European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.2620212 Volume 5 │ Issue 12 │ 2019

THE MIDDLE SCHOOL MATHEMATICS TEACHERS' SUBJECT MATTER KNOWLEDGE: THE CONTEXT OF DIVISION OF FRACTIONS

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Abstract:
In this study, the middle school mathematics teachers' knowledge of division of fractions was investigated through the strategies that they proposed and the problems that they posed for division of fractions. The data was collected from 22 middle school mathematics teachers through the task and semi-structured interviews. The data of this case study was analyzed by content analysis approach. According to the findings, 5 alternative strategies were proposed by 22 teachers to divide fractions. Among these strategies, the most commonly used was invert and multiply strategy and the least are converting fractions into decimal and converting to equation. In relation to the problems that the teachers posed, the findings revealed that more than half of the teachers could not propose appropriate problems. In other words, the teachers had difficulty in generating an appropriate story problem to illustrate division of fractions. These results and implications are discussed and the recommendations are presented in accordance with the findings of the study.

Keywords: teacher knowledge, subject matter knowledge, division of fractions, alternative strategies, problem posing

1. Introduction

In the 21st century, there has been great interest in creating mathematically rich and intellectually challenging environment to immerse students in learning mathematics (Stein & Lane, 1996; Boaler & Staples, 2008). In order to create such a classroom environment, it is not enough to provide teachers rich mathematical tasks and several manipulatives (Stein, Remillard, & Smith, 2007). Rather, it is thought that implementing mathematically rich and intellectually challenging mathematics instruction requires drawing upon and managing various resources at the same time, including teacher beliefs and knowledge, student skills, attitudes and previous knowledge and

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curriculum materials (Cohen, Raudenbush, & Ball, 2003). Among these resources, teacher knowledge has been stated as an important resource that influences the outcomes of teaching and students learning (Borko & Putnam, 1996; Mewborn, 2003). Therefore, the interest in understanding and describing teachers' knowledge continues in mathematics education society. Researchers have claimed that both the quality of the mathematics teaching and student learning depends on teachers' knowledge (Ball, Hill, & Bass, 2005). Because of the importance of teachers' knowledge on students' achievement, there is a need for more and in-depth understanding of teachers' knowledge required for teaching. Accordingly, various frameworks were constructed as a means to understand the complex construct of the teachers' knowledge.

1.1 Theoretical Framework

Shulman (1986), who is the pioneers of teachers' knowledge, categorized it into 3 components: subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knowledge (CK). Shulman's (1986) SMK involves knowing the facts, truths and concepts, explaining the reasons for learning these concepts, and relating the concepts within and without the discipline. The second component, PCK, consists of knowing "*the ways of presenting and formulating the subject that make it comprehensible to others*" and "*an understanding of what makes the learning of topics easy or difficult*" (p.9). Lastly, Shulman (1986) defined CK as the knowledge of program developed for the teaching of particular subjects at a particular level.

Expanding Shulman's framework, Ball, Thames and Phelps (2008) presented a framework which is only for mathematical knowledge for teaching. According to Ball and her colleagues, Shulman's SMK is composed of Specialized Content Knowledge (SCK), Common Content Knowledge (CCK) and Horizon Content Knowledge (HCK). Although SCK is unique to the teaching, CCK is not exclusive to teachers. More specifically, every adult can have well-developed CCK whether s/he is a mathematics teacher or not. But, SCK is unique to the teacher who engages in teaching mathematics. The characteristics of the SCK are representing mathematical ideas, providing mathematical explanations and procedures with their justifications, and deciding whether the student's methods are generalizable to other. Lastly, horizon knowledge is defined as "*an awareness of how mathematical topics are related over the span of mathematics included in the curriculum*" (Ball et al., 2008, p.403). On the other hand, Shulman's PCK is identified as knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). Ball et al. (2008) stated that KCS is the combination of knowledge of content and knowledge of students, KCT is the combination of knowledge of content and knowledge of teaching and KCC is combination of knowledge of content and knowledge of curriculum.

