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SPATIAL VISUALIZATION TRAINING USING COMPUTER-AIDED CROSS SECTIONS OF SURFACES

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Abstract:
The aim of this study was to improve pre-service teachers' ability to infer cross-sections of geometric solids with Wolfram demonstrations and Mathematica in Analytic Geometry II course. The study was conducted with third year students studying Elementary Mathematics Education. In this study, the pre-test--post-test control group design was used. Both before and after the procedure, both groups were administered the Santa Barbara Solids Test (SBST) developed to measure their ability to mentally visualize the cross-section that results from the intersection of a cutting plane and a geometric solid. During the procedure, the participants in the control group were mainly asked to complete the graph drawings of geometric solids by just using plane cross-sections in paper-and-pencil format whereas those in the experimental group were supplemented with computer aided instruction in addition to these paper-and-pencil activities. In addition, the students in the experimental group were asked about their opinions on the procedure. The results showed a statistically significant increase in average achievement for both groups. On the other hand, this increase in the averages was significant for all the indices, and therefore for all problem types, of the experimental group SBST whereas it was significant only for the embedded orthogonal and embedded oblique items and for all problem types of the control group ($p<0.05$).

Keywords: cross-sections of surfaces, spatial visualization, mathematics pre-service teacher, Wolfram demonstrations

1. Introduction

Ability to relate two-dimensional representations and three-dimensional representations of geometric objects or to draw two-dimensional representations of three-dimensional objects on paper requires spatial visualization. In order to visualize a geometric solid three-dimensionally and draw a two-dimensional representation of it,

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one should first be able to identify the cross-section that results from the intersection of a cutting plane and a geometric solid (Cohen, & Hegarty, 2007). Inferring cross-sections of geometric solids and understanding and interpreting the spatial properties of these sections is required in a number of areas such as engineering, medicine, biology and geology as well as teaching. Therefore, mental visualization of the cross-sections of geometric solids and drawing their representations when necessary is an important ability that needs to be improved (Cohen, & Hegarty, 2012). In order to develop this ability in students, it first needs to be developed in teachers, who are designers of teaching-learning environments. Like in many countries, in Turkish educational system, the subject of geometric solids within geometry learning covers the ability "*to identify and construct the cross-section that results from the intersection of a cutting plane and a geometric solid*" as a learning outcome (MEB, 2009). This learning outcome involves observing what type of areas the cross-sections would be when appropriate models of geometric solids are taken and cut with a plane that is parallel, perpendicular or at any angle to their bases and determining the surfaces that can be formed by rotating plane geometric areas (e.g. rectangle, right triangle, etc.) around a side. In addition, the curriculum of departments of Elementary Mathematics Education as determined by the Turkish Higher Education Council includes surface area and volume calculations with definite integrals in Analysis I course (requires inferring cross-sections), graph drawing in multivariate functions in Analysis II course (requires inferring cross-sections), and surfaces in space, surface graph drawing, conics, finding cross-sections of conics and planes in Analytic Geometry II course (YÖK, 2006). It is a common problem to identify how a surface given in any way would appear. Analyzing this problem requires finding some of the properties of the given surface and drawing a graph of the surface or, in other words, representing the surface with a shape. The most challenging task at this point is inferring the cross-sections of the surface. Once the cross-sections of the surface are mentally visualized, it is relatively easier to mentally visualize the surface as a whole by considering their combination and, therefore, to draw its graph. Intersection of a surface with a plane is a planar curve. This curve is called cross-section of a surface with a given plane. The cross-sections of a surface can easily be found especially with planes parallel to coordinate planes. In general, the graph of a surface can be found with a sufficient number of cross-sections.

