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THE SUCCESS OF PRIMARY SCHOOL 4th GRADE STUDENTS IN THE "DIGIT SYSTEM" CONCEPT IN NATURAL NUMBERS

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Abstract:
The purpose of the present study was to examine the success of fourth grade primary school students in the "digit system "concept in natural numbers. The research was conducted with a mixed model. A total of 300 students who attended the fourth grades of two primary schools in Elazig city center, Turkey participated in the study. A test that consisted of 28 questions that were developed by examining the relevant literature and the mathematics curriculum of 2018 were used in the study. The reliability of the test was calculated by using the KR-20 formula, and the Reliability Coefficient was calculated to be .885. The data that were collected were analyzed by employing descriptive statistical methods together with the Content Analysis method. The results showed that 4th grade students had serious difficulties in understanding the "digit system" concept and that they had misconceptions in this respect. The overall success level of the students was found as 58%. The sub-learning area in which students had the highest difficulty levels was determined as "Subtraction in Natural Numbers", and the sub-learning area in which they were most successful was determined to be "Multiplication in Natural Numbers". The students reached the correct result at the highest level by using the calculation strategy and at the lowest level by using the estimation strategy. Again, the students had difficulty at the highest level in "writing the numbers given in an unusual way". Again, according to these findings, the highest mistake type was detected in the questions on the "digit system" (9 different mistakes), and the least error type was detected in the questions about showing the numbers using symbols and models (2 different mistakes). Depending on the results obtained from the study, suggestions were made to better teach the concept of the "digit system", which can be considered as the basis of mathematics education.

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Keywords: "digit system", primary school mathematics education, strategy, mistake

1. Introduction

A full understanding of the "digit system" concept develops during the primary and secondary school processes (Van De Walle, Karp & Bay-Williams, 2014: 187). The "digit system" or the "digit value", which enables students to read and write very large and very small numbers with symbols with ease, is one of the most important features, and is also among the most abstract concepts of the number system and arithmetic we use in which a number takes its value according to its position in the numbers set. The "digit system", or the "digit value" can be defined as the value taken by the numbers according to their location in a number set (Tosun, 2011: 24). Thompson (2000) reported that most children could think of the "digit system" concept at a very early age; however, confusion on this issue continued for many years. Garlikov (2000), on the other hand, examined the studies conducted on the "digit system" concept and reported that children in America did not learn the "digit system" concept in an effective way. Again, Thompson and Bramald (2002) asked a question on kilometers to the students, and asked them to tell how many kilometers would be covered in case the indicator showed 06142 at first and then 06299. A total of only 24% of the 144 students (2^{nd}, 3^{rd} and 4^{th} graders) were able to answer this question correctly.

According to Dinc, Artut and Tarim (2006), the teaching of the "digit system" concept during teaching numbers and operations is one of the biggest difficulties that children ever face. Ross (1985) conducted a study and examined the development of the "digit system" concept in 2^{nd}-5^{th} graders. They conducted the study with 60 students, and each student was asked about the similarity between the numbers in a two-digit number set and the amount of objects. It was observed that although most of the students in the study knew that 25 represented twenty-five objects, they did not know that "2" represented "20" and "5" represented the remaining "five objects". As a result, it was concluded that students should have knowledge of the "number" concept and of the relationship between "piece and whole" to understand the "digit system" concept. According to the results obtained in the study that was conducted by Kamii (1988), the rate of students who said that "1" meant "10" in 16 was 0% in the 2^{nd} grade, 33% in the 4^{th} grade, and 50% in the 5^{th} grade. This shows that students have difficulties in distinguishing the "number" and "digit" value of the number set.

Rusch (1997) conducted a study to evaluate the understanding of the digit value concept of teacher candidates. A means of assessment was developed to be used in this study, and was used to obtain information on digit values unlike the operational skill. In the study, teacher candidates participated in a course, and the means of assessment was applied to them before and after this course. The results showed that many of the teacher candidates started the course with a superficial digit value approach, and completed it by realizing that this superficial understanding was far from the understanding they needed. Dinc, Artut and Tarım (2006) tried to determine how well 1^{st}-5^{th} grade students could learn the digit value concept, and what kind of mistakes were made by those who could not learn it. The data they obtained showed that only 1.5% of the students could show 10 counting bars for "1" in the decimal digit of the number "16". They also observed that 98.5% of the students could not show this number correctly in the first stage; however, after a clue was given, 46.2% corrected the error and gave correct answers. In terms of gender, on the other hand, the difficulties were found to be similar. The representation of "zero" by children occurs relatively later than other representations in the number system. It is not difficult for children to represent the concept "nothing" with "0"but it is difficult to use "0"as a digit value (Sharma, 1993). Many children think that when it is used to represent a zero digit, zero means nothing. For example, "zero" represents "nothing" or an empty set alone but the zero in number "10"shows that there is no "1". The "zero" in the number "108"shows that there are no decimals (Olkun & Ucar, 2007: 94).

A total of seven students from 8th grade students participated in the study conducted by Kaplan (2008) to examine the perceptions of students on digit and digit values as separate concepts for each student. The results of the study showed that many of the students thought that the digit value was a place, and the concept of the digit value was the product of a multiplication. In addition, it was also observed that when the students explained their ideas about the digit concept and digit value in a number system that is different from the decimal number system, they continued their habits that came from the decimal number system. Macdonald (2008) conducted a study and examined the misconceptions of 7th grade students on digit values in decimal numbers. In this study, it was seen that a 7th grade student answered the question "What is the digit value of 8 in 6.781?" as "one tenth". The student, who thought that the structure of whole numbers was also valid for decimal numbers, explained the first digit after the comma as "one in first". According to the study, this stems from the fact that students do not understand that the digits of the decimal part in a number should be staged as "one tenth", "one hundredth", "one thousandth", etc.

In this study, the purpose was to examine the success status of 4th grade students in terms of digit values in natural numbers. For this purpose, the success levels of primary school 4th grade students in the digit value concept, and how they solved questions regarding the digit value concept, were examined.

