

Since the probability of at least two students sharing a birthday is

$$P(A) = 1 - P(A')$$

$$P(A) = 1 - 0.4927027656 = 0.5072972343 \cong 50\%$$

It can be seen that the answer obtained from simulation is really close to the theoretical answer. Correction of the simulation, of course, will be improved with more trials. If the class size is 23, the probability of two people sharing a birthday is 0,507 or about 50%.

Also, the probability of at least two people sharing a birthday can be examined by changing the number of people in the group. For example, in Figure 8, when the group number is 50, it can be seen how strikingly increases the probability.

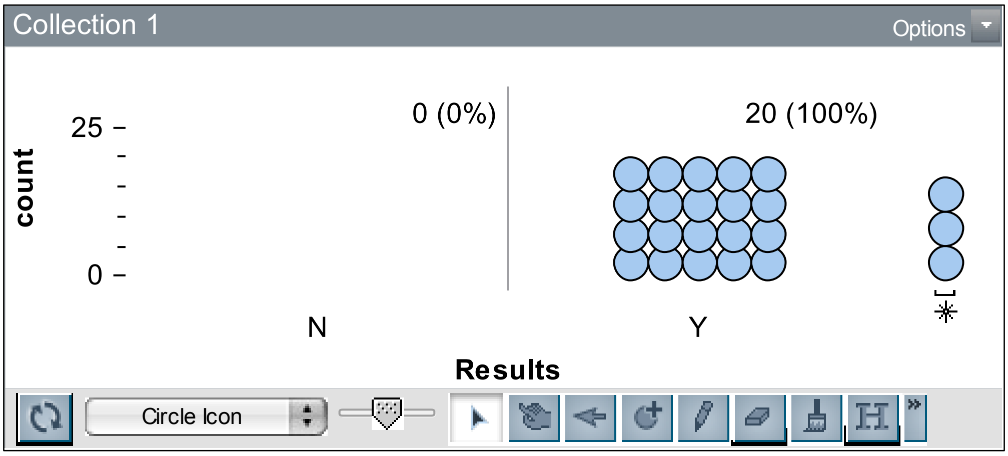


Figure 8: Birthday Problem simulation results (50 people and 20 trials)

In Figure 8, according to the simulation results of 50 people and 20 trials, the experimental probability result was obtained as 100%. When this probability was calculated as the theoretical calculation for 23 people, we obtain 0.970. It can be said that the experimental and theoretical probability results are approximate to each other.

4.5 Prospective Teachers' Views on Using Simulation

Direct quotations from some prospective teachers' views on the use of simulation are presented below.

PT23: *"We experimentally and theoretically analysed both Cereal Box and Birthday Problem in the study. We saw that the facts are different from what we have thought."*

PT13: *"Simulation makes solving problems very easy."*

PT3: *"In simulations, observations can be made by changing the number of trials, which is a huge advantage."*

PT41: *"We focused on using simulation models for problems, doing experiments, visualisation of the results and calculation processes. At the same time, we compared experimental and theoretical solutions. I think it was quite different and effective."*

PT34: *"I saw the power of the software and the possibilities it provided to solve the problems."*

PT14: *"This software is quite handy because of the fact that it does not need coding ability, it also virtualised the data colourfully, and it is easy to use and accessible to teachers and students."*

PT8: "*Simulated images are visualized with tables and graphics. We figured out what it means to make it happen.*"

PT19: "*I learned to solve problems with computers*"

PT22: "*I saw that theoretical and experimental solutions are close together.*"

As it can be understood from their views on the use of simulation, the opinions of prospective teachers on the use of simulation are quite positive. They have expressed some of the advantages of using simulation.

5. Discussion

In this study, it was once again seen that the answers to the problems involving uncertainty situations were not consistent and satisfactory. For this reason, it is necessary to develop the intuitions of the teachers and learners. Along with improvements in technology, the range of software is growing. The software offers advantages in terms of dynamism speed and scope [4; 10]. One of them is simulations. Simulations are a teaching methodology that is accepted in many disciplines. The data comes from a database and gets collected by a series of trials that can be used to analyse them. Thus, technology can now provide students with the opportunity to conduct real studies of real problems. There are two common ways of using a modelling in statistical thinking. The first one is choosing, designing or using the suitable modelling to answer the research question to generate the data. The second one is, defining variability and explaining it, creating a statistical model for the collected data from a trial, survey or available data [26]. In this study, the researcher focused on both stages while the prospective teachers focused on the second stage. Since the created simulations are models of the real world process within themselves, they have designed a learning environment for a better understanding of inputs, internal processes and process outputs.

Look, Think and Act cycle was adopted in this action research. This research cycle is a self-assessment study that allows teachers and researchers to better understand and develop their own practice and will add to the body of action research in teaching mathematics. Most importantly, it can lead to valuable insights into improving the incorporation of educational technology into probability teaching through social activism. When we view our classrooms as living, dynamic learning environment, we can, as teachers, make sense of what might otherwise seem chaotic or meaningless. This viewpoint enables us not only to describe and explain what is happening around us, but also to use our findings to influence emerging problems and difficulties in our classrooms or our larger communities.

6. Conclusions

This study concludes that simulations could be very efficient tools to understand to stochastic processes. Here are some of the advantages of using simulation in this study, simulation use visualizes the changes in the test outputs, it contributes to the development of ideas about randomness and change, experiments using simulations make it easy to make consistent estimates for the solution of problems. How to get information from sources allows you to think about the theoretical reasons why a model is appropriate and why the results are that way. Simulation allows the number of trials to be changed so that a small number of trials as well as a large number of trials can be observed. The simulations provide many possibilities. It provides the students with the atmospehere for forecasting, discussion and deduction. Indeed, it has been emphasized that computer-based simulations will provide a powerful mathematical basis for future teachers, and that modern mathematics teaching will offer opportunities to meet all important demands such as analyzing and representing real situations, problem solving, and mathematical reasoning [1]. Findings obtained from prospective teachers also support these views.

6.1 Research Limitations

1. This research is limited with 46 prospective mathematics teachers.
2. The research is limited to two probability problems which require estimation, observation and explanation about the probability.
3. The use of simulation is limited to TinkerPlots Sampler tool.
4. Selected problems are suitable for experimental and theoretical calculations.

6.2 Future Research Directions

In this study, simulations created with TinkerPlots software were used to calculate the frequency of occurrence of an event and the average of these experiments according to the results of many experiments. No matter which tools are used for simulations (MINITAB, Fathom, R software, TI-Nspire, TinkerPlots etc.) all of them offer unique opportunities to students are taught, which other methods do not. With the use of simulation, students can more easily understand the relation between mathematical knowledge and real life as well as the experimental and theoretical probability. Simulations create different learning environments for teachers and students and simulations on the concept of probability provide more contribution to the students and teachers than the process in which pencils and papers are used. Simulations are an effective tool, especially in teaching experimental probability. From the results of this study, it is suggested to use simulations to solve the problems at every level from the elementary school to the university in order to understand the concepts of probability and to develop the perceptions of the students about the stochastic processes better.

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