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PEDAGOGICAL-PSYCHOLOGICAL ASPECTS OF DEVELOPING THE SUBSTITUTION THINKING

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Abstract:
The primary objective of the educational process is the development of the thinking of learners. This paper deals with the development of learners' thinking in mathematics, in the transition from the acquisition of calculation algorithms to the acquisition of the algorithmic rules. Algorithmic rules need to be understood and it means the need to change the way of teaching. A substantial change in teaching is the use of question-answer system. This system is described in the paper in terms of pedagogy and psychology, and all these facts lead to comprehensive view of the application of the system in teaching mathematics. In the next part of the article we describe the features of learners thinking. When the method of substitution is taught properly, these features enrich the learners 'thinking. The method of substitution is the first algorithmic rule.

Keywords: thinking, algorithm, question, answer, method of substitution

1. Introduction

Development of human civilization goes hand in hand with the development of human thinking. Progress is not a repetition of well-known, but testing of new possibilities and finding the ways to replace current practices. When creating additional experience, the existing knowledge is overcome and human skills are more developed. Also new educational goals should always be, based on the individual abilities of the children, more challenging than the previous ones. Otherwise, we learn nothing new. At present, the human development is associated, while respecting his/her individual and social needs, not only with the improvement of his/her mind, but increasingly with the development of critical thinking and creativity.

It is necessary to enable people to build on extensive internal and external context so that they can think critically, act and work creatively. Internal context is

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characterized by mental and physical dispositions of a human being; external context is filled with the natural and social environment.

To think critically means to grasp the idea, examine it thoroughly, subject it to take a detached scepticism, compare it with other ideas and then make decision for answer, opinion and resolution (Klooster, 2000 Grecmanová, Urbanovská, 2007, p. 15). The human brain is better versed in diverse terrain from which it chooses the stimuli and information. The important things are associations, relationships, respect and progress, already accepted by J. A. Comenius, from simple to complex, from close to the remote, from the known to the unknown, from the concrete to the abstract and so on.

These principles are also important for Bloom's taxonomy of educational objectives (1956) and Tollinger´s taxonomy of educational tasks (1976). Bloom's taxonomy of educational objectives (1956) encourages the acquisition of knowledge and the development of intellectual abilities from the lowest to the highest goals. At the beginning it is important to remember that we have a chance to understand the content, and then we are able to apply, analyze, synthesize, evaluate and finally create. To achieve these objectives there are helpful learning tasks that Tollinger (1976) also categorized by degree of difficulty, ranging from commemorative acquiring through a simple mental processes of knowledge (detection, enumeration and description of facts, procedures, processes, relationships among them, comparison, abstraction, generalization etc.) to tasks requiring complex mental operations (transformation, interpretation, induction, deduction, verification etc.) and jobs to the communication of knowledge (drawing review, summary, content, project, etc.) or jobs requiring creative productive thinking (applications, problem solving situations, questions and tasks creating, tasks of discovering by observation and reasoning). When solving problems, it is necessary to understand their tasks, e.g. what are the main characters, fill in, join expressions, suggest a solution, what is the relationship, apply, split, compose, justify what happens - when etc. Mostly it is not possible to work only mechanically, without understanding the meaning of learning. Mechanical learning tasks should serve as a means to meet the higher targets.

Algorithm should be only revised, algorithmic regulation should be understood. Also in the teaching of mathematics there is not the final aim to teach learners some procedures to be used according to the type of task, but teach them to seek a solution to a specified task using already acquired mathematical knowledge and skills. It is proper to teach them to think how to continue in a solution, to disallow them to count automatically only "memorizing" learned process. The learners should be able to see forward, visualize how the solving of the task will develop, and how they will do this or that step. Very common mistake is that the learners think how the task was solved during the last lesson, what the procedure was used. Mathematics teaching is directed to the realization that solving the problem (not just mathematical) does not lie in the repetition of learned, but it is a process during which solution is created.

The process of solving the problem requires considerable creativity and flexibility, but also tolerance for ambiguity of other procedures. To be able to solve the problem well, we have to identify it correctly in the first phase, then define it properly and establish adequate mental representation.

