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FOURTH GRADERS AND NON-ROUTINE PROBLEMS: ARE STRATEGIES DECISIVE FOR SUCCESS?

Yeliz Yazgan[i]
Assist. Prof. Dr., Elementary School Department,
Uludag University Faculty of Education, Turkey

Abstract:
This study aims to determine the explanatory and discriminative powers of non-routine problem solving strategies used by fourth graders. Six problems were asked to 240 pupils. After scoring answers between 0 and 10, bottom and top segments of 27% were determined based on total scores. Lastly, all scripts of students in these segments were re-scored with regard to strategy use. Multiple regression and discriminant analysis were utilized to evaluate data.
Results showed that strategies explain 84% of the problem solving success, and their importance order is as follows: look for a pattern, work backward, make a systematic list, make a drawing, guess and check and simplify the problem. Strategies which play a significant role in distinguishing students are look for a pattern, make a systematic list, work backward, simplify the problem, make a drawing, respectively. Further researches using more problems, and examining students of different grades are suggested.

Keywords: non-routine problems, problem solving, problem solving strategies, fourth grade students, students' success

Introduction

We are consistently confronted with problems on a daily basis such as deciding which kind of shoe is more suitable for us, reaching an address we have ever been, planning a birthday party, and deciding for whom to vote in an election etc. Whether they are

[i] Correspondence: Uludag Universitesi, Egitim Fakultesi, Ilkogretim Bolumu 16059, Nilufer, Bursa, Turkey, e-mail: yazgany@gmail.com, GSM: +90 538 6751775

mathematical or not, these examples share a common core: *"A problem arises when a living creature has a goal but does not know how this goal is to be reached"* (Novick and Bassok, 2005; p. 321). Therefore, problem solving has been described as what you do when you don't know what to do (Bilstein, Libeskind & Lott, 1996).

Since it has so much importance in our daily live, it is inevitable to say that problem solving is the cornerstone of school mathematics, and without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge, and skills severely limited [National Council of Teachers of Mathematics (NCTM), 2000]. Besides, mathematics education communities commonly agree that teaching problem solving means teaching non-routine problems as well as routine problems. Actually, a large body of literature about mathematical problem-solving show that non-routine problems are the kind of problems which are most appropriate for developing mathematical problem-solving and reasoning skills, and development of the ability to apply these skills in real-life situations (e. g; Polya, 1957; Schoenfeld, 1992; Cai, 2003; London, 2007). Because, routine problems can be solved using methods familiar to students by replicating previously learned methods in a step-by-step fashion.

The followings are examples of one-step and two-step routine problems respectively:

- *A bag contains 2 dozen cookies. Andy bought 3 bags. How many cookies did she get?*
- *A book store purchases 3 packages of books, and each package includes 12 books. If price of one book is 8 liras, how much do all books cost?*

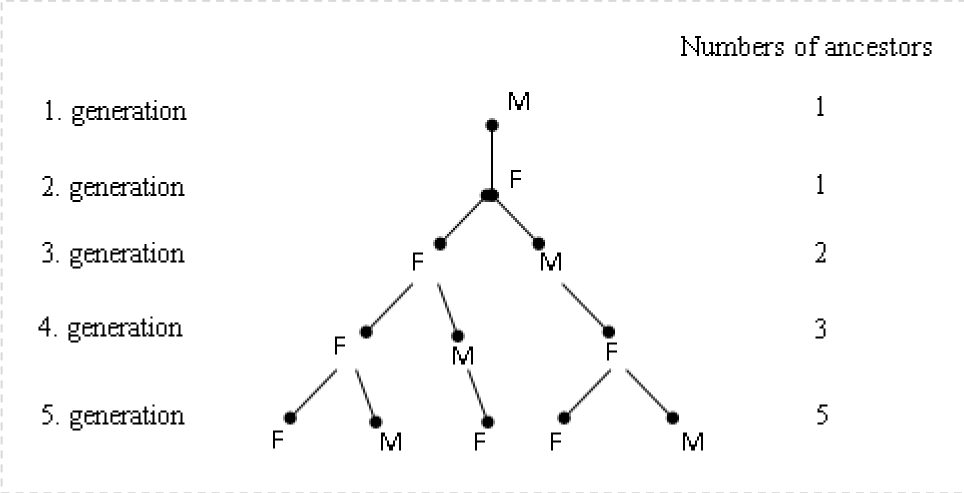
Non-routine problems are problems for which there is no predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or worked-out examples (Woodward, Beckmann, Driscoll, Franke, Herzig, Jitendra, Koedinger, & Ogbuehi, 2012). Importantly, non-routine problems need reasoning and higher-order thinking skills and often go beyond procedural skills (Kolovou, van den Heuvel-Panhuizen & Bakker, 2009).