Another framework that is special to mathematics teaching is called as Knowledge Quartet presented by Rowland, Huckstep and Thwaites (2005). Rowland and his colleagues defined Knowledge Quartet as "Mathematical Knowledge in Teaching" instead of "Mathematical Knowledge for Teaching" in order to state the difference between their model and Ball's model. Foundation, transformation, connection and contingency are the categories of Knowledge Quartet. Foundation is related to teachers' content knowledge, beliefs about mathematics and the purposes of mathematics education. They emphasized that teachers do not always present this type of knowledge during instruction. Knowledge needed to plan teaching and to use during the instruction is called as transformation. Rowland et al. specified that the teacher who have this knowledge know the best ways of transferring their knowledge to help the students learn content meaningfully. The third dimension of Knowledge Quartet is connection which is related to making connections between the topics and procedures, and the sequencing of topics of instruction within and between lessons. Last dimension, contingency, refers to the teachers' in-the-moment decisions about unexpected events.

This study focused on teachers' subject matter knowledge which was adopted from Shulman's framework. More specifically, this study aims to investigate middle school teachers' subject matter knowledge through their knowledge of alternative solution strategies and their knowledge of different representations such as posing story problems.

1.2 Related Studies on Teachers' Subject Matter Knowledge

Many researchers focused on the role of teachers' SMK in students' mathematics learning and they emphasized that it is an important determinant of mathematics teaching and learning (Fennema & Franke, 1992; Hill, Schilling, & Ball, 2004; Tirosh & Graeber, 1991; Tirosh, 2000). For instance, Even (1990) stated that a teacher who has adequate content knowledge helps the students achieve a conceptual understanding of the subject matter. On the other hand, a teacher who has misconceptions and deficiencies in subject matter is likely to transfer those to their students. As a result of her study, she resulted in that teachers' knowledge of functions was weak and fragile. In another study, Huang and Kulm (2012) aimed to explore the knowledge of function of pre-service middle school mathematics teachers. The researchers concluded that the pre-service teachers failed to solve quadratic/irrational equations, made mistakes in solving problems using the integration of algebraic and graphic representations of functions, and performed poorly in selecting appropriate perspectives and using representations of the concept of function. Additionally, Ball (1990a) analyzed pre-service teachers' understanding of division with fractions and the results showed that the teachers had a narrow understanding about the topic. The pre-service teachers applied the invert and multiply rule however they did not know the underlying reasoning behind the rule. Indeed, the important things for effective teaching are discussing the meanings of the concepts, making relationships between the concepts and the procedures, and explaining the concepts to the students with the logic of the rules rather than applying the rules (Ball, 1990a). Similarly, Ball (1990b) investigated teachers' knowledge of division. As in her previous study (1990a), the results showed that although many pre-service teachers could produce correct answers, they could not explain the underlying principles and meanings of division algorithm.

In the literature, there are many studies which investigated teachers' subject matter knowledge of various mathematics (Carreño, Ribeiro, & Climent, 2013; Contreras, Batanero, Diaz, & Fernandes, 2011; Even, 1993; Even & Tirosh, 1995; Huang & Kulm, 2012; Livy & Vale, 2011; Pino-Fan, Godino, Font, & Castro, 2013). The results of these studies showed that the teachers' subject matter knowledge were not adequate to teach mathematics efficiently. In order to get more detailed and broader information of teachers' subject matter knowledge of mathematics, there is a need to conduct studies which aims to investigate teachers' subject matter knowledge from different point of views. In examining teachers' knowledge of mathematics, one of the critical areas of inquiry and analysis is the way in which the different representations that they present to the students. As Ball (1990a) stated, in order to represent the subject in multiple ways, the teachers need to know alternative solution strategies and generate different problems that require deeper subject matter knowledge. Also, Lin (2004) and Ticha and Hospesova (2009) emphasized that teachers' problem posing ability is one of the indicators of their knowledge of mathematics. From this point of view, the aim of current study is to investigate teachers' knowledge by examining the solution strategies that middle school mathematics teachers proposed to solve the division of fraction and the problems that the teachers posed. It is important to solve the problems with different solution strategies and to pose various problems. Since if the teachers are able to pose different problems and have broader knowledge of alternative solution strategies, then they reflect this knowledge to their students (Castro-Rodríguez, Pitta-Pantazi, Rico, & Gómez, 2016; Wahyu, Amin, & Lukito, 2017). In this way, the students will have knowledge about problems different than the problems presenting in the textbooks and alternative solution strategies different from the standard strategies.