2. Method

The study was conducted in two public schools in the city center of Elazig, Turkey. The research group was selected with the Purposeful Sampling method and the Convenient Sampling method by considering the number of students and teachers of the schools. All of the 4th grade students were invited to participate in the present study, and 300 students agreed to participate in the study voluntarily with their parental consents.

A success test for the digit value concept in natural numbers was developed to collect the data in the study. The preparation of the questions in the test was based on the mistakes of students emphasized in relevant literature considering the 17 acquisitions mentioned in the Primary School 4th grade mathematics curriculum of 2018.

Although according to the curriculum the digit concept is taught as of the 1st grade of primary school, there are several reasons for the selection of 4th grade primary school students in the study. Firstly, the acquisitions and skills of the digit concept are taught from the 1st grade in primary school according to the Elementary School Mathematics Teaching Program (2018). However, the researcher aimed to see the general development of the primary students by selecting the 4th grade students as participants because the learning objectives become more specific and detailed when 4th grade begins. Finally, 4th grade students were thought to be the most appropriate group for primary school level study because they would act better in thinking and problem solving compared to younger students.

In the test, there were four acquisitions on the digit concept in the natural numbers sub-learning area. In addition to these acquisitions, the researcher added the following acquisitions to the natural numbers sub-learning area; "The student shows the four-digit numbers by using the model"; "s/he expresses how many decimals are obtained from how many units, and how many hundreds are obtained from how many decimals"; and "s/he notices the relations between the numbers and patterns in the hundreds' table". The reason why these acquisitions were added was that these acquisitions were included in the Primary School 1st, 2nd and 3rd grade mathematics curriculum showing digits with models and expresses them in different ways. Similarly, two acquisitions were included in the sub-learning area of natural numbers in addition operations; two acquisitions were included in the subtraction operations in the natural numbers sub-learning area; two acquisitions were included in the multiplication operations in the natural numbers sub-learning area; and four acquisitions were included in the division in natural numbers sub-learning area in the draft achievement test. Examples of the questions in the test are given in Appendix 1. Necessary information was given to the primary school 4th grade teachers in the selected schools after the necessary permissions were obtained from the Directorate of National Education. The study was conducted by classroom teachers under the supervision of the researcher in all classes at the same time in the Spring Semester of the 2018-2019 Academic Year. The Achievement Test was applied as 14+14 questions with 10-minute intervals in two class hours considering the number of the problems that fourth graders could solve in one class hour. The application lasted 80 minutes in total. The students were asked to solve each problem by writing or drawing.

Expert opinions, relevant literature and the acquisitions in the digit concept in the 4th grade natural numbers curriculum of primary schools were made use of to ensure the content validity of the test. To ensure content validity, the developed test was presented to one professor and two associate professors working in the Class Teachers Education Department, and to two doctorate instructors working in the Education Programs and Teaching Department for expert opinion. The reliability of the test was calculated by using the KR-20 formula, and the Reliability Coefficient was calculated to be .885. Data analysis was carried out according to an answer key that was prepared together with the researcher and a field expert. The data were analyzed with the ITEMAN Program by scoring each correct answer with 1, and each false and empty answer with 0. Then, the correct and wrong solutions of the students were examined. In this way, it was determined which strategies students used to reach the correct answer and what mistakes they had made if the solution was wrong.

3. Findings

In this section, the results that were obtained from the Achievement Test, which was applied to 300 students to determine the achievement levels of primary school 4th grade students in terms of the digit concept, are given in line with the sub-themes determined by the researcher.

3.1 Findings on the problems on reading and writing numbers

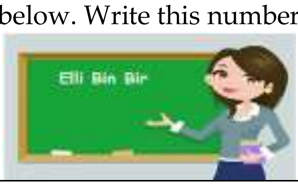
There were 28 questions in the test intended to determine the level of understanding of the digit concept in natural numbers of primary school 4th grade students. Problems 1 and 2 were intended to test reading and writing. The students were asked to solve and also to explain each problem. The frequency and percentage values of the answers that were given by the students are given in Table 1. In this respect, 89% of students were able to read and write a five-digit number and the digits of a six-digit number, and also to determine and analyze the digits in the first question; and 87% of the students were successful in doing these in the second question.

Table 1: Frequency and percentage distributions of the answers given to the problems on reading and writing numbers

	Correct answer		**Incorrect answer**		**Empty answer**	
	f	%	f	%	f	%
1st Problem	267	89	33	11	0	0
2nd Problem	261	87	39	13	0	0

The examples of correct and incorrect/inadequate answers that were given by the fourth grade students to the problems on reading and writing numbers are given in Table 2.

Table 2: Sample student answer

<table>
<tr><td colspan="2">Question 1: Our teacher has written in words the reading of a number on the board below. Write this number below.

</td><td colspan="2">Question2: What is the number that has 9 in the hundreds of thousands digit, 8 in the thousands digit, 2 in the decimals digit, 7 in the units digit, and that is 0 in the sum of the decimals of thousands and hundreds digits? Show it to Murat by writing it on the dotted line below.</td></tr>
<tr><td>The answer accepted as correct</td><td>Students' strategies</td><td>The answer accepted as correct</td><td>Students' strategies</td></tr>
<tr><td>50.001</td><td>Naming
(Naming a verbal statement by writing it in</td><td>908.027</td><td>Naming
(Naming a number whose digits are given by writing it in correct digits)</td></tr>
</table>

	numbers)		
Answers that were accepted as incorrect/insufficient	**Students' mistakes**	**Answers that were accepted as incorrect/insufficient**	**Students' mistakes**
5000001 510001	Expansion of the number (Expansion of the number as a result of using 0 and 1 more)	9780627	Defining both the digit number incorrectly and placing the numbers in incorrect digits.
5001	Reducing the number (Reducing the number as a result of using 0 insufficiently)	98027	Reducing the number (Reducing the number as a result of using 0 insufficiently)
		900827	Placing the numbers in incorrect digits

All of the students who answered the questions on reading and writing numbers in a correct way answered the question by using the Naming Strategy. The students made two different mistakes in the first question, and three different mistakes in the second question. The common mistake of the students in both questions was to reduce the number as a result of using 0 in a misplaced way.