To have a more detailed opinion about what a non-routine problem is, following problem and its solution can be revised:

"A male bee is born from an unfertilized egg, a female bee from a fertilized one. So, in other words, a male bee only has a mother, while a female bee has a mother and a father. How many total ancestors does a male bee have going ten generations back?"

This problem can be solved with the help of the diagram in Figure 1. In this diagram F means female and M means male.

Figure 1: Solution of the non-routine problem sample



Without completing this diagram until the tenth generation, solution can be reached by using the pattern among numbers of bees in every generation: Add the first and second number (1 + 1 = 2) to get the third number, add the second and third number to get the fourth number (1 + 2 = 3) and so on. According to this pattern, solution of the problem is 55.

Non-routine problem solving strategies can be defined as procedures used to explore, analyse, and probe aspects of non-routine problems in an attempt to formulate pathways to a solution (Nancarrow, 2004). They play a very important role in the mathematical process experienced by students while solving non-routine problems. Results of recent studies have provided evidence for the use of strategies as a means to enhance problem solving (Elia, Van den Heuvel-Panhuizen, & Kolovou, 2009). In the literature, the most outstanding non-routine problem solving strategies are as follows:

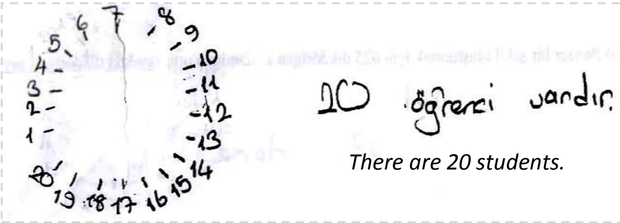
Act it out, look for a pattern, make a systematic list, work backward, guess and check, make a drawing or diagram, write an equation or open sentence, simplify the problem, make a table, eliminate the possibilities, use logical reasoning, matrix logic, and estimation. (Altun, Bintas, Yazgan & Arslan, 2007**;** Herr & Johnson, 2002; Leng, 2008; Posamentier & Krulik, 2008; Posamentier & Krulik, 2009).

However, most renowned ones that are in use at elementary school level will be presented here:

a) Make a drawing or diagram: This strategy includes using supportive representations to solve problem. It provides a way to depict the information in the problem and make the relationships apparent. In the following, there is a problem that *making a drawing* strategy seriously facilitates to reach the answer in solving process, and a fifth grader's script about the solution of the problem (Figure 2):

Students in a class are standing in a circle; they evenly spaced and are numbered in order. The student with number 7 is standing directly across from the student with number 17. How many students are in the class? (Bilstein, Libeskind, & Lott, 1996; p.49)

Figure 2: A solution including *"make a drawing or diagram"* strategy



As understood from what the student did, drawings do not need to be detailed or exact. The only things that need to be drawn are what is essential in solving the problem (Reys, Suydam, Lindquist, & Smith, 1998)

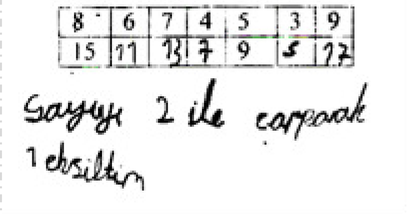
b) *Look for a pattern:* Finding patterns includes items or numbers that are repeated, or a series of events that repeat. It enables persons to reduce a complex problem to a pattern and then use the pattern to derive a solution. Often the key to finding a pattern is to organize information. Following problem is a good example for which *look for a pattern* strategy can be utilized solely:

*An explorer found some strange markings on a cave wall. Can you find and complete the pattern? (*Reys, Suydam, Lindquist, & Smith, 1998; p.77).

5	6	1	3	0	8	16
11			7			

In Figure 3, a fourth grade student's answer to this problem is presented. The student explained the pattern that he found by writing.