In order to examine teachers' knowledge of solution strategies and the problems that they posed in depth, the scope of the study was narrowed down. Division of fraction was chosen even though there were research studies which investigated teachers' knowledge of division of fractions. However, these studies did not focus on the strategies that the teachers proposed different than invert and multiply rule and the problems that they posed related to division of fraction. Additionally, many research studies focused on knowledge of pre-service teachers (Ball, 1990a; 1990b; Contreras, Batanero, Diaz & Fernandes, 2011; Even, 1993; Even & Tirosh, 1995; Huang & Kulm, 2012; Isiksal, 2006; Livy & Vale, 2011) even though pre-service and novice teachers generally do not have a robust knowledge of mathematics (Magnusson, Krajcik, & Borko., 1999; Shulman, 1987). In this sense, the practice of experienced teachers would provide valuable information related to teachers' knowledge about the alternative solution strategies and the problems that they posed related to division of fractions.

1.3 Division of Fractions

Division of fractions is one of the most challenging topics for students (Fendel, 1987; Tirosh, 2000) because of the nature of both division and fraction. Division is the least understood one among the four operations and the fractions are regarded as the most complex number (Ma, 1999). NCTM (2000) stated that one of the reasons for this difficulty might be teaching the division of fractions with invert and multiply strategy which is the least understood standard algorithm. Also, some researchers claimed that the teachers' understanding of division of fractions and the way they teach are the main reasons for students' difficulty in division of fractions (Ma, 1999; Wahyu et al, 2017). In this sense, teachers' subject matter knowledge becomes an important factor to make division of fraction as the most understood operation. However, the previous studies concluded that the teachers, like the students, had difficulty in division of fractions. The teachers could divide the fractions using invert and multiply strategy; however they could not explain the underlying principles of this strategy (Ball, 1990b; Olanoff, Lo, and Tobias, 2014). Although some of them knew the reasons for inverting and multiplying the divisor, they could not explain the reasons to their students conceptually (Borko et al., 1992). Moreover, Tirosh (2000) stated that the pre-service teachers did not know any alternative strategies and they thought that dividing numerators and denominators strategy is not correct to divide fraction. In contradistinction to these studies, Ma (1999) reported that Chinese teachers had conceptual understanding and generated alternative solution strategies such as converting fractions to decimal strategy, distributive law, common denominator, etc. As a result of her study, Ma emphasized that the teachers should have adequate knowledge and should be ready to present multiple ways such as different representations, alternative solution strategies, variety of problems to help students understand the division of fractions. From this point of view, in order to reveal whether the middle school teachers have enough knowledge to make the students more understandable about the division of fractions, the teachers' subject matter knowledge was examined in the scope of the alternative strategies that they proposed and the problems that they posed. More specifically, the research questions of the study were as follows:

1) What are the alternative solution strategies the middle school mathematics that middle school teachers propose for division of fractions?
2) To what extent are these teachers successful at posing a problem related to division of fractions?

2. Method

2.1 Design of the Study

The study aims to investigate middle school mathematics teachers' knowledge of division of fractions through the alternative strategies that they proposed and the problems that they posed. More specifically, the aim is to explore the teacher's knowledge in depth rather than determining the level of teachers' knowledge. For this reason, case study method, one of the qualitative research approaches, was used to collect the data since Creswell (2007) stated that the purpose of case studies are to get a richer and deeper description about the case or cases by examining them within a bounded system.