3.2 Findings on showing the numbers by using symbols and models

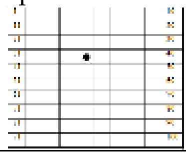
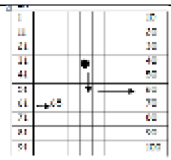
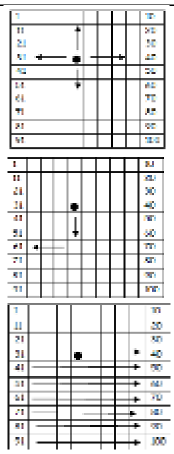
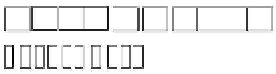
In the test, questions 3 and 6 were intended to measure the ability of students in representing the numbers using symbols and models. The frequency and percentage values of the answers given by the students are displayed in Table 3. According to these data, 51% of the students were successful in showing the digit values of the numbers by using some models and in showing the patterns and relations in the hundreds tables in the 3rd and 6th questions.

Table 3: Frequency and percentage distributions of the answers given to the questions on showing numbers by using symbols and models

	Correct answer		Incorrect answer		Empty answer	
	f	%	f	%	f	%
3rd question	153	51	147	49	0	0
6th question	153	51	135	45	12	4

The examples of the answers that were accepted as correct and as incorrect/insufficient given by primary school 4th graders to the questions on showing numbers using symbols and models are presented in Table 4.

Table 4: Sample student answer

Question 3.  How do you complete the number above given in the hundreds' blocks by using the hundred and ten blocks? Show below by drawing.		Question 6.In the following hundreds tables; … represents the number = 35. How do we find the result of 35+ 28 by moving the ball to the right, left, up and down? Show it on the table by drawing. 	
The answer accepted as correct	**Students' strategies**	**The answer accepted as correct**	**Students' strategies**
10 x ☐ +☐ = 1010 To obtain 1010,it is necessary to use ten 100's blocksandone 10's block.	Representing - Calculating (Expressing the given natural number by using standard representations and by making use of the four operations)	 If I go down 1 square, it becomes 45; if I go down 1 square more, it becomes 55. Then I move towards right for 8 squares in 8 units, and it becomes 63.	Representation (Reaching the result by making use of the patterns and connections indicated by the given representation.)
Answers that were accepted as incorrect/insufficient	**Students' mistakes**	**Answers that were accepted as incorrect/insufficient**	**Students' mistakes**
☐+☐ + 0 =1010 If I add one hundred's block and one ten's block, it makes 1010.	Confusing the models (Thinking as if the hundred's block were the thousand's block)		Not knowing the patterns in the hundred's table
 I draw 10 hundreds block, and then I draw 10 decimal blocks	Confusing the models (Thinking as if the decimal block were unit block)		
 If I draw ten from 10's block, I find the result.	Missing modeling (Not using decimals blocks)		

The students who gave correct answers to the questions on showing numbers by using symbols and models did so by using representation and calculation strategies. The students made two different mistakes in the in the 3rdquestion.However, all of the students who gave incorrect answers in the 6thquestion did so because they did not know the patterns in the hundred's tables.

3.3 Findings on the questions on understanding the relationship between digits

In the test, questions 4, 5, 7, 8 and 9 were on understanding the relation between the digits. The frequency and percentage values of the answers of the students given to these questions are given in Table 5. According to these data, 45% of the students succeeded in rounding numbers to the nearest decimal, sorting numbers using big/small symbols, and stating numbers in different ways as units, decimals and hundreds in question 4; 51% succeeded in doing these in question 5; 76% students in question 7; 42% students in question 8; and 58% students in question 9.

Table 5: Distributions of the frequency and percentage of the answers to the questions on the relationship between digits

	Correct answer		**Incorrect answer**		**Empty answer**	
	f	%	f	%	f	%
4th question	135	45	135	45	30	10
5th question	153	51	129	43	18	6
7th question	228	76	72	24	0	0
8th question	126	42	165	55	9	3
9th question	174	58	120	40	6	2

The examples of the answers of the fourth graders that were accepted as correct or incorrect/insufficient given to the questions on the relationship between the digits are presented in Table 6.

Table 6: Sample student answer

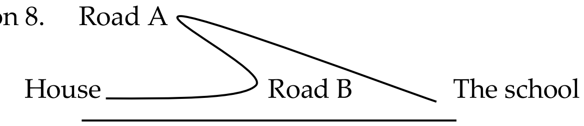
Question 4. If I am 341, and if I have 22 decimals, how many hundreds do I have left? Show it on the dotted area below by writing.			
Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
3 4 1 - 2 0 2 ——— 1 2 1 22 decimals make 220, if I subtract 220 from 341, I find 121, and there are 1 hundred, 2 decimals, and 2 thousands in 121.	Calculation (Finding the result by subtracting)	3 hundreds. Because, if I divide 341 by 100,I find 3.	Not including all the data in the operation (not considering 22 decimals)
		3 4 1 + 2 2 ——— 3 6 3 300 units 3 4 1 - 2 2 ——— 3 1 9	Not being able to write the number given in an unusual way (writing 22 decimals as 22 units)
Question 5. How many decimals can be in 658? Show it by writing on the dotted area below.			
Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
658 | 10 60 | 65 tane 058 - 50 8 I divide 658 by 10 to find how many 10s are there	Calculation (Finding the result by dividing)	There are maximum 58 decimals in 658	Not analyzing the number (Thinking as if the number were given in decimals by combining the units and decimals digit)

in 658.the result was 65.		There are maximum 60 decimals in 658 because. 6 hundreds means 60.	Not including all data in the operation (Considering only the hundreds digit)
		There are maximum 5 decimals in 658. Because, there is 5 in the decimals digit.	