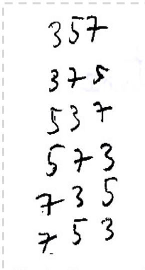
Figure 3: A solution including *"look for a pattern"* strategy



I multiplied the number by 2, and then subtracted 1.

c) *Make a systematic list:* This strategy is exactly what the name implies: making an organized list that requires some method of counting things or making certain that every possibility has been covered. Sometimes simple counting of the different ways or situations would be impossible. As can be seen in the following answer of a sixth grade student to "*How many three digit numbers can be formed from the digits 3, 5, and 7, if each digit can only be used once?*" problem (Figure 4), an organizational scheme guarantees that all thing required for counting have been included, and none were counted twice (Van de Walle, 1994; p.50).

Figure 4: A solution including "*make a systematic" list* strategy

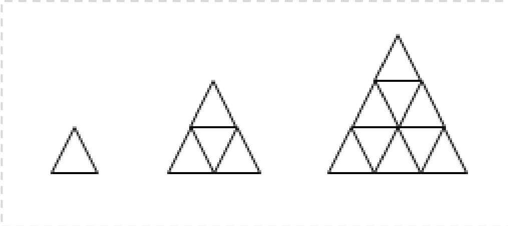


d) *Guess and check:* When using this strategy, a person makes a reasonable guess, checks the guess, and revises the guess if necessary. By repeating this process someone can arrive at a correct answer that has been checked. Using this strategy does not always yield a correct solution immediately but provides information that can be used to better understand the problem and may suggest the use of another strategy.

In Figure 5, an eighth grade students' solution to the question of *"Tolga's team entered a mathematics contest where teams of students compete by answering questions that are worth either 3 points or 5 points. No partial credit is given. Tolga's team scored 44 points on 12 questions. How many 5 point questions did the team answer correctly?* (Bilstein, Libeskind, & Lott, 1996; p. 49)" is presented.

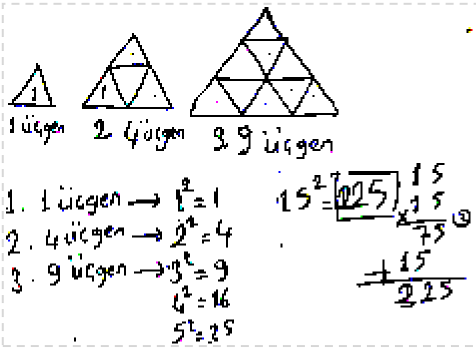
Figure 5: A solution including *"guess and check"* strategy

e) *Solve a simpler or similar problem:* When problems that seem to be overwhelming due to their complexity and/or large numbers are encountered, it may be useful to try the same problem with much smaller numbers of fewer conditions. In this situation, one of two things will happen: (1) solving the easier problem may help to see a way of original problem; or (2) there is a chance that working a series of simpler problems, starting with the easiest possible and systematically increasing the difficulty, may lead to a pattern or generalization that will solve the original problem (Van de Walle, 1994; p.52). For the question presented in the following, this approach may really work: *Each* *of following shapes consists of small triangles like the first one. How many triangles do you need to make fifteenth shape?*



A fifth grade student' solution represented in Figure 6 is completely convenient to the essence of *solve a simpler or similar problem* strategy.

Figure 6: A solution including *"solve a simpler or similar problem"* strategy



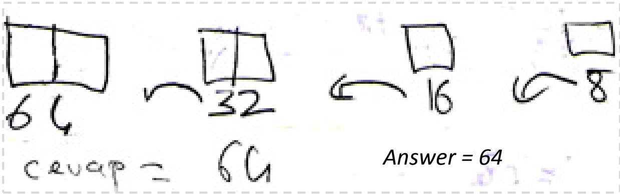
Üçgen= triangle

f) *Work backward:* Some problems are posed in such a way that people is given the final condition of an action and are asked about something occurred earlier (Reys, Suydam, Lindquist, & Smith, 1998; p.80). In another words, the work backward strategy is useful when a series of events takes place and we know the result but need to determine the condition at the beginning. If the events are arithmetic operations, the task is to reverse these operations.

In other problems a series of unknown events results in some condition, and the task is to find out what the sequence of events is. If not all steps are obvious, occasionally the last one is relatively clear. From the last step you can then get to the next to last, and so on (Van de Walle, 1994; p.51), like a fifth grader did to solve the following question (Figure 7):

A bus driver started to drive from the terminal and dropped half of his passengers off at every bus stop he stopped by. Meanwhile, no passenger got on the bus. After stopping by 3 bus stops there were 8 remaining passengers on the bus. How many passengers got on the bus at the terminal?"