2.2 Participants

The participants were selected through the purposeful sampling method since the participants were selected from among the people from whom the most knowledge can be gained, can be accessed easily and with whom the most time can be spent (Merriam, 1998). In this direction, 22 middle school mathematics teachers, whose boundaries were having at most 10 years teaching experience, were selected as participants. The pseudonym such as T1, T2,..,T22 were given to each participant instead of using their real names.

2.3 Data Collection

The data were collected in two stages. In the first stage, a task consisting of 2 questions prepared by the researcher was applied to all participants. In the first question, middle school teachers were asked to solve the division of fraction, which is 4/15÷2/3, using as many strategies as they can. In the second question, it was requested the teachers to pose a problem which can be solved by 4/15÷2/3. After applying the task, the semi-structured interviews were conducted with all teachers to get more in-depth knowledge about the strategies that they proposed to solve the division algorithm. Also, the reasons for writing such a problem were explored during the interview.

2.4 Data Analysis

The data was analyzed through content analysis approach which is a method to categorize the data into similar categories (Strauss and Corbin, 1990). In the first stage of the data analysis, participants written responses were examined and coded by two researchers. In the second stage, the transcripts of the semi-structured interviews were analyzed in order to clarify participants' responses and get more information about their knowledge. At the end of both stages, the categories were emerged as a result of the agreement of both coders. Thus, the inter-rater reliability was also ensured.

3. Results

In this study, the middle school mathematics teachers' knowledge of division of fractions was investigated. More specifically, the strategies that the teachers proposed to solve $\frac{4}{15} \div \frac{2}{3}$ and the problems that the teachers posed which can be solved by $\frac{4}{15} \div \frac{2}{3}$ were examined to reveal teachers' knowledge of division of fractions.

3.1 The Alternative Strategies of Middle School Teachers for the Division of Fraction

During the data collection process, it was asked middle school teachers to solve the given algorithm related to division of fraction using as many strategies as they can. Based on the analysis of the data, 5 different strategies were proposed by 22 mathematics teachers to divide fractions. The frequency of strategies was presented in Table 1.

Table 1: The Frequency of Strategies That Proposed by the Teachers

Strategy	**f (%)**
Invert and Multiply	20 (91)
Common Denominator	13 (60)
Dividing numerators and denominators among themselves	8 (36,4)
Converting fractions into decimal	1 (4,5)
Converting to equation	1 (4,5)

As it can be realized from Table 1, the most common strategy that the teachers prefer to use while dividing the fractions was invert and multiply strategy. Among 22 teachers, 20 of them suggested this strategy to divide fractions. During the interviews, the reasoning of invert and multiply strategy was asked to these teachers. The surprising thing is that many of them did not know the reasoning behind why the divisor is inverted and multiplied. Regarding this, one teacher claimed that invert and multiply strategy is the definition of division of fractions without explaining the justification of how the division of fraction can be defined and what is the relationship between the invert and multiply strategy and the definition of division of fraction. Additionally, 16 teachers (out of 20) stated that invert and multiply strategy is the most reliable way to divide the fractions since it can be generalized to all problems related to division of fractions. However, these teachers did not know how it can be generalized to all problems and why it is the most reliable strategy. When asked about the reason for why this strategy was reliable, T21 stated the following:

"I believe that it is reliable since it is the general rule for division of fraction. When I was a student, I had learned the division of fractions by this strategy. Also, this strategy is presented all mathematics textbook. For this reason, I consider that it is the most reliable and efficient way to divide fractions."

On the other hand, 3 middle school mathematics teachers addressed that division is the inverse operation of multiplication. In relation to this, T6's explanation is presented below:

"Division by a number means multiplication by its reciprocal. Thus, while dividing $\frac{4}{15}$ by $\frac{2}{3}$, we can multiply $\frac{4}{15}$ by $\frac{3}{2}$. The logic behind invert and multiply strategy is this."