Question 7. Round 59099 to the nearest decimal digit. Write your answer to the dotted area below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
59.100 In case the number in the units digit is 0,1,2,3,4, and if the number is 5 and over, it is rounded to an upper number, I did this for this reason.	Prediction (Applying the "Rounding Rule")	60.000	Not knowing the "Rounding Rule" (Considering the biggest digit when rounding, or rounding to the nearest hundred digit instead of the nearest decimal digit)
		59.000	Not knowing the "Rounding Rule" (Rounding to the next lower hundredth)
		590100	Not knowing the "Rounding Rule" Writing the nearest decimal value directly

Question 8.



Ayse uses Road A on her way to school, and Road B on her way back home from school. Ayse describes the length of Road A as "1 hundred + 22 decimals "meters; and describes the length of Road B as"32 decimals + 5 units" meters. Which road is longer for Ayse, the departure road or the return road? Explain it by writing to the dotted place below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
A=100+ 22 x 10 + 0 = 320 makes 1 hundred (100). 22 decimals make 220, we add them, the result is 320,I mean, road A is 320 m long. B=32 x 10 + 5 = 325. Here, there are 32 decimals, which makes 320. If I add units to it, it makes 325m. I mean, road B is longer.	Re-naming-Comparison (Finding two numbers given in an unusual way according to their being big or small)	Road A 1220 Road B is longer than road 325 A.	Not being able to write the number given in an unusual way (writing 1 hundred digit as 1 thousand digit)
		Road A, because there is a hundred.	Not including all the data to the operation (Considering the number values in the digits in an independent manner)

Because, 325 is bigger than 320 by 5 units.		A= 122 B=37	Not including all the data to the operation (writing 32 decimals as 32 units)
Question 9. Which of the above numbers is smaller than 2 decimals and 54 units? Circle these numbers and write why they are small.			
Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
12, 41 and 52;since the numbers in the decimals digit here are smaller than 7, 74> 12, 41, 5274<89	Re-naming-Comparison (Finding the two numbers given in an unusual way being big or small) 2 decimals + 54 units =74	54+2=56➔ 56>12,41,52	Not being able to write the number given in an unusual way (writing 2 decimals as 2 units)
		20+54=74 There are 4 decimals in 74. However, since there are 2 units and 2 decimals in 12, 41, 52, 74>12,41,52	Not knowing the digit value (Confusing the place of units and decimals)
		Since it says54 units, 12, 41 and 52's units are smaller than 4. 54>12,41,52	Not knowing the "Comparison Rule" (Not knowing what to consider when comparing the numbers as small/big)

Fourth graders answered the questions on the relationship between the digits correctly by using calculation, re-naming, comparison and prediction strategies. The students used the same strategies when they solved the 4th, 5th, 8th and 9th questions. The students made two different mistakes in the 4th, 5th and 8th questions; one different mistake in the 7th question, and three different mistakes in the 9th question. In general, most of the mistakes of the students in the questions on the relationship between the digits were mostly not including all data in the operation, not being able to write the number given in an unusual way, not analyzing the number, not knowing the "Rounding Rule", not knowing the digit value and not knowing the comparison rule.

3.4 Findings on the questions on four operations in natural numbers

Questions 10, 15, 19, 21, 22, 23, 24 and 25 in the test were about four operations in natural numbers. The frequency and percentage values of the answers of the students for these questions are given in Table 6. As seen in Table 6, 87% of the students gave correct answers in the 10th question; 64% of them gave correct answers in the 15th and 25th questions; 62% of them gave correct answers in the 19th question; 56% of them gave correct answers in the 21st question; and 60% of them gave correct answers in the 22nd, 23rd and 24th questions.

Based on this finding, it is possible to argue that students are successful in adding and subtracting two four-digit numbers, multiplying a three-digit number by a two-digit number, dividing a three-digit number by a two-digit number, dividing a four-digit number by a one-digit number, and dividing a five-digit number with the last digit being 0 by 10 and the multiples of 10.

Table 7: Frequency and percentage distributions of the answers of the students given to the questions on four operations in natural numbers

	Correct answer		Incorrect answer		Empty answer	
	f	%	f	%	f	%
10th question	261	87	30	10	9	3
15th question	192	64	105	35	3	1
19th question	186	62	105	35	9	3
21st question	168	56	120	40	12	4
22nd question	180	60	120	40	0	0
23rd question	180	60	117	39	3	1
24th question	180	60	114	38	6	2
25th question	162	64	120	30	18	6

The examples of the answers that were accepted as correct and the answers that were accepted as incorrect/insufficient of primary school 4th graders on four operations in natural numbers are presented in Table 8.

Table 8: Sample student answer

Question 10. 1358 +1276 Solve the addition given here, and explain how it is solved.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
1358 +1276 2634 I start on the right and add towards the left. Since8+6=14, I write 4 from 14 to the units digit, and transfer 1 decimal to the decimals digit. 5+7=12, here, there were ten waiting, It made 13. I write 3. decimals to the decimals digit. I transfer the 10 decimals as 1 hundreds to the hundred digit.	Calculation (Finding the result by adding)	1358 +1276 2624 8+6=14, I write 4 from 14. I write 2 from 12 in 5+7=12. There was 1 waiting in 3+2=5. If I add 1 to it, it makes 6. 1+1=2. The result is 2624.	Not knowing how to make the operation (Transferring missing decimals)
		1358 +1276 2644 8+6=14, here, I write 4 from 14. 5+7=12, here, there is 2 waiting, it makes 14. 3+2=5, here, there was 1 waiting. It makes 6. 1+1=2. The result is 2644.	Not knowing how to make the operation (Transferring excessive decimals)
		1358 +1276 3634 8+6=14. In 14,I have 1 waiting. 5+7=12, here, if I add 1, it makes 13. I write 3 from 13. Again, I have 1 waiting here. 3+2=5, here, I have 1 waiting. It makes 6. 1+1=2, and I have 1 waiting. The result is 3634.	