Consequently, it is necessary to formulate appropriate strategy for solving the problem, including the processes of thought analysis, synthesis, divergent and convergent thinking. Strategies consist of decomposing a complex issue to the elements and meaningful parts, of refolding into a new meaningful whole, of producing a variety of ideas and alternative solutions, so that we can select or create the best solutions. In subsequent phases, it is equally important to organize information that we have available, plan the time management, efforts and resources that are available, and subsequently monitor and control the intermediate results and finally evaluate the process in identifying new realities and challenges. In the process of searching for a solution of a mathematical problem, it is important to join two areas of knowledge which learner gain gradually. The first area consists of theoretical knowledge and the second area is basic calculation algorithms (modification of the algebraic expressions, solving equations, inequalities and their systems etc.). In the case of solving mathematical calculation task, the first step will be the analysis of defined problem, associated with identifying the problem and assigning the appropriate theoretical knowledge. Subsequently (if desired) the task is mathematized, and then the equation, inequalities or their systems are created. The process of task solving will culminate with using adequate calculation algorithm and verifying the correctness of the result.

Based on the successful task solving, the individual acquires the algorithm of solutions, i.e. the exact sequence of particular steps that lead to the solution of the problem. It means that a learner has "an open door" to the solutions of similar, corresponding problems. R. Sternberg (2002) talks about the isomorphic problems. As each individual imposes more and more algorithms in his/her cognitive structure, we would expect that he is able to solve even wider range of problems. Not always, however, a person is able to see the similarity of problems. According to Sternberg (2002) it may be demanding for school children to see the structural similarities between different verbal tasks that are specified in different situational scenarios. A lot depends on whether the problem is properly represented and whether it is properly defined. Then, there can be positive transmission, which facilitates solution of the new problem. At transferring analogy, similarity or a close resemblance to the structure of relationships is proving very important. When an individual has acquired knowledge well anchored, when he/she is active in the process of acquiring, he can compare and find similarities and differences, all these facts help to positive transfer. People may have problems with the awareness of the similarity of the problems, if we do not draw their attention to this similarity. So even children do not often realize the connection and similarity of problems solved in various school subjects (mathematics, physics or chemistry, history, literature etc.).

However, it may also lead to negative transmission, when already acquired algorithms make the further learning process more difficult. We encounter with this phenomenon in dealing with others, thus distinct, dissimilar and not corresponding problems. Previously acquired algorithms can hinder creative, original approach. It can lead to stick to one (or more) of the learned algorithm, which is inappropriate, unwanted, ineffective in this situation. Reproduction and the use the old algorithm is counterproductive. In dealing with the new challenges it is not therefore necessary the reproductive thinking, but productive, creative thinking, detachment from the existing ways of solution, of existing associations and ability to see the problem and its components from new perspective.

So called "incubation" help the elimination of obstacles in the form of negative transfer. It means to forget the problem, push it aside, not to think about it for the certain time. It helps its subconscious processing. Incubation phase is considered a necessary part of the problem solving process in which we give up the unimportant ideas, get new incentives, and the old associations and algorithms weaken.

Algorithms occur in two forms in the maths. Calculation algorithm is considered to be a series of precise and unambiguous steps. Their successful transfer enables the successful solution of the task. Everyone who learns the appropriate sequence of steps will reach the accurate result. The utility of calculation algorithms, however, is limited to the same types of tasks. The acquired algorithms will later become part of algorithmic rules, which individual operations have no formal character. Rather, they have content character associated with understanding their sense. If an individual knows why he performs the particular operations, and why in that sequence, it will lead to the solution of tasks. Algorithmic regulations are often learned by learners as the algorithms. Thereby they lose their versatility and the possibility to use in different types of tasks. The different ways of using calculation algorithms and algorithmic rules in the teaching of mathematics can prevent their equating. The primary aim in calculation algorithms is to know how to repeat the algorithm. The appropriate method to teach them is the reproductive (didactic) method. The individual operations that form algorithmic prescription are necessary to be understood, therefore, the heuristic method can be selected to teach them. In this method, a teacher is in the role of moderator, who directs the formation of a solution. He uses appropriately formulated and directed questions as the tool.

2. Transformation of a learner –perceiver to learner- thinker by question answer the system

Creative approach to mathematics, and not just to mathematics, consists in finding answers to questions. And the questions should not be directed only to the past but above all to the future. In terms of learning psychology, the relationship between asking questions and thinking is reversible. Also, the questions formulated to solve problems in mathematics stimulate mental activity; affect the way and the depth of thinking. Differently formulated questions activate various cognitive processes and mental processes of different levels of difficulty.