Figure 7: A solution including "*work backward*" strategy



According to Tiong, Hedberg and Lioe (2005), the most recognizable and immediate difference between non-routine problem solving strategies and algorithms is that non-routine problem solving strategies do not guarantee solution, while algorithms (if the procedures are followed exactly) guarantee a solution. Another and perhaps more important difference between the two is that algorithms come with very specific procedures, while non-routine problem solving strategies have non-rigorous suggestions about what should be done. Also, algorithms are usually applicable to a certain type of question in a specific topic area, while non-routine problem solving strategies are generally applicable through types of question and topic areas. This raises another important characteristic of non-routine problem solving strategies, that of *transferability.* The most important reason to learn them is that because they can help individuals solve problems in unfamiliar topic areas and expand their point of view.

In spite of the above mentioned importance of non-routine problems and non-routine problem solving strategies, their place in math education in Turkey is limited. In the general explanations at the beginning of elementary school mathematics curriculum developed by Ministry of National Education (MoNE), *guess and check, make a systematic list, make a drawing, work backward, simplify the problem* strategies are merely mentioned, and just *look for a pattern* strategy were taught under the title of "Patterns and Tessellations" (MoNE, 2004). Findings of nationwide research reveal that non-routine problems, and strategies used to solve them, are not introduced to students extensively in textbooks nor learning environments (Altun et al, 2007; Yazgan et al., 2013).

Importance of the Study and Research Questions

Most research about non-routine problem solving is aimed at examining students' currently used skills and attitudes on this subject without any intervention (e.g. Hok-Wing & Bin, 2002; Wong & Tiong, 2006; Muir, Beswick, & Williamson, 2008; Wong, 2008; Elia, et al., 2009; Salleh & Zakaria, 2009; Mabilangan, Limjap & Belecina, 2012). There are also some studies examining effects of an instruction on non-routine problem solving skills of students (e.g. Follmer, 2000; Ishida, 2002; Nancarrow, 2004; Yazgan et al., 2005; Johnson & Schmidt, 2006; Lee, Yeo, & Hong, 2014). Another group of studies is focused on place of non-routine problems and strategies in mathematics textbooks and syllabi (e. g. Tiong et al., 2005; Kolovou et al., 2009; Marchis, 2012;). Grade levels of these studies vary from primary school to high school and their results can be summarized in three points: (i) many students consider that non-routine problems are more complicated and difficult than routine problems, and students may not initially believe that the non-routine problems are mathematical since they are not familiar with this kind of problem, (ii) providing students with a framework for the use an application of strategies is beneficial and increased students' level of confidence, (iii) only a very small proportion of the problems included in the textbooks is non-routine. In some textbooks series these problems are completely absent.

Still, studies investigating the impact of each of non-routine problem solving strategies on the problem solving success of students are really rare. Moreover, none of them are at fourth grade level (age 9-10). However, determining importance order and distinctiveness of each strategy in terms of problem solving success may bring some important outcomes out and they can be used to arrange textbooks and learning environments. Therefore, this study first aims to draw a general picture about strategy use of fourth graders, then (and more importantly) investigates the role of each strategy in explaining problem solving success and in discriminating between successful and unsuccessful students at fourth grade level.

Thus, research questions are as follows:

1. *What is the overall situation about usage of non-routine problem solving strategies at fourth grade level?*
2. *What is the role of each non-routine problem solving strategy in explaining the problem solving success?*
3. *Which non-routine problem solving strategies are more effective in discriminating between successful and unsuccessful students at fourth grade level?*

Related Studies

Studies by Hok-Wing & Bin (2002) and Elia et al. (2009) will be mentioned more in-depth here because of three reasons: (i) they were also at fourth grade level, (ii) they aimed to seek student's existing abilities on non-routine problem solving without any instruction like this study; (iii) their methodologies were similar to the present study with regard to allotted time for tests, and number of questions which tests included.

Distinct from this study, participants of Hok-Wing & Bin (2002)'s study were from four regions of China, and their findings showed that different and low performances on the choice of problem-solving strategies of pupils from four regions had been measured. Generally, the *make a drawing* strategy was successfully used by the students, while usage of *look for a pattern* was unsuccessful.