Question 15. 6000
-3009 Solve the subtraction given here, and explain how you solve to Ahmet by writing.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
10-9=1 9 0-9 9 0-9 5-3=2 2991 The bigger number cannot be subtracted from a smaller number, so, I take 1 decimal from the decimals digit. If it is not possible, I take a hundred from the hundreds digit, if it is not possible, I take a thousand from the thousands digit.	Calculation (Finding the result by subtracting)	6000 -3009 3091 We cannot subtract 9 from 0. We take a decimal from the neighbor. If we subtract 9 from 10, we have 1. We have 9 here. If we subtract 0 from 9, we have 9. If we subtract 0 from 0, again, we have 0. If we subtract 3 from 6, we have 3.	Not knowing how to make the operation (Transferring a decimal to the units digit from the thousands digit)
		6000 -3009 2001 It is not possible to subtract 9 from 0. I go to the neighbor, but, since they are 0, I go to 6. If we subtract 9 from 10, we have 1. There was 5 here. If we subtract 3 from 5, we have 2. The result is 2001	Not knowing how to make the operation (Transferring decimals to the units digit from the units digit by ignoring the hundreds and decimals digits)
		6000 -3009 2891 It is not possible to subtract 9 from 0. We need to go to the neighbor and take a decimal. Since there are 0 in the decimals and hundreds digits, we go to the thousands digit. We take from the thousands digit and give it to the hundreds digit. Then we take it from there, and give it to the decimals digit. We take it from there and give it to the units digit. We have 10-9=1. There is 9 left in the decimals digit. There is 8 left in the hundreds digit. If we subtract 3 from 5, we have 2. The result is 2891.	Not knowing how to make the operation (Subtracting two hundreds from the hundreds digit in the subtracting operation)

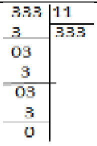
890
X 99
Question 19. Solve the operation to the left, and explain how you solve by writing below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**

8 9 0 x 9 9 8 0 1 0 8 0 1 0 8 8 1 1 0	Calculation (Finding the result by multiplying)	8 9 0 X 9 9 0 1 0 7 3 8 8 7 3 8 9 0 9x0=0, 9x9=81, 81; here, I write 1. 9x8=72, 72; here, I write 2. Then, I go to the other side. 9x9=81; here, I had 7 waiting, and it became88. 9x8=72; here, we had 1 waiting, it made 73. When we add them, it is 73890.	Not knowing how to make the operation
		8 9 0 X 9 9 8 0 1 0 8 0 1 0 1 6 0 2 0 9x0=0, 9x9=81; here, we write 1. 9x8=72 becomes 80 when added 8. Since all numbers are the same, here it becomes 8010. When we add them, we find 16020.	Not knowing how to make the operation (Not knowing digit transfer in multiplication)
		8 9 0 X 9 9 1 6 2 8 0 9x0=0, 9x9=81, 81; here, I write 8. 9x8=72; here, I write 2 from 72. I multiply8 by 2, and obtain 16. The result is 16280.	Not knowing how to make the operation (Writing the results obtained side by side)

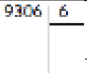
Question 21. 333 \| 11 Solve the division on the side, and explain how you solved it to Ali.

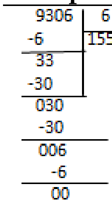
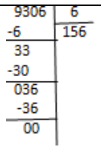
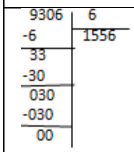
Sample answer that was accepted as correct	Students' strategies	Sample answer that was accepted as incorrect/insufficient	Students' mistakes
333 | 11 -33 | 30 003 Since we start division from the left, firstly, since there is no 11 in 3, I look into 33. Since there are 3 times of 11 in 33, we obtain 3x11, 33 and 33-33=0. Since there are no11 in 0, we take 3 below, and since there are no 11 in 3, I add 0 to the division section.	Calculation (Finding the result by dividing)	333 | 11 33 | 33 003 - 33 30 There are 3 times 11 in 33.3x11=33. The remaining 3 is taken down. There are 3 repetitions. Again, 33 we find. That's it.	Not knowing how to do the operation (When the small number is not divided by the bigger number, not knowing how to continue the operation)
		333 | 11 33 | 3,2 0030 - 22 8 There are no 11 in 3, therefore, we look for 11 in 33. We find 3 when we divide 33 by 11. 3x11=33. When we subtract, we have 0. Since there are no 11 in 3, we put 0 next to it, then, to continue to the operation, we put a comma to the division section. There are two 11s in 30, and we have 8 here. We cannot go on any more.	Not knowing how to make the operation (Not knowing when to put 0 and comma to the division section)

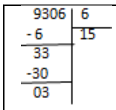
		 There are 3 times 1 in 3. If we subtract 3 from 3, we have 0. We take 3 down. We should always do the same operation.	Not knowing how to make the operation (Dividing each digit separately)

Question 24. 11000÷110 Explain how this operation can be solved in a short way by solving it.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
11000÷ 110= 100	Calculation (Finding the result by the rule of deleting 0)	11000 ÷ 110= 1100 11000÷ 110= 10 11000÷110= 110	Not knowing how to make the operation (Not knowing how to divide 10 and 10s in a short way)

Question25. -  Explain how this operation can be solved in a short way by solving it.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
 There are one 6 in 9. We write 1 to the division section. 1x6=6. If we subtract 6 from 9, we have 3. Since there are no 6 in 3, we take the other 3 down. There are 5 times 6 in 33. The operation goes on like this.	Calculation (Finding the result by dividing)	 If we divide 9 by 6, we have 1. We have 1x6=6. If we subtract 6 from 9, we have 3. We take the other 3 down. There are 5 times 6 in 33. We subtract again, we will have 3 again. We take 6 down again. There are 6 times 6 in 36.	Not including some numbers in the operation (Forgetting the 0 in the decimals digit of 9306)
		 We find 1 if we divide 9 by 6.We have 1x6=6. If we subtract 6 from 9, we have 3.We take the other 3 down. There are 5 times 6 in 33. 5 times 6 is 30. When we subtract it, we have 3 again. 0 goes down, and we have 5 times 6 in 30. We write 6 here.	Not including some numbers in the operation (Writing the 6 in the unit digit of 9306 to the division section without including it in the operation)

	 If we divide 9 by 6, we have 1. We have 1x6=6. If we subtract 6 from 9, we have 3. We take the other 3 down. There are 5 times 6 in 33. We have 5x6=30. We subtract 30 from 33, and have 3.	Not including some numbers in the operation (Not including the units and decimals digits of 9306 in the operation)

According to the findings in Table 8, all of the students who gave correct answers to the questions on four operations in natural numbers reached correct results by using the calculation strategy. Again, fourth graders gave incorrect answers to the questions because they did not know how to do four operations, and also, they did not include some numbers of the given number.