So a lot depends on what type of questions a teacher in mathematics asks and whether he initiates the process of learners' independent creation of right questions. A teacher communicates what is important according to him, what he appreciates, by using questions (Grecmanová, Urbanovská 2007, according to Steele, Meredith, 1991). Teacher´s questions that require only a reproduction of the learned material (mathematical processes), leave learners know that it is useless to think independently. But the questions, that motivate to reflection, speculations, making proposals, alternatives and careful consideration of the various options, increase the level of learners´ thinking. Learners learn that their thinking is valuable and inspiring, and therefore it is possible to use different ways to come into conclusion.

When looking for ways to efficient and emerging questioning in mathematics we can be inspired by Sanders´ reprocessing of Bloom's taxonomy of questions. In accordance / compliance with Bloom´s taxonomy of educational objectives, which have already been highlighted, as well as questions organized from the lowest level of questioning and thinking (questions requiring a literal response, fact recollecting, remembered reproduction) through questions of interpretation (requiring meaning understanding, understanding the causes of phenomena , understanding the logical connections), application and analytic questions to the highest degree of intellectual activity, which is seeking the answers to synthetic and evaluational questions, requiring creative problem solving and formulating their own judgments. There is the real place for any type and level of questions in complex educational process, because they lead to the investigation of the problem at different levels, to the involvement of various types of cognitive processes and thinking.

To develop critical and creative thinking that is the aim in mathematics, the key questions are interpretative questions. They incite inquisitive and speculative thinking. Application questions that give learners the opportunity to find other possible uses, to solve problems, uncover new uncertainties and analytic questions, enabling clarify the problem, divide each component aspect of the problem, essential and secondary characters, and belong to the key questions. Synthetic questions, looking for alternatives, stimulating creative problem solving, requiring independent thinking, are very important, as well as evaluational questions, which encourage taking a stand on the judgment according to their own evaluation criteria (Grecmanová, Urbanovská 2007, according to Steele, Meredith, Temple, Walter, 1998). As a result of the asked questions, the learners begin to think independently, use all previously acquired knowledge creatively and defend their own opinions and ideas. In addition, such questions develop the ability to anticipate, to formulate hypotheses, to suggest alternative processes and ultimately increase learner interest in the acquisition of new knowledge, looking for new avenues and opportunities, the deeper exploration of relationships and connections.

It is important, for the developing way of questioning, to know how to ask the right question at the appropriate time, to choose the appropriate ratio between closed questions and open questions that force learners to think, to choose challenging and interesting questions, and such questions that evoke thought-inducing productive activity. Such questions lead to the independent discovery of relationships and connections, principles of organization and principles of solution.

Well chosen questions help creative approach to the problem solving.

The questions are divided into several groups according to the structure of mathematical task solving. The first group consists of questions aimed at identifying the problem and the correct classification of the structure of mathematics. The second group consists of questions that help the learner to recall the already acquired knowledge, which could contribute to tasks solving. The third group of questions is critical for tasks solving. This includes questions aimed at making the procedure of solving itself solutions. Based on these questions we not only choose the solution, but also evaluate the solution if it is reasonably practicable. It must be assessed whether it leads to the final result of the task. The last group consists of the questions, asked after solving the task. They head towards the future. These questions help to assess the novelty of solutions as well as their potential use in other types of tasks.

Even the putting the questions and searching for answers to these questions has its own specifics in mathematics. First of all, it should be noted that the question, which is a part of the defined mathematical task, is not the only question that needs to be answered. The learner should be aware that he will first need to answer several sub-questions. The calculation results are often the answer to the question. But the result of the last calculation may not be the answer to the question in the task assignment. So at the end of task solving there is a necessity of the interpretation of result. This means that the learner has to verbally formulate the numerical result and verify that it fulfils the task conditions. Another specification of the mathematical task solving is selecting the following process of solving. The learner has an idea about the next step of solving. He/she needs to know how to evaluate whether a given step is in the right direction and at the same time, whether it is mathematically correct. The learner should evaluate the accuracy or inaccuracy of his/her step himself. This is an important reflection the person who learns something new. We think that the teacher should not be a judge, who will decide instead of the learner. The learner should be responsible for his/her learning process, so he should be aware that his/her ideas basically created a new elementary problem. Then he/she realizes that it is necessary to leave the solution of the defined problem and fully devote to the solving of the created elemental problem. The procedure is particular, from parts to the whole. In summary, solving math problems is divided into several elementary tasks by a pair of question - answer. Basically, we can talk about the role of atomization. Each atom, which comprises the task solving, is substantially a single task. If the teacher teaches how to ask questions and seek answers to these questions, teaches learners to understand mathematics. But not only to understand the math, but to solve problems in different areas of life using the thinking and their knowledge and skills. Applying constructivist teaching methods help develop learners critical thinking.