Table 1 shows that another strategy that more than half of the middle school teachers presented was common denominator strategy. Thirteen teachers among 22 teachers stated that this strategy can be an alternative way to divide the fractions. Among these teachers, T1 noted that:

"This strategy is not common in our school. At first, we did not learn division of fractions with this method. We learnt invert and multiply strategy. However, I think that common denominator strategy is another efficient strategy and it can be generalized to all

problems related to division of fractions. Now, I propose my students to use it while dividing the fractions."

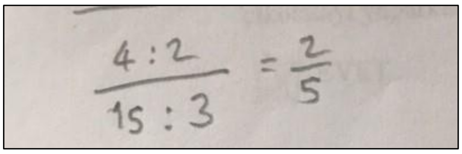
Another teacher, T12 explained that:

"The only strategy in our textbooks was invert and multiply strategy, now common denominator strategy is presented in the textbooks. This strategy might be made more sense than invert and multiply strategy since it is similar to adding the fractions. The students might confuse when the denominators must be common, when it is not necessary. With this strategy, they may think that they can equate the denominators before doing basic algorithms of fractions always."

As well as connecting the division of fractions with addition of fractions, a few teachers made relationship between division of fractions and division of whole number with common denominator strategy. In relation to this, T8 stated that:

"When the denominators are equated, the division of denominator will be 1 and the denominator become identity element. Therefore, we only divide numerators which are whole numbers. In this case, one whole number is divided by a whole number. In this way, the students will not have difficulty in doing division operations with fractions."

Apart from these strategies, 8 middle school teachers stated another strategy which is named as dividing numerators and denominators among themselves. In this strategy, the numerators and denominators of the dividend and divisor are divided separately. One of the teachers' solutions, T14, is presented as an example below.



In the interview, it was asked to 8 teachers whether it is valid for all problems or not. The excerpt from interview of T14 is as follows:

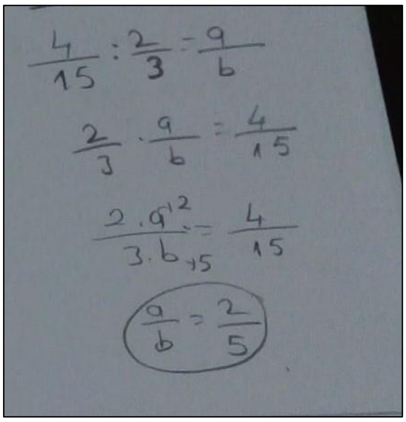
"I haven't thought before. Hıııımmm..I should think a few minute. If I divide $\frac{6}{14}$ by $\frac{2}{7}$, the result will be $\frac{6:2}{14:7} = \frac{3}{2}$. There isn't any problem with the strategy. I want to do one more division algorithm with fractions. In this case, the first fraction is $\frac{5}{12}$ and the second one is $\frac{2}{7}$. When I apply dividing numerators and denominators among themselves, the numerator and denominator of dividend is not divisible by those of divisor. Ooo…I cannot use this strategy to divide all fractions."

As it can be realized from the excerpt, all teachers realized that dividing numerators and denominators among themselves is not generalizable to all fractions.

As an alternative strategy for division of fractions, only one teacher, T13, specified converting fractions into decimal strategy and added:

"The division of fractions can be done with converting the fractions into decimals. Then, the division algorithm with decimals can be performed. In this example, it might be challenging since both the divisor and dividend is repeating decimals and division of repeating decimals can be difficult for students. Otherwise, it will be one of the easiest ways to divide the fractions."

The last strategy that T4 was proposed to divide the fractions is converting to equation strategy. In this strategy, the student converts the division operation into algebraic equation. T4's solution is presented below.