3.5 Findings on solving problems on the digit concept

Questions 11, 12, 13, 14, 16, 17, 18, 20, 26, 27 and 28 in the test were regarding the digit concept. The frequency and percentage values of the answers given by the students to these questions are given in Table 9. According to the findings given in Table 8, the correct answers provided by the students are as follows: 53% of 4th graders gave correct answers in the 11th and 16th questions, 49% in the 12th question, 57% in the 13th question, 58% in the 14th question, 42% in the 17th and 26th questions, 38% in the 18th question, 62% in the 20th and 28th questions, and 45% in the 27th question. Based on these findings, it is possible to argue that primary school 4th grade students are successful in solving problems that require addition, subtraction, multiplication and division in natural numbers.

Table 9: The frequency and percentage values of the answers given by the students to these questions

	Correct answer		**Incorrect answer**		**Empty answer**	
	f	%	f	%	f	%
11th question	159	53	138	46	3	1
12th question	147	49	150	50	3	1
13th question	171	57	120	40	9	3
14th question	174	58	126	42	0	0
16th question	159	53	135	45	6	2
17th question	126	42	165	55	9	3
18th question	114	38	180	60	6	2
20th question	186	62	105	35	9	3
26th question	126	42	165	55	9	3
27th question	135	45	165	55	0	0
28th question	186	62	114	38	0	0

The examples of the answers of primary school 4th grade students that were accepted as correct or incorrect/insufficient for problems related to the digit concept are presented in Table 10.

Table 10: Sample student answer

Question 11. How many decimals will make Ali obtain 87404 by adding to 87354? Show it by solving in the dotted area below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
87404 - 87354 ――― 00050 I start the operation from the right side with the unit's digit. Since 4-4 is 0, and it is not possible to subtract 5 from 0 in the decimals digit, I take 1 hundred from 4 in the hundreds digit and transfer it to the decimals digit. Since there are 1 hundred and 10 decimals, we have 5 decimals when we subtract 5 decimals from 10 decimals. Since there are 3 hundreds in the hundreds digit, it we subtract 3 hundred from 3 hundred, we have 0. We have 0 if we subtract 7 thousand from 7 thousand and 8 ten-thousand from 8 ten-thousand. The result is 5 decimals.	Calculation (Finding the result by subtraction)	We must add 50 decimals.	Confusing the multiplication value and the digit value.
		87404 -87354 09050	Not knowing how to make the operation (Erroneous decimals analysis)
		87404 -87354 00100	Not knowing how to do the operation (Not reducing decimals from the hundreds digit)

Question 12. 8a3a is a four-digit number. If the sum of digit values of "a "sis 202,what is the number value of "a"? Explain this operation by solving it.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
100 x a + 1 x a = 101 x a =202 a =2 8 x1000+ a x 100+ 3x 10+a 8000+100a+30+a=100a+a=2 x100+ 2a=2	Re-naming (Finding the result by analyzing the number)	2022 2101a =101 002 -2 0 Since there are two "a"s, and since they are equal to each other, we divide202 by 2.	Not being able to define the number value.
		2022+2=4a=4	
		101+101=202a=101	

Question 13. The list of some products and their prices in a market are as follows.

Product	Price
	4 decimals 11 units TL
	50 units TL
	1 decimals 3 units TL

In this context, how much will a person who buys 1 a clutch of eggs, 1 kg yogurt and 1 package flour will pay? Explain the operation by solving it below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
Eggs 4x10+11= 51 Flour =50 Yogurt =1 x10 +3 =13 we add them together. 50+51+13=114	Re-naming-Calculation (Finding the result by adding the numbers given in usual and unusual way)	1 a clutch of eggs 411,1 package of flour 50, 1 kg yogurt 13411+50+13= 474	Not being able to write the number given in an unusual way (Considering 4 decimals as 4 hundreds)
		Eggs 51, Flour500, Yogurt 13 500+51+13=564	Not being able to write the number given in an unusual way (Writing 50 units as 50 decimals)
		4,11 5,0 + 1,3 10, 41	Not being able to write the number given in an unusual way (Confusing with decimals)

Question 14. For which two of the above products will Ayşe pay more to the market? Explain it by solving this problem below

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
Eggs 4 decimals + 11 units=51 Flour 50 units=5051+50=101. For this reason, if I buy these two products, I will pay more.	Re-naming-Calculation (Finding the result by adding two numbers given in an unusual way)	Eggs 51 and yogurt 13.	Insufficient answer
		4.11+1.03=5.14 Eggs and yogurt	Not being able to write the number given in an unusual way (Confusing with decimals)

		114+51+50=215	Not being able to write the number given in an unusual way (Writing 4 decimals and 11 units as 11 decimals and 4 units)
Question 16. Ahmet has 426 liras. He gave 13 ten liras and 6 one Liras. How much money does Ahmet left? Explain it by solving.			
Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
13x10= 130, 130+6=136, 426-136=290 13 ten Liras makes 130 Liras. And oh, there is also 6 thousand Liras. If I add them, it makes 136 Liras. If I subtract this from Ahmet's money, I will find how much money he has left. The result is 290 Liras.	Re-naming-Calculation (Finding the number given in an unusual way, finding the result with operations)	426 413 -13 - 6 413 407	Not being able to write the number given in an unusual way (Writing in the form of 13 decimals and 13 units)
		26-13=13 If we subtract 13 decimals from 26 decimals, we have 13decimals. We also subtract 3 units from 6 units. 6-3=3	Not including all of the numbers in the operation.
		There are 40 decimals in 426. 40-13=27 27-6=23	Not analyzing the number (Not including the decimal in the decimals digit)
Question 17. If the decimal and hundred digits of 1896 change, how does the value of the number change? Explain this by solving below.			
Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
There is 9 in the decimals digit, and there is 8 in the hundreds digit. If we replace them, we have 1986. 1986-1896=90 Here, we have a reduction.	Calculation (Finding the result by subtracting)	1896-1698=198	Confusing the place of the digits (Replacing the units digit and hundreds digit)
		1986-1896=100	Not knowing how to make the operation
		1986-1896= 80 Here, we have an increase.	