One of the problems of mathematics learning is when a learner is convinced that if he/she knows the task solution, he/she also knows how to solve it. We think that in this case, the learner can just repeat the procedure of task solving and just for the same or a similar assignment. The learner is able to solve the task if he can justify the individual steps (atoms, particles) of the solution. The individual steps are the result of a process „question – answer“. Then we can say that the learner understands the task solution. The teaching of mathematics is primarily aimed at developing the learners´ thinking. One of the objectives is to create the concept of mathematics as the one, inter-connected whole in the mind of the learner. The result of the teaching of mathematics should be a learner, who is able to create a solution of the adequate task based on the acquired knowledge and skills, using the question - answer system.

3. Substitution thinking as a new form of approach to tasks solving

Mathematical cognition is based on the first stage of acquiring basic theoretical knowledge related mainly to the need to learn the basic calculation algorithms. New theoretical knowledge will be added in the next stage. Already learned calculation algorithms, or their modifications, are mostly used when we solve the tasks. The stage of mathematical cognition is a space for construction of mathematical knowledge with the „question-answer system". This system motivates the learner to become the co-creator, and later even the creator of the task solution.

The basis of the effectiveness of teaching the „question -answer system"is correct and common understanding not only of the mathematical concepts. The correct choice of teacher´s questions can reveal different misconception created in the minds of learners. The misconceptions are the obstacles of successful solving of the mathematical tasks. The „question – answer system"also has preventive effect. Teaching the „question - answer system" means a dialogue and common thinking over the problem. Not only a teacher but also the learners express their opinions about the task solving. A teacher follows the gradual creation of new concepts of the terms and emerging misconception can now be removed.

Nowadays the learners are in the position of the recipient of the knowledge during the lessons. It is quite difficult to motivate learners to think and seek answers. We consider it appropriate at the beginning of the task solving to choose questions, for which it is assumed that learners already know the answer. Such oriented questions have strong internal motivational character. Learners realize that they already have the knowledge that could be used in the task solving and they could therefore be successful in the solutions. The questions should be very specific with the simple answer. To motivate learners to answer the teacher's questions it is very important to remove the negative sense of wrong answers. It is necessary to create the atmosphere of equivalence of good and bad answer. Each response or proposal for the next step of the solution is always the subject of discussion between the teacher and the learners and among learners themselves. The teacher should not act as a "judge" to decide on the correctness or incorrectness of the answers. Rather, the teacher should act as a moderator of the solution and again, using the appropriate questions, facilitates learners to the evaluation of correctness or incorrectness of answers or suggestions for further solutions. The teacher should show the learner how to work with an error as a natural part of learning. This is the way how the learners learn to evaluate their own thoughts and ideas. At the same time the teacher removes the misconception that only that learner is good at mathematics who immediately finds the right procedure of solution. The misconception is gradually replaced by the notion that an important tool on the road to mathematical success is the ability to ask questions properly and patiently.

Questions can be divided into several groups according to the phase of task solving. At the beginning of task solving, the teacher "gathers" already acquired knowledge about the task by asking the appropriate questions. "What do I know, what can I say to that?" Then, the questions come, like "How to use what I know about solving the task? " The answer is largely equations, inequalities or their systems. These types of questions follow: "How to solve them?" The answer is the use of a suitable calculation algorithm. The final question is: "What does it mean? " The answer is the interpretation of the calculation results in terms of task assignment. We get the basic skeleton of questions: What do I know? What will I use? What method of solving will I use? How will I interpret the result of the calculations?