As it can be realized, the teacher wrote the result of division of fraction as an algebraic equation. Then, he converted division algorithm into multiplication algorithm by equating the multiplication of divisor and quotient to the dividend. Lastly, he solved two equations, 2.a= 4 and 3.b=15, to find the result which is represented as $\frac{a}{b}$.

As a result, 22 middle school teachers proposed 5 different strategies to divide the fractions. Although the most common strategy was invert and multiply strategy, converting fractions into decimal strategy and converting to equation were the least common strategies that middle school teachers addressed.

3.2 The Problems Posed by Middle School Teachers for the Division of Fraction

An adequate content knowledge requires being able to represent the subject appropriately and in multiple ways such as story problems, pictures, manipulatives, graphs and tables (Ball, 1990a). From this point of view, middle school mathematics teachers' content knowledge was examined through the problems that they posed related to the division of fractions. The teachers asked to pose a problem which can be solved by $\frac{4}{15} \div \frac{2}{3}$. Table 2 shows the categories of the problems that middle school teachers posed related to division of fractions.

Table 2: The Problems That Middle School Teachers Posed Related to Division of Fraction

Categories	**Frequency (%)**
Unable to pose a problem	4 (18.19%)
Inappropriate problems	8 (36.36%)
Appropriate problems	10 (45.45%)

As a result of the data analysis, it is found that 4 teachers could not pose a problem. During the interview, it was asked them to write a problem however they stated that the problem cannot be written with this algorithm. The excerpt from the interview of T17 was as follows:

"We cannot write any problem with these fractions since $\frac{4}{15}$ is smaller than $\frac{2}{3}$. How can we divide the smaller one to the bigger one. Therefore, I cannot pose any problem."

Additionally, 8 teachers (36.36%) wrote a problem solved applying the given algorithm but the context of the problems was not appropriate. As an example, the problem that T2 posed was presented below:

"How many bags of 2/3 kg are needed to put into 4/15 kg chickpeas?"

The problem can be solved with dividing $\frac{4}{15}$ into $\frac{2}{3}$. However, the number of bags cannot be a fraction.

Therefore, this problem was not appropriate story problem for the given algorithm. Similar problems were also posed by other teachers and the result of the division algorithm represented bottle, people, cups etc. Due to the fact that these can be represented by whole number, teachers' problems were regarded as inappropriate problems.

On the other hand, 10 teachers among 22 teachers (45.45%) posed appropriate problem in terms of the algorithm and the context. The problem of T10 was presented as an example below.

"Sedat divided his birthday cake into 15 slices. He separated 4 slices of the cake for his family. Then he ate $\frac{2}{3}$ of 4 slices. In this case, find the amount of cake that Sedat ate."

Five problems coded as appropriate problems were similar the given example which were asked the amount of small part within the big part. Additionally, the rest of 4 problems were very similar to each other. As an example, the problem that T18 posed was as follows:

"The phone has a 4/15 charge. How many hours will it take to end the charge if it is reduced by 2/3 per hour?"

As it can be realized, these findings indicate that more than half of the middle school mathematics teachers in this study had difficulty in generating an appropriate story problem to illustrate division of fractions.

4. Discussion

The aim of this study was to investigate middle school mathematics teachers' subject matter knowledge in term of the alternative strategies that they proposed to divide the fractions and the problems that they posed related to division of fractions. Based on the analysis of the data, five strategies were addressed by the teachers to divide $\frac{4}{15}$ by $\frac{2}{3}$. It is not surprising that the most popular strategy was invert and multiply strategy which was started by 91% of the participants. Although the teachers solved the given algorithm with invert and multiply strategy, they did not know why the divisor is inverted and multiplied. In this sense, it was concluded that the teachers focused on the particular rules rather than tending to search for the underlying meanings. One of the reasons for this might be that the division of fractions is mostly taught with invert and multiply strategy without teaching the underlying reasoning of the strategy and without making the students think the reasons for inverting and multiplying. Therefore, the teachers in this study most probably learnt dividing the fractions procedurally and used invert and multiply strategy when they were students. Furthermore, the teachers may assume that stating the rule is tantamount to presenting justification of the reasoning behind the rule. Similarly, Ball (1990b) emphasized that the prospective teachers got stuck in explaining the invert and multiply rule because of their inadequate knowledge. Also, in the study of Tirosh (2000), the prospective teachers knew how to divide the fractions, namely they used invert and multiply strategy, but could not explain the procedure. Similarly, Ma (1999) resulted in that the U.S. teachers had deficiency in explaining invert and multiply strategy. Contrary to the U.S. teachers, Chinese teachers had conceptual knowledge about the reasoning behind the procedure. They justified the division of fractions by stating that "*dividing by a number is equivalent to multiplying by its reciprocal*" (Ma, 1999, p. 49). Similar to Chinese teachers, the Turkish teachers in this study presented alternative strategies for division of fractions.