Question 18. My father bought a refrigerator and a television to our house, and paid6541 Liras. Since he paid 1 thousand and 13 hundreds to the seller as the price of the television, how much is the price of the refrigerator? Explain it by solving.

Sample answer that was accepted as correct	Students' strategies	Sample answer that was accepted as incorrect/insufficient	Students' mistakes
1 thousand= 1000 1 hundreds= 100 100x 13=1300 1300+1000=2300 6541-2300= 4241	Re-naming-Calculation (Finding the number given in an unusual way, and finding the result with operation)	1000+130=1130 I add 1 thousand and 13 hundreds. If I subtract 6541,I find the price of the refrigerator. 6541-1130=5411	Not being able to write the number given in an unusual way (Writing 13 hundreds as 13 decimals)
		1000+13=1013 Since the price of the television is 1 thousand and 13 hundreds, I add them together. 6541-1013=5528 Then, if I subtract it from 6541,I find the result.	Not being able to write the number given in an unusual way (Writing 13 hundreds as if it were 13 units)
		6541-1013=5428 Since there are 1 thousand and 13 hundreds, which makes 1013 Liras, I subtract 1013 from 6541.	Not being able to write the number given in an unusual way and making mistake in the operation (Writing 13 hundreds as 13 units, and reducing decimals in the hundreds digit more than necessary)

Question 20. There are a total of 30 boxes in the greengrocery of Ali. Although there are 100 apples in 29 of these boxes, there are 59 apples in the 30th box. How many apples are there in the greengrocery of Ali? Explain it by solving below.

Sample answer that was accepted as correct	Students' strategies	Sample answer that was accepted as incorrect/insufficient	Students' mistakes
29 x100=2900 Since there are 59 in the last box, I add 59 to 2900. 2900+59=2959	Calculation (Finding the result by multiplication and addition)	29 x 100= 290 Since there are 100 apples in 29 boxes, I multiply by 100 and then, add 59. Because there are 59 apples in 1 box. 290+59=349	Not knowing how to make the operation (Not knowing how to multiply by 100 in a short way)

		100 x 29 000 + 1200 12000 + 59 12059 100 x 29 900 + 2 902	Not knowing how to make the operation

Question 26. A grandfather wants to divide 3207 liras evenly to his 3 children. How should this grandfather do this sharing? Explain it by solving below.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
3207 | 3 -3 | 1069 020 -18 27 - 27 00 There are 1 three in 3. 1 times 3 is 3. I subtract, and have 0. There is no 3 in 0. There is no 3 in 2. For this reason, I leave 0 here. Then, if I bring the 0 above, I have 20, and then I divide it by 20.	Calculation (Finding the result by dividing)	3207 | 3 -3 | 169 020 -18 27 27 00 I divided by 3 because there are 3 classes.	Not knowing how to make the operation (Not knowing when to add 0 to the division section)
		3207 | 3 -3 | 167 020 18 27 21 6	
		3207 | 3 -3 | 1690 020 -18 27 27 00	

Question 27. Ali grouped the eggs firstly in tens and obtained 500 boxes. Then he realized that the number of the boxes was too much, and decided to group the boxes in 100s. How many boxes will Ali have now? Explain the operation by solving.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**
10 x 500= 5000 5000 ÷ 100= 50	Calculation (Multiplication and finding the result by dividing)	500÷100= 5. 500 If I divide the box by 100, how many boxes can I have? The result is 5	Not including all data in the operation
		500 x 10= 5000 5000÷100= 500	Not knowing how to make the operation (Not knowing how to make division shortly with 100)
		10 ÷ 5= 2	Insufficient answer

Question 28. We want to distribute 125 books we have to the classes as 2 decimals and 5 units. To how many classes can we distribute these books? Explain it by solving.

Sample answer that was accepted as correct	**Students' strategies**	**Sample answer that was accepted as incorrect/insufficient**	**Students' mistakes**

125 | 25 -125 | 5 000 2 decimals and 5 units=25	Re-naming-Calculation (Finding the number given in the usual way, and finding the result with proper operation)	125 | 5 -125 | 50 000	Not including all data in the operation
		125 | 25 -120 | 6 000	Not knowing how to make the operation
		125 | 7 -7 | 17 55 -49 6 2 decimal+ 5 units=7	Not being able to write the number given in an unusual way (Writing 2 decimalsas2 units)

According to the findings in Table 10, primary school 4th grade students gave correct answers on the questions that were related to digit values by using calculation and re-naming strategies. Students gave incorrect answers to the questions due to the following reasons: Not being able to write the number given in an unusual way, not knowing how to do four operations, confusing multiplicity and number values, not knowing the value of the number, not including all the digits of the given number in the operation, confusing the places of the number, including the data that are not given in the question, not analyzing the number given, and insufficient answers.

4. Result and Discussion

Although this study, in which the level of knowledge of primary school fourth grade students on the digit concept in natural numbers and the ways of solving the questions were examined, was limited with the findings obtained from the sampling and data collection tool, it yielded very important results. When the correct answer rates of the students on the digit concept in natural numbers are considered, it may be argued that more than half (58%) of the students have good knowledge on the digit concept in natural numbers. However, when the correct answer rates of the acquisitions are examined in terms of 75% learning level in the study, it is seen that primary school 4th grade students have not achieved many acquisitions. The achievements that had a correct answer rate of 75% or more were K1 (89%), K2 (87%), K3 (76%) and K8 (87%). Although K1, K2 and K3 acquisitions are in the Natural Numbers sub-learning domain, K8 acquisition is in the Addition in Natural Numbers sub-learning domain. When this finding is considered, it is seen that primary school fourth grade students have serious difficulties in understanding and using the digit concept in natural numbers in an effective way in operations. Kamii (1988) investigated grades 1, 3 and 4 reporting that none of the 1st grades, 33% of the 3rd grades, and only 50% of the 4th grades could give correct answers to the digit concept. The results of our study are in line with the results reported by Kamii (1988). In their study, Thompson ve Bramald (2002) concluded that 4 students were very good at their step value, 14 students were good, 28 students were middle and 46 students were above middle.