Study carried out by Elia et al. (2009) is differentiated at the point of subject being focused. Authors investigated strategy flexibility as well, and they proposed and investigated two types of strategy flexibility: inter-task flexibility (changing strategies across problems) and intra-task flexibility (changing strategies within problems). Findings showed that these students rarely applied strategies in solving problems. Among strategies, the trial-and-error (guess and check) strategy was found to have a general potential to lead to success. The two types of flexibility were not displayed to a large extent in students' strategic behaviour. However, on the one hand, students who showed inter-task strategy flexibility were more successful than students who persevered with the same strategy. On the other hand, intra-task strategy flexibility did not support the students in reaching the correct answer.

Although it was carried out with teacher trainees, study which is most related to the present study was done by Altun and Sezgin-Memnun (2008). Unlike this study, participants were given problem solving instruction. However, authors also tried to find out which strategies are the leading indicators of problem solving skills. Multiple regression analysis revealed that the relative importance order of the problem solving strategies in terms of their effects on the problem solving success is as follows: *simplify the problem, look for a pattern, reasoning, make a drawing, make a systematic list, guess and check, and work backwards.* According to discriminant analysis results, the strategies of *reasoning, work backwards, make a drawing, make a table* and *simplify the problem*, respectively had a big impact in separation of successful and unsuccessful participants. The analysis also confirmed that 80% of the problem solving success could be explained by the problem solving strategies.

Method

Research Design

The research design of this study is of the descriptive type. This type of research illustrates the situation at the time of the study. Namely, it involves gathering data that describe events and then organizes, tabulates, depicts, and describes the data collection. Descriptive research can be either quantitative or qualitative. As preferred in this study, it involves collections of quantitative information that can be tabulated along a continuum in numerical form, such as scores on a test or the number of times a person chooses to use a-certain feature of a multimedia program, or it can describe categories of information such as gender or patterns of interaction when using technology in a group situation (Glass & Hopkins, 1984).

Participants

Two hundred and forty four students in fourth grade participated in the study from eight different primary schools governed by MoNE (Ministry of National Education) in Bursa/Turkey. Participating schools were chosen by using simple random sampling. Participants did not have any experience with non-routine problem solving in their school life before the current study, since this kind of problems have not been emphasized in mathematics education in Turkey.

Information about Data Collection Instrument and Procedure

To measure the problem-solving success of students, a paper and pencil test (Problem Solving Test – PST) comprised of six non-routine open-ended problems was constructed by the author. Problems in the test were intended for using the strategies of *make a systematic list, look for a pattern, work backward, simplify the problem, make a drawing, and guess and check* strategies (see Appendix). Selection of these strategies were derived from previous studies, projects and books (Follmer, 2000; Altun et al., 2007; Leng, 2008; Posamentier & Krulik, 2009; Elia et al., 2009; Lee et al., 2014) dealing with strategies that can be learned and used at fourth grade level.

PST was conducted by the author and students were given 45 minutes to complete it. But, if a student needed, he/she could have extra time. Students were encouraged about to write down all their thoughts while they were solving the problems.

All students' answers for each problem were scored between 0 and 10 by categorizing them under the titles of *correct, little mistakes stemming from calculation errors or inattentiveness, insufficient answers despite understanding the problem and taking the right action, wrong answers* and *no answer.* Every student was scored between 0 and 60 points. The author and another researcher evaluated answers independently and the given points were compared to each other. If there was a big difference between points given to a student solution, the solution was re-examined together by the scorers to determine a common score. The focus was on the correctness of the students' problem solving processes. Strategy usage was not taken into consideration at this point.

Based on scores for each question in the PST instrument, Cronbach's alpha was used to assess the reliability of the instrument. Generally, the lower limit for Cronbach's Alpha should be .70 (Robinson, Shaver, and Wrightsman, 1991). The alpha coefficient was computed as .713 for the PST indicating acceptable reliability for the instrument.

To assess validity of the PST, factor analysis with Varimax rotation was performed based on the same scores as used for the Cronbach alpha reliability. Before proceeding with factor analysis, it was necessary to assess the sampling adequacy. This was done using the Kaiser-Meyer-Olkin (KMO) and Bartlett's test for Sphericity. The level of significance of KMO and the Bartlett's test for Sphericity should be significant at the .05 confidence level (Hair, Black, Tatham and Anderson, 1998). Table 1 below shows that the results of the tests meet acceptable levels that permit proceeding with the factor analysis.