As Chinese teachers, the Turkish teachers divide the fractions using converting the fractions into decimals strategy and dividing the numerators and denominators among themselves. However, these strategies were not widely used in Turkish mathematics classrooms. The reasons for this might be that dividing the numerators and denominators among themselves cannot be applied for all problems unless the denominator and numerator of dividend is the multiple of those of divisor. On the other hand, this strategy is presented in the Turkish textbooks (MoNe, 2018) but most of the teachers did not know it. This can be interpreted as although the textbooks present variety of strategies, the teachers ignore the textbooks and the alternative strategies. They may only focus on the strategy which was valid for all problems. Regarding converting fractions into decimal strategy, apart from one teacher, other teachers could not present this strategy even though Ma (1999) stated that one of the popular ways of division of fraction was using decimals. In her study, one third of the Chinese teachers divide the fractions by converting the fractions into decimals. Ma emphasized that while converting the fractions into decimals makes the problem easier, converting the decimals into fraction makes the decimal problem easier, too. Contrary to Ma's thought, the Turkish teachers did not think that the decimals make the fraction problems easier. Parallel to the teachers' thought, this is not presented in the Turkish curriculum and the textbooks (MoNE, 2018). In other words, Turkish curriculum and textbooks did not present converting the fractions into decimals as an alternative way for division of fractions, however converting the decimals into fractions is one of the widely used strategies for division of decimals presented in the curriculum and textbooks. Due to the fact that the curriculum and textbooks do not address using decimal strategy as an alternative strategy for division of fractions, the teachers may not have used this strategy while dividing the fractions. Apart from these strategies mentioned above, more than half of the teachers solved the given algorithm using common denominator strategy which was also presented in the previous studies (Ma, 1999, Son & Crespo, 2009; Van de Walle, Karp, & Bay-Williams, 2013), and in the Turkish curriculum and textbooks (MoNE, 2018). As some teachers stated in this study, the teachers participated in the previous studies specified that while using common denominator, the division of fraction is converted into division of whole numbers. Because of this, the teachers claimed that common denominator strategy is the easiest strategy to divide fractions. According to them, the students do not have to memorize a rule, like invert the divisor and multiply it with the dividend and convert the fractions into decimals. They reported that all students know the multiplication of the whole numbers which is done to equate the denominators and then all of them know the division of whole numbers.

The interesting finding of this study is the converting to equation strategy. According to accessible literature, this strategy has not been presented in the previous studies. In the present study, this strategy was stated only one teacher. In order to convert the division of fractions into equation; she represented the answer of the problem as an algebraic statement and then solved the equation. This strategy is efficient way to divide the fractions and it is generalizable to all problems. Moreover, it is related to the solving the equations. Rather than memorizing the rule, the students connect knowledge of division of fractions with knowledge of solving equations. Additionally, common denominator, dividing numerators and denominators among themselves strategy and converting fractions into decimals strategy requires connecting different subject (division of whole numbers, division of decimals) with division of fractions. From this point of view, these strategies may provide students to learn division of fractions conceptually. For this reason, teachers need to know the alternative strategies in order to teach the students and make the solution of the problems more effective for students. Also, teachers' knowledge of alternative solution strategies is obligatory for teachers to address the reasoning about students' invented strategies (Fennema et al., 1996; Franke, Kazemi, & Battey, 2007; NCTM, 2014). In conclusion, the teachers' subject matter knowledge has a significant role in order to provide conceptual learning for students.