Studies have shown that the use of computer technology helps improve the teaching and learning processes of mathematics (Karakaş, 2011; Ersoy, & Akbulut, 2014; Andrade-Arechiga, Lopez, & Lopez-Morteo, 2012). According to Wu and Chiang (2013), "*Although instructors and students are often encouraged to use computer devices to promote effective teaching and learning, most instructors and students still struggle to use traditional methods, such as altering from pictorial drawings drawn by hand to learn the orthographic views*" (Wu & Chiang, 2013, p.29). Wu and Chiang (2013)'s research data shows the application of 3D computer animations results using 3D computer graphics also demonstrates a better visual comprehension for students, especially when objects are constructed by the complicated features (Wu & Chiang, 2013, p.28). The ever-increasing presence of computers has affected mathematics education, and it resulted in the development of software in this field. Computer Algebraic Systems (CASs) occupies a very important place among such software. CASs has become an essential tool in making the teaching and learning process become more meaningful (Dost, Sağlam, & Uğur, 2011). Since the early 1980's, there are commonly used several CASs in mathematics education, such as Mathematica, Maple, Mathcad, and Mathlab. These mathematical software packages have been equipped with visualization tools that can be used to display calculated results in 2D and 3D and CASs in mathematics education provides an impetus for educators to use visualization techniques (Mzoughi, Herring & Foley, 2007). The reasons of using Mathematica for this research are its user friendliness and the ease of drawing interactive 3D graphics. In addition, Wolfram Demonstrations Project is interactive mathematical demonstrations created using Mathematica (Petrusevski, Dabic, & Devetakovic, 2009). Wolfram Demonstration Project (http://demonstrations.wolfram.com) is an online collection of free and interactive drawings that has been rapidly growing in many fields such as mathematics, science, engineering, arts, etc. These demonstrations include improved and two- or three-dimensional graphs of mathematical functions. Each demonstration also includes animated previews and webpages with summaries of subjects and links for further information. The site's search interface is the way of finding content. The mathematics section is subdivided into applied mathematics, geometry, pure mathematics et al. All of demonstrations were completed with Mathematica open source code, so users with Mathematica can edit the open source code to modify a demonstration and Mathematica code is readable, so learning to create original demonstrations by reading the source code of existing ones is also relatively easy (Maclachlan, Bolte, & Chandler, 2009, p.108). "Cross sections of Quadratics Surfaces" and "Plane Sections of Surfaces" demonstrations are available from the freely downloadable Wolfram Demonstrations (http://demonstrations.wolfram.com/PlaneCrossSectionsOfTheSurfaceOfACone/ and http://demonstrations.wolfram.com/PlaneSectionsOfSurfaces/). The demonstrations chosen for the subjects facilitate interactive control of the participants and allow them to do the required operations by discovering interaction and changing the parameters of the model.

Undergraduate education plays a key role in the learning process because mathematics teachers gain their knowledge in their subject area regarding the learning outcomes in the curriculum during their undergraduate education and there are some points in the instruction of three-dimensional geometry that need to be improved. Identifying the cross-sections that result from the intersection of a cutting plane and a geometric solid is important for manipulating it three-dimensionally and drawing a two-dimensional representation of it. In this regard, the aim of this study was to improve pre-service teachers' ability to understand, interpret, and mentally visualize the properties of the cross-sections of geometric solids and to draw their representations when necessary by means of Mathematica, a computer algebra system with effective applications of numerical calculations and graph drawings, and Wolfram Demonstrations, an online collection containing interactive drawings prepared with Mathematica.

2. Material and Methods

In this study, the pre-test--post-test control group design was used. Both before and after the procedure, both groups were administered the Santa Barbara Solids Test (SBST) as a pre-test. The reason why a pre-test was administered was the assumption that the students' prior knowledge and learning experiences about the research subject, inferring plane cross-sections of geometric solids, could affect the research results. During the procedure, the participants in the control group were mainly asked to complete the graph drawings of geometric solids by just using plane cross-sections in paper-and-pencil format whereas those in the experimental group were supplemented with computer aided instruction in addition to these paper-and-pencil activities. After the procedure, both groups were re-administered the SBST as a post-test. In addition, the students in the experimental group were asked about their opinions on the procedure.

2.1 Participants

The study was conducted in the spring term of 2012-2013 with third year students studying Elementary Mathematics Education at Eskişehir Osmangazi University. A total of 20 volunteer students, 10 in the experimental group and 10 in the control group, participated in the study. The experimental and control groups were selected randomly.