Table 1: KMO and Bartlett's test results

Kaiser-Meyer-Olkin Measure of Sampling Adequacy		.74
Bartlett's Test of Sphericity	Approx. Chi-Square	158.51
	Df	15
	Sig.	.000

Factor analysis results suggested the existence of two factors. The eigenvalues showed that the first and second factors explained 33.39% and 19.41% of the variance of PST scores. Collectively these two factors explained over 52% of the variance. As seen from the rotated component matrix (Table 2), factor loading of each item is greater than .45. Thus, it can be said that problem solving success was properly measured by items of PST. In other words, the administered PST test was validated.

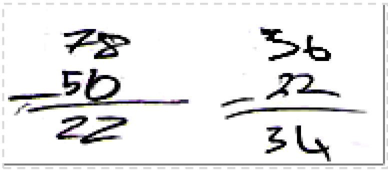
Table 2: Rotated component matrix

Item	Factor 1	Factor 2
3	.72	.07
1	.68	-.26
2	.67	.34
5	.64	.24
6	-.04	.85
4	.40	.46

To answer the second and third research questions, (except the scoring system explained above), all student scripts were evaluated with regard to strategy use again. Instead of scoring each question separately, each student's paper was considered as a whole and usages of each strategies dealt with in this study were coded as 2 (correct and effective usage of strategy), 1 (incomplete usage of strategy), and 0 (no usage of strategy). So, every student had a point about usage of each strategy.

In Figure 8, one student's answer to sixth question *(In an exam, each right answer was scored as 1 or 2. A student had 56 right answers and got 78 point. How many one-point questions did he answer correctly?)* In PST is represented to explain scoring processes of problem solving success and strategy use thoroughly. This question could be solved by using *guess and check* strategy, but the student did not prefer to use it. Actually, his reasoning was quite different and elegant. He firstly supposed that all right answers were scored as 1. Than he thought that he needs 22 extra points to get 78 points in total, meaning that he has to convert points of 22 questions from 1 to 2. Thus, he found that number of two-point questions was 22, and there were 34 one-point questions. Since his reasoning and answer was completely right, this students' problem solving success score for this question was 10. But since he did not use *guess and check* strategy for any questions in PST, his score about use of this strategy was 0.

Figure 8: A participant's answer to sixth question in PST



To account for inter-coder agreement (based on coding about strategy use), Cohen kappa coefficient for inter rater agreement was calculated and it was found as .81. Then, means and standard deviations about each strategy were computed. To reveal how functional the strategies were in the problem solving success, multiple regression analysis was executed by using strategy scores (independent variables) and total problem solving success scores (dependent variable) for each student.

In order to see which strategies were more useful in discriminating between successful and unsuccessful students, first bottom and top segments of 27% were determined according to the total problem solving scores the students achieved in the PST. The group at the bottom consisting of 27% of the students was composed of those who had got a score of 9 or lower, and the group at the top consisting of another 27% of the students achieving higher than 27. There were 63 students in each of bottom and top groups. Later on, discriminant analysis was carried out based on the strategy and total PST scores of students.

Results

Means and standard deviations about each strategy were used to have an overview about strategy usage levels of participants (n=240). According to results shown in Table 3, it can be said that usage of *look for a pattern* and *make a systematic list* are quite close to the average. However, low means for the use of other strategies may be an indicator of the fact that students are not very familiar with these strategies.

Table 3: Some descriptive statistics on usage of strategies

Strategies	Mean	Std. Deviation
Make a systematic list	.82	.88
Look for a pattern	.95	.90
Work backward	.39	.72
Simplify the problem	.23	.61
Make a drawing	.36	.56
Guess and check	.16	.43

Multiple regression analysis was employed to determine each strategy's contribution to problem solving success with regard to the second research question. The results of the regression analysis are given in Table 4.