In this study, teachers' subject matter knowledge was investigated in terms of the problems that they posed in the context division of fraction. It is surprising that more than half of the teachers could not generate an appropriate problems with the given division algorithm. Some of them said that it was unable to pose a problem and the rest of them tried to pose a problem, however they could not pose any problems. This result let me to conclude that most of the middle school mathematics teachers had not adequate knowledge to generate a problem. Similar results were stated in the previous studies and they revealed the relationship between teachers' knowledge and their problem posing abilities. For instance, Ball (1990a) and Chapman (2002) emphasized that teachers' problem posing abilities is highly correlated with teachers' content knowledge. Also, Lin asserted that teachers could improve their problem posing abilities with their adequate content knowledge. More specifically, if the teachers cannot pose any problem, then it may be resulted in that their knowledge of mathematics is not sufficient to pose any problem. On the other hand, 45.45% teachers could pose appropriate problems. This means that these teachers had adequate knowledge to be able to represent the division of fraction appropriately with story problems.

The present study investigated the middle school mathematics teachers' subject matter knowledge in the scope of alternative strategies that they proposed to divide the fractions and the problems that they posed related to division of fractions. Based on the results, the study has some implications for teachers, teacher educators and curriculum developers. First, it is no beyond doubt that the teachers transfer their knowledge to their students. The more the teachers know the subject in conceptual way, the more their students know it in conceptual way. The teachers should have knowledge about the reasoning of the strategies even if they learnt them as a rule when they were a student. They should explain why this rule is meaningful and why it can be applied to the problems. Otherwise, students will not learn the subjects conceptually. In addition, it would be significant for teachers to know variety of solution strategies related to division of fractions to teach students. In this way, they expand students' knowledge of division of fractions and they give opportunities to students to select the strategy which is the most easiest and meaningful for them. Otherwise, all students have to solve the problems with only one strategy. Moreover, the teachers' problem posing ability is also important to pose different and challenging problems and to create opportunities to students pose any problems related to the subjects. In this way, teachers help students develop their mathematical understanding, enhance their creativity, improve their mathematical reasoning, discover the relationship among the mathematics concepts and formulate existing situations to the new situations (English, 1998; Silver, 1994; Stoyanova, 2003; Ticha & Hospesova, 2009). Also, while posing problems, the teachers improve their content knowledge through making relationship between the mathematical concepts, terms and the numbers. When the importance of teachers' knowledge of strategies, and knowledge required to pose problems is considered, it is revealed that the teacher educators have the responsibility to educate more knowledgeable teachers. Furthermore, in order to improve their knowledge of the alternative solution strategies of any subject of mathematics, the teachers may need to see them in the curriculum and the textbooks. Thus, curriculum developers and textbook authors need to provide alternative strategies and problem posing activities in the textbooks to help the teachers improve their knowledge.

Although the present study provided interesting findings that contribute to the literature and teaching practices, there is still more to do. In the present study, it was studied with middle school teachers. It is suggested that similar studies could be conducted with prospective teachers in order to reveal the knowledge of future teachers. In this sense, if their knowledge is not adequate to teach the division of fractions conceptually, then the teacher educators can teach the subject again and make them more knowledgeable. Also, similar study could be carried out with students in order to investigate what kind of strategies they develop and what kind of problems they pose. Moreover, future studies could be carried out with different mathematics topics to portray larger picture of students, teachers and prospective teachers' knowledge. As a final point, data might be collected via classroom observation to examine the strategies that they teach and the problem posing activities that they applied. In this way, information about how they present their knowledge related to both alternative strategies and problem posing during their teaching could be gained.

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