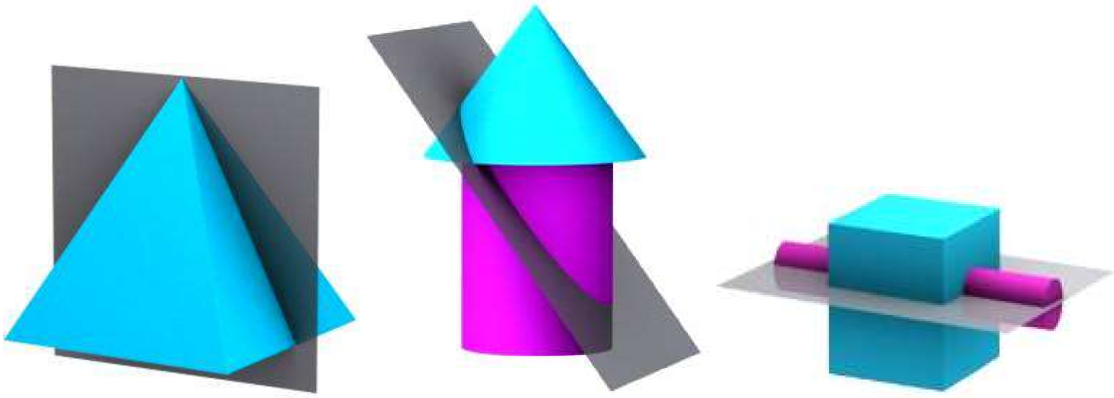
2.2 Data collection tools

The participants were administered the Santa Barbara Solid Test (SBST) at the beginning and end of the study. The SBST was developed by Cohen and Hegarty (2007) in order to measure the ability to mentally visualize the cross-section that results from the intersection of a cutting plane and a geometric solid. The test consists of 30 multiple-choice items. In these items, three-dimensional structures are classified as Simple, Joined and Embedded whereas cutting planes were categorized as Orthogonal and Oblique. Table 1 shows the distribution of questions in the 30-item test.

Table 1: Distribution of the questions in the Santa Barbara Solids Test

Cutting plane / Geometric structure	Orthogonal plane	Oblique plane (horizantal and vertical)
Simple item	1,4,13,19,28	7,10,16,22,25
Joined item	2,5,11,14,17	8,20,23,26,29
Embedded	6,12,18,21,24	3,9,15,27,30

Figure 1 shows respective examples of the questions in Simple Orthogonal, Joined Oblique and Embedded Orthogonal types in the SBST.



(a) (b) (c)

Figure 1: a) Single orthogonal, b) Joined oblique, and c) Embedded orthogonal figures from the Santa Barbara Solids Test

2.3. Procedure

Both before and after the procedure, in computer-based format, both the experimental group and the control group were administered the Santa Barbara Solids Test (SBST) developed to measure individual differences in spatial visualization ability that involves mentally visualizing the cross-section that results from the intersection of a cutting plane and a geometric solid.

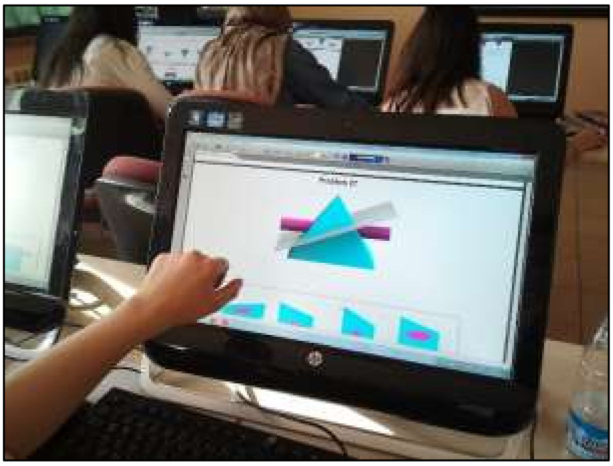


Figure 2: Computer-based pre-test procedure

This procedure consists of two stages. In the stage 1, both groups took part in the same activities. Firstly, the students were given paper-and-pencil activities requiring them to find the equations of certain plane cross-sections based on surface equation, draw these plane curves and obtain the surface graph. The researchers provided the participants with the necessary theoretical knowledge. In both the experimental group and the control group, activities involving surface equations and graph drawing were carried out with worksheets and in paper format in the classroom (see Figure 3).