In the phase of acquisition of basic calculation algorithms of solving equations, inequalities and their systems is a space for teaching question - answer system limited. Learners do not need to ask the above questions. At the stage of the mathematical knowledge acquisition they learn that to solve the task, it is sufficient to assign the correct calculation algorithm. However, there are a number of equations and inequalities, which cannot be solved in this way. These are the equations and inequalities solved by the method of substitution. They can become the first step in teaching algorithmic rules by system question - answer. It will cause that the mathematical thinking of learners will take a new direction. The learners gradually begin to realize that the task can be divided into several sub-tasks whose solution is more or less independent of the rest of the task. They recognize the opportunity to transform the task to the task solution from different area of mathematics. The teachers should include the tasks solved by substitution more frequently and at the time of the training of basic calculation algorithms, the new form of learners mathematical thinking – substitution thinking is formed. The name "substitution thinking" is derived from the teaching method "substitution". The teaching of substitution stands on the transition from the calculation algorithms to the algorithmic rules requiring a change of thinking. The learners with advanced substitution thinking are aware of the fact, that it is not important to remember the process of task solving but solving principles. Properly developed substitution thinking leads to creative tasks solving, to putting the right questions and to seeking the correct answers.

As essential elements of the substitution thinking may be considered:

1. The ability to split the task into elementary parts - atomize task solving. The process analysis is a fundamental thinking operation, which is necessary for further handling with the mental contents. In many cases, the learner after reading the task assignment tries to assign any of the learned ways of solving to the task, and this way leads to a final task solution. If he fails to assign the learned process of solving to the task he does not even start solving the task. The most common reason given in the questionnaire was: "We did not solve such a task in mathematics lesson." Learners, whose substitution thinking was developed, are not primarily looking for the total solution of the task but they determine the partial objectives of solutions. They realize that after reaching their stated objective of solution they will stand before a new, often easier task than was in the assignment, and, at the same time, they came closer to total task solving. Such an approach also leaves creative tension that increases the level of motivation to complete the task. They perceive the task solution as their own creative problem and at the same time they built a mathematical confidence to be able to tackle the new tasks with which they did not meet in mathematics lessons.

2. To realize, that solving math problems might not be monothematic - it is the ability to combine information from several parts of mathematics in solving one task. In achieving the above mentioned first level of mathematics objectives the learners get the idea that the solutions of mathematical task are monothematic. They learn, for example, to make basic operations with rational numbers, modification rational expressions, solving linear and quadratic equations etc. To acquire the necessary basic skills they deal with multiplicity of monothematic tasks. And at this stage of mathematics education it is recommended to remind learners of, for example, for solving linear equations they use the knowledge and skills they have acquired when they fixed the expressions. A monothematic view of task solving is restrictive in further study of mathematics. Based on the advanced substitution thinking the learner knows that in solving tasks the knowledge and skills from different areas of mathematics can be, and often it is also necessary, to import knowledge and skills. The connection between efficiency of the learning process and troubleshooting with the acquired knowledge from various fields is formulated as one of the general rule of the learning process, as the rule of the transfer. To support positive impact of previously acquired knowledge, skills, experience, it is important to allow and encourage the evocation of the findings, for example, by noticing of the possibility of their use. A non-monothematic view to solving the tasks is closely linked with the ability to atomize the task.

3. To find effective ways of task solving - to simplify or transform the role. While practicing numeracy a learner often meets with the term “simplify” right in the assignment. The results of such tasks are shorter writing assignments. If there is no word “simplify” in the assignment, the learners often “forget” to simplify during task solving. They do not realize that simplify means to create a mathematical object with which the work is more effective. The learners with advanced substitution thinking approach solving the task creatively and try to optimize the task solution, to find the easiest way to solve the specified task. Using the substitutions is a powerful tool in optimizing the task solution.

4. To identify the primary problem of the phase of task - solution and then solve it. In applying this element of the substitution thinking the learner solely focuses on the problem that he set. Subsequently, the learner faces the new task, from which the difficulty was “removed”. It was the difficulty that impeded on the way to an overall task solution. Such solution phasing splits it into the autonomous several stages, which, in many cases, might be atomized. With this attribute of the substitution thinking the learner is able to determine his/her own goal, which is usually different from the objective to be achieved by the assignment.

5. Being able to import the knowledge and skills from different areas of mathematics in solving problems - pass on the basic idea of solutions to different type of problem. The learner who has got this mathematical competence is able to solve mathematical tasks easily. The learner is not limited by barriers of mathematics, to which the task according to assignment belongs. He realizes that in mathematics we often meet with different names for the same operations. The learner with developed substitution thinking understands, for example, that to look for intersections of the graph of the quadratic function with the coordinate axis x means to solve the quadratic equations. And the obtained roots are x-coordinates of the searched intersections. Many maths problems can be effectively solved by such a mathematical detached view, which assumes the combination of different areas of mathematics in the mind of the learner.