Table 4: Multiple regression analysis results

Strategies	B	Std error	β	t	p	Dual correlation
Constant	4.14	.53		7.77	.00	
Make a systematic list	4.57	.35	.34	13.06	.00	.46
Look for a pattern	6.73	.38	.51	17.70	.00	.70
Work backward	5.31	.45	.33	11.74	.00	.57
Simplify the problem	2.73	.52	.14	5.22	.00	.36
Make a drawing	2.90	.59	.14	4.90	.00	.46
Guess and check	4.71	.71	.17	6.57	.00	.17

According to the dual correlations between the problem solving success and problem solving strategies, it can be seen that the highest correlation coefficients belong to the strategies of: *look for a pattern* (.70), *work backward* (.57), *make a systematic list* (.46) and *make a drawing* (.46). Correlation coefficients which belong to *guess and check* and *simplify the problem* strategies (.36 and .17) indicate that there is a positive but weak relationship between each of these strategies and problem solving success. Generally speaking, there is a significant relationship between strategies used and problem solving success ($R = .92$, $R^2 = .84$, $p = .00$). Strategies used in this study as independent variables, explain almost 84% of problem solving success.

According to the standardized regression coefficients, the order of relative importance of strategies in terms of their effects on the problem solving success is as follows: *look for a pattern*, *work backward*, *make a systematic list*, *make a drawing, guess and check* and *simplify the problem*. After analysing t-test results from the regression coefficients, it appears that all strategies had a decisive role in explaining problem solving success. The regression equation related to the success in problem solving is as follows: "problem solving = 4.14 + 6.73 *looking for pattern* + 5.31 *work backward* + 4.57 *make a systematic* list + 4.41 *guess and check"* + 2.90 *make a drawing* + 2.73 *simplify the problem.*

Table 5: Wilks' Lambda and F values of bottom and top groups

Strategies	Wilks' Lambda	F
Make a systematic list	.47	141.41*
Look for a pattern	.07	1638.52*
Work backward	.53	111.33*
Simplify the problem	.76	38.23*
Make a drawing	.91	12.15*
Guess and check	.99	1.00

*Significant at .05 confidence level.

According to the results of discriminant analysis (considering the low values of Wilks' Lambda and high value of F), strategies that play a significant role in distinguishing successful and unsuccessful students were: *look for a pattern, make a systematic list, work backward, simplify the problem* and *make a drawing*, respectively (See Table 5). The *guess and check* strategy did not have any significant contribution. With the help of the discriminant function, a classification with an accuracy rate of 98.4% was achieved.

Discussion

Non-routine problems which are usually not specific to any mathematical topic, have no fixed procedure for solving, and require the use of one or more strategies to solve. Moreover, they are especially challenging for many primary school students, since they require an integration of several cognitive processes such as accounting for all possibilities, visualizing relationships, etc. (Lee, Yeo, & Hong, 2014). Non-routine problem solving skills and strategy use of fourth graders were scrutinized in this study. In this context, findings suggest that fourth graders have difficulty in solving non-routine problems without being exposed to specific training. This result is in line with results found by Hok-Wing and Bin (2002) and Elia et al. (2009). The marginal place of non-routine problems in Turkish textbooks and mathematics curriculum determined by the MoNE could be an explanation for this situation. According to means computed for each strategy, usage of two strategies was remarkable in the present study. These were *look for a pattern* and *make a systematic list.* Relatively high application levels of *look for a pattern* strategy can be explained by the fact that it is the only strategy which is elaborated in primary schools in Turkey. In Hok-Wing and Bin (2002)'s study, use of *make a drawing* strategy was prominent, while *guess and check* strategy was the most broadly successful strategy in Elia et al. (2009)'s study. It seems that the most popular strategies for students in fourth grade vary across countries depending on educational systems or curriculum emphasis.

In regard to findings about the second and third research questions, the R^2 value found through the multiple regression analysis demonstrates that in general, knowledge of strategies explains 84% of problem solving performance. In other words, students have to know about different problem solving strategies. Besides, 98.4% accuracy in classification according to discriminant analysis results suggests that strategies have a dominant and decisive role in determining novice and expert problem solvers. Altun and Sezgin-Memnun (2008) achieved the similar results in this sense. However, the most important contribution of this study is the detailed information on the role of each strategy in explaining success, and in discriminating between successful and unsuccessful students at fourth grade level. Even though almost all strategies had statistically significant roles, three strategies were conspicuous: *look for a pattern, make a systematic list*, and *work backward.*