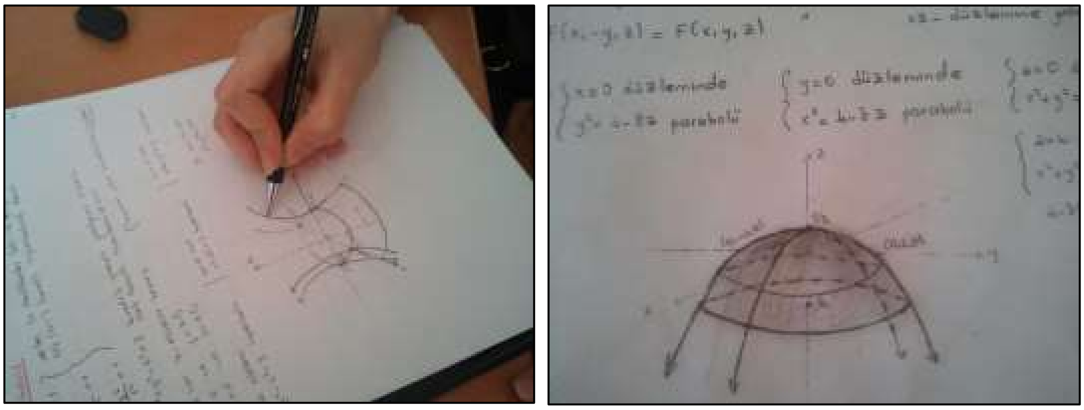


Figure 3: A sample drawing activity in the stage 1

In addition, the students shared their drawings on the board with their friends, they eliminated their possible errors and completed the activities in an interactive way (see Figure 4).

Figure 4: A student doing the activity on the board

In the stage 1, surface drawing activities to be done by inferring cross-section in paper-and-pencil format were presented in four class sessions.

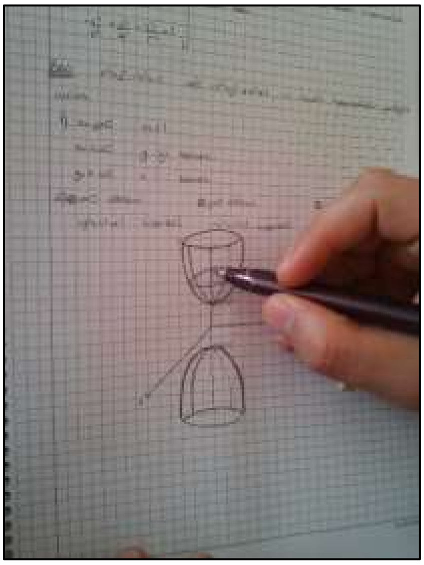


Figure 5: A sample control group activity in stage 2

In the stage 2, the participants in the control group were mainly asked to perform different surface equations and complete graph drawings in paper-and-pencil format in the classroom (see Figure 5) whereas those in the experimental group were asked to do activities prepared based on appropriate presentations in Mathematica and Wolfram Demonstration project in a computer room.

On account of the dynamic visualization and advanced interface of Mathematica, a Computer Algebra System (CAS), graph drawing activities were supplemented with computer aided instruction. Since the activities required the participants to have basic operational knowledge of Mathematica software, they were given a two-hour training on Mathematica after the procedure groups were determined. Following the activity pages prepared with Mathematica and surface visualization presentations, the participants interactively took part in discovery activities in the computer room. In this way, the participants had the opportunity to infer cross-sections by viewing a geometric solid from all perspectives. Surface graphs were drawn in Mathematica software and plane cross-sections were studied on these graphs (see Figure 6 and Figure 7).

ContourPlot3D[{z = 4 (x^2 + y^2), z = 8}, {x, -2.5, 2.5}, {y, -2.5,   2.5}, {z, 0, 50}]

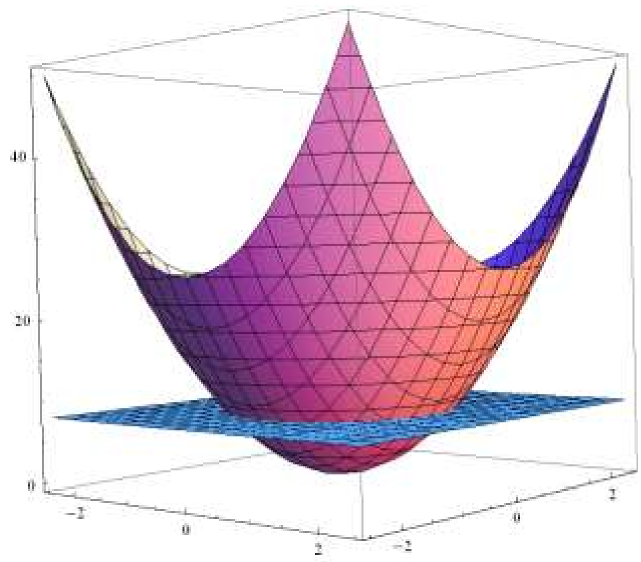


Figure 6: A sample screenshot showing the plane cross-section of a surface

ContourPlot3D[-x^2/a^2 - y^2/b^2 + z^2/c^2 = 1, {x, -5, 5}, {y, -5,   5}, {z, -5, 5}]]