6. To choose the most effective task solution among several options - The purpose of introducing the substitution is to simplify the task, make the solution more efficient. Often the introduction of the substitution is not required, and it is up to the choice of the solver whether or not the substitution will be used. The learners who are taught to prefer the substitution, know, that thanks to the substitution they can simplify the task, and they are supposed that the substitution algorithm is perfectly mastered by them. During lengthy calculations the question emerges in the mind of the learners: “Would it not be possible to solve the task in an easier way?” They are used to searching and creating not only mechanical counting. Then, they often find new, more effective solutions. They can even surprise the teacher with these solutions. Many of them can imagine in their mind where the particular paths of the solutions lead. This idea makes it easier to select another process of the task solution. Thanks to this advanced mathematical imagination the learner is competent to see the substitution expression before it is actually created by modifications.

It is clear from above, that the substitution thinking combines the elements of creative and critical thinking. When we develop the substitution thinking, constructivist teaching approach can be applied, when teaching goes through the three phases. The phases are full of various questions and challenges and aim at fulfilling the educational goals. These phases are referred as: evocation - meaning realization - reflection. The learners are encouraged to active approach in solving problems in evocation. The teacher stimulates their thinking with questions: What do you know about this task, what do you think you know, what your associations are, when the task is defined which questions does the task evocate, what would you like to learn about the problem. Learners are thus motivated to find a solution of the task, and it is not possible without their participation and thinking. In the second phase there is the acquisition, classification, systematization and structuralization of new knowledge and the creation of links with previous knowledge or experience. There we suppose the reconstruction of existing thought maps as a graphic cognitive structures, and it continues in the third phase - reflection. Reflection also brings serious findings about how the questions were answered, what responses were confirmed and which ones were excluded, what new the learners learned in problem or task solving, what procedure they used and how they were thinking about the task. Learners deal with meta thinking.

If the teachers apply the above described three-phase model of learning together with adequate activisation and developing methods in the particular mathematic lesson, they can constantly monitor the learners' learning progress and lead them in the whole process.

4. Conclusion

Psychologically, the thinking is one of the cognitive functions of personality and it means a function of higher order. Thinking itself builds on the results of the operations of lower cognitive functions such as perception and imagination. Although thinking builds on the sensations and ideas; but it is related to the relationships and dependencies between them, they are not perceptible nor imaginable and they mediate more broadly, deeply, precisely - awareness and understanding of the world. (Košč, L., 1972) Education should be directed to the development of personality of the learners and to each of its sites. In maths lessons the teaching, also with regard to its abstractness, focuses on the development of learners' thinking. It focuses on detection and awareness of the relationships and dependencies between phenomena, and thus makes it possible to explore the essential and universal facts and also reveals new, often very complex features of reality. Knowledge of relations between phenomena and their generalization by thinking, enable their effective use in solving the different tasks. A teacher has the great possibility to form and develop learners' thinking. In teaching mathematics at high schools, it is important to transform a learner-perceiver, acquiring calculation algorithms, to learner-thinker, acquiring algorithmic rules.

This process should begin at the time when the teacher explains solving the equations and calculation algorithms and algorithmic rules (substitution method) start to overlap. The teacher, using the question-answer system, teaches learners, that it is no longer enough just to perceive the lesson interpretation of the curriculum, but it is also necessary, when acquiring new knowledge, to think. Thinking is associated with understanding, and, at the same time le a learner realizes that new knowledge is often created by connecting already acquired knowledge. This is the way how the mathematical confidence of a learner, as the creator of solution, is formed. It forms mathematical learners'confidence. The above-described substitution thinking presents the key elements of thinking skills necessary for independent creation of the task solution. This way of thinking is the result of purposeful teacher work and frequent assigning tasks that are solved by the method of substitution. The goal is not the knowledge how to use the substitution, but the learners thinking should obtain the attributes described above. Substitution thinking goes beyond mathematics and affects solving the tasks of daily life. The substitution thinking also economizes the process of perception, setting aside and generalizing substantial or significant in some way from the process. So the learners´ learning becomes more effective.

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