When the correct answer percentages are accepted as the success rates, the sub-learning area in which the students were successful at the highest rate was Multiplication in Natural Numbers with an average correct answer rate of 62% (f=186). The sub-learning area in which primary school fourth grade students had the most difficulty with an average correct answer rate of 49% (f=147),was the Subtraction Operation in Natural Numbers The average correct answer percentages of Natural Numbers and Addition in Natural Numbers are the same (61%; f=183). Again, the average correct answer of students in the Division in Natural Numbers is 56% (f=168).

In general, the most successful achievement of primary school fourth grade students was the K1 acquisition, with an 89% (f=267) correct answer percentage. The most difficult acquisition was K11 with a correct answer percentage of 44% (f=132). Again, the question in which the students were successful at the highest rate was the 1st question with a correct answer percentage of 89% (f=267). The question the students had the highest difficulty was the 8th, 17th and 26th questions with a correct answer percentage of 42% (f=126). Dinc, Artut and Tarım (2006) reported that only 1.5% of 728 primary school fourth grade students could show 10 counting bars for "1" in the decimals digit of 16.

When the solution methods in the correct answers given by primary school 4th grade students to the questions in the success test were examined, it was seen that these were naming, re-naming, representation, prediction, comparison and calculation. Although students achieved the correct result mostly by using a single strategy in the question, they reached the correct result by using two strategies together in some questions. While the most commonly used strategy by the students was the calculation strategy, the least used strategy was the prediction strategy. When the strategies that were used by the students were examined in terms of the sub-learning domains, it was seen that the primary school fourth grade students mostly used the naming strategy in the Natural Numbers sub-learning domain, they had the correct result by using the calculation strategy in Addition, Subtraction, Multiplication in Natural Numbers and Division in Natural Numbers sub-learning domains.

Again, it is possible to argue that the correct forms of answers given by primary school 4th grade students to the The Achievement Test questions were mostly based on the usual rules taught at school (rounding rule, big/small rule, etc.). The students mostly found the correct answer by using a single and similar strategy, and the students applied the rules based on the digits that were taught at school rather than using student-invented strategies.

When the wrong ways of solving the questions in The Achievement Test of the 4th grade students are examined, it is seen that the students gave incorrect answers to the questions because of seventeen different mistakes. The students made more than one mistake in each question. The students had difficulty mostly in "writing the number given in an unusual way". According to these findings, again, the highest error type was detected in the questions that were on the digit concept (9 different mistakes), and the lowest error type was detected in the questions that required showing numbers by using symbols and models (2 different mistakes). Engelhardt (1977) identified eight different errors in his study in which he administered 84 arithmetic tests to 194 students. Students have difficulty with concepts 0 and 1 when calculating at an early age (Engelhardt, 1977). This is much more so in transactions involving the concept of 0 (Cockburn & Parslow-Williams, 2008). Children with zero and one concepts also frequently cause conceptual difficulties (Bamberger, Oberdorf and Schultz Ferrell 2010; Cockburn and Litter, 2008; Engelhardt, 1977; Van de Walle, Karp and Williams, 2014). Önal (2017) stated that the error made at the highest level within the mistakes made by the students was the error of "placing the digits in the wrong place" by 19.75%. When the mistakes made by the students according to the sub-learning domains were examined, it was seen that students had difficulties mostly in "writing a number that is given in an unusual way" in the Natural Numbers, Addition in Natural Numbers and Subtraction in Natural Numbers sub-learning domains. However, students gave incorrect answers to the questions that were related to "not being able to do four operations" in the Multiplication in Natural Numbers and Division in Natural Numbers sub-learning domains. Yorulmaz and Önal (2017) studies revealed that students made more mistakes in adding the eldest in the picking process, breaking the tenner in the subtraction process, scrolling the digits in the multiplication process, and assigning zero to the division in the splitting process.

Kubanç and Varol (2017) in their study, second and third grade students 'multiplication process; subtraction instead of multiplication, addition instead of multiplication, aggregate and subtraction rules to multiply generalization, continuous processing without scrolling steps and 0 and 1 with the multiplication rule to understand the most common misconceptions in questions that require multiplication. Varol and Kubanç (2015) in their study, the second and third grade students started the splitting process by generalizing the right-to-Start Process Rule, which is valid in addition to the addition, subtraction, and multiplication processes, and the splitting process by generalizing the right-to-start process, as in the addition and subtraction process, and generalizing. Erbaş, Çetinkaya and Ersoy (2009) stated that students' mistakes were mostly arithmetic or transactional. In his study of 140 students, Wallece (1984) stated that the most common mistakes were: not dominating basic addition and multiplication events; not understanding digit value and numbering, and confusion about subtraction and renaming. Again, Brown and Burton (1978) addition and subtraction students in the process of the mistakes they made; in total, the columns independent of each other, thinking the application to the properties of the extraction process the collection process, to attach the value of the zero digit, add the digits to the end of the collection process as in the production process, aggregate production process and at the same time as you add digits to the end of the process to the next column to the total ones ignore digits, the numbers in the figure as assessment, to be issued the number from the left alignment, the biggest steps from borrowing, expressed in the form of borrowing from scratch. The results of the study support the results of the earlier study.

Based on all these findings, when the wrong solution methods of the 4th grade students given to the questions in The Achievement Test are examined, it is seen that the students made similar mistakes. All of the mistakes that students made were caused by not understanding the digit concept. Considering these results, the following recommendations can be made on the things that must to be done in teaching environments. Firstly, teachers must examine the methods and techniques that may be used to explain the digit concept more effectively. Then, they must also determine the mistakes and deficiencies of students in this subject by using appropriate measurement tools. Finally, students must be encouraged to use more than one strategy to solve questions.

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