Strategies of *make a drawing*, *simplify the problem*, and *guess and check* were not used effectively by either low or high achieving students. Using the s*implify the problem* strategy, students should first solve simple versions of the problem and then look for patterns and make generalizations. It seems that the sophisticated structure of this strategy makes it difficult for participants to use. This may be an explanation for the ineffectiveness of the *simplify the problem* strategy in accounting for achievement and on differentiating between good and poor problem solvers. Nevertheless, contrary to expectations, most of the students did not prefer to make drawings which would help them in visualizing and solving the problem. This situation can stem from the fact that students are not encouraged by teachers to illustrate their thoughts by using pictures or simple drawings while they are solving problems. Lastly, *guess and check* was the weakest and least effective strategy. According to Elia et al. (2009), this strategy does not entail high cognitive demand and it is widely used in a variety of mathematical and everyday situations. However, results of this study exhibit an incongruity in this sense. It is apparent that g*uess and check* is often dismissed in math education in Turkey as an unproductive or less sophisticated approach to solving problems.

Determining importance order and distinctiveness of each strategy in terms of problem solving success may bring some important outcomes which can be used to arrange textbooks and learning environments. First, since each of six strategies sought in the current study has meaningful contribution to success, it can be concluded that all these strategies are feasible to work at fourth grade level. High potentials of *look for a pattern, make a systematic list*, and *work backward* might indicate that these strategies can be highlighted. However, other strategies should not be neglected. On the contrary, impotent strategies such as *guess a check* should be given more time so that they can be also be used effectively by all students.

Despite the fact that this study provides many meaningful conclusions, further research is necessary. Firstly, the study included only six non-routine problems, and using more different types of non-routine problems would provide more robust evidence for the findings of this study. Maybe routine problems can also be involved to provide more detailed information about the general problem solving success of students.

Thinking processes of students were not analysed in detail in the present study. Because this study was based on students' written answers and students have difficulties in writing down their thinking processes. Complementary qualitative methods such as clinical interviews, observations and videotaping of pupils solving problems could be used to collect data about students' cognitive processes. In addition, examining students of different grades age and school year level, mathematical abilities, and educational systems would be more illuminating.

In summary, no matter how essential strategies may be as a component of problem solving performance, it is evidence that they do not tell the whole story (Schoenfeld, 1983). Nevertheless, if a primary goal of instruction is to develop students' ability to think strategically so that they have a problem solving disposition including the confidence and willingness to take on new and difficult tasks, non-routine problems should be studied from the first grades of elementary school. Teachers may benefit from non-routine problems that force students to apply what they have learned in a new way. Furthermore, teachers should increase the variety of strategies which they present to the students by considering grade level, and difficulty level of the strategy.

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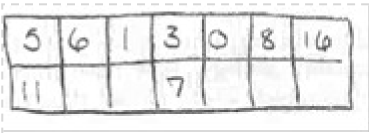
Appendix

Problem Solving Test (PST)

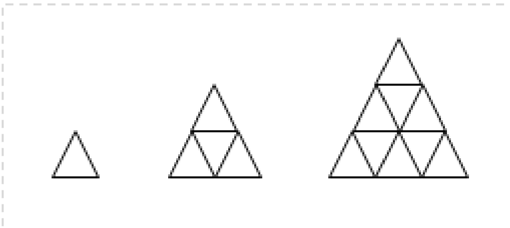
1. At a pizza restaurant, there are 4 toppings for pizza: Mushroom, sausage, onion, and green pepper. How many different ways can you make a pizza with 2 toppings? *(Make a systematic list)*

2. While walking in a cave, a tourist saw the following code on the wall. He recognized a relationship between numbers in each column. But there were missing numbers in the code. Complete the code and explain the relationship. *(Look for a pattern)*



3. A bus driver started to drive from the terminal and dropped half of his passengers off at every bus stop he stopped by. Meanwhile, no passenger got on the bus. After stopping by 3 bus stops there were 8 remaining passengers on the bus. How many passengers got on the bus at the terminal? *(Work backward)*

4. Each of following shapes consists of small triangles like the first one.



How many triangles do you need to make fifteenth shape? Explain what you thought. *(Simplify the problem, look for a pattern, make a drawing).*

5. At a birthday party, every child shakes hands with every other child. If there are 7 people at the party, how many handshakes take place? *(Simplify the problem, look for a pattern, make a drawing).*

6. In an exam, each right answer was scored as 1 or 2. A student had 56 right answers and got 78 point. How many 1-point questions did he answer correctly? *(Guess and check)*