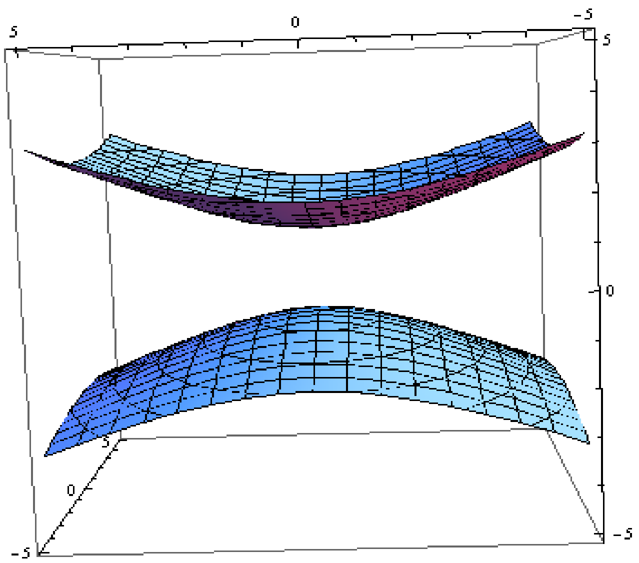


Figure 7: A sample screenshot showing the curves obtained from plane cross-sections on the surface

Also, in the stage 2, the experimental group worked with appropriate examples chosen from the online collection with free and interactive drawings in Wolfram Demonstration Project, which was prepared with Mathematica source codes. The freely downloadable Wolfram Demonstrations entitled: "Cross sections of Quadratics Surfaces" and "Plane Sections of Surfaces" available at http://demonstrations.wolfram.com/PlaneCrossSectionsOfTheSurfaceOfACone/ and http://demonstrations.wolfram.com/PlaneSectionsOfSurfaces/. The demonstrations chosen for the subjects examined in this study facilitated interactive control of the participants and allowed them to do the required operations by discovering interaction and changing the parameters of the model (see Figure 8).

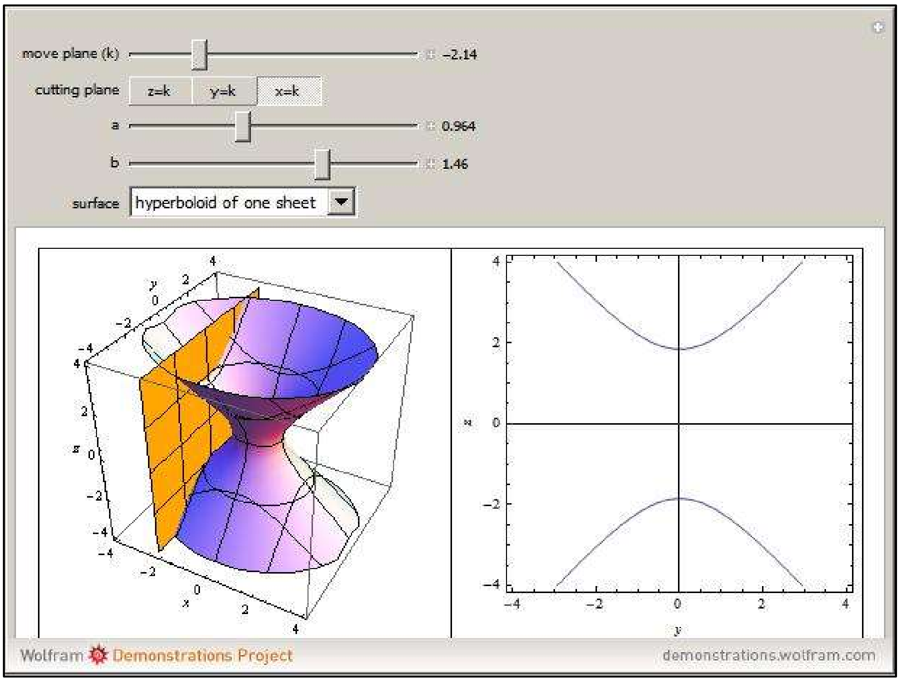


Figure 8: Cross-section of the hyperboloid of one sheet

Since Wolfram Demonstrations are designed as dynamic webpages with animated previews of each special surface, the participants had the opportunity to access these pages and see the different cross-sections on each surface in a dynamic way. Figure 9 shows a Wolfram screenshot displaying the cross-section curve "hyperbole" resulting from the intersection of hyperbolic paraboloid surface and z=-0.87 cutting plane and the corresponding status of the surface and intersection plane according to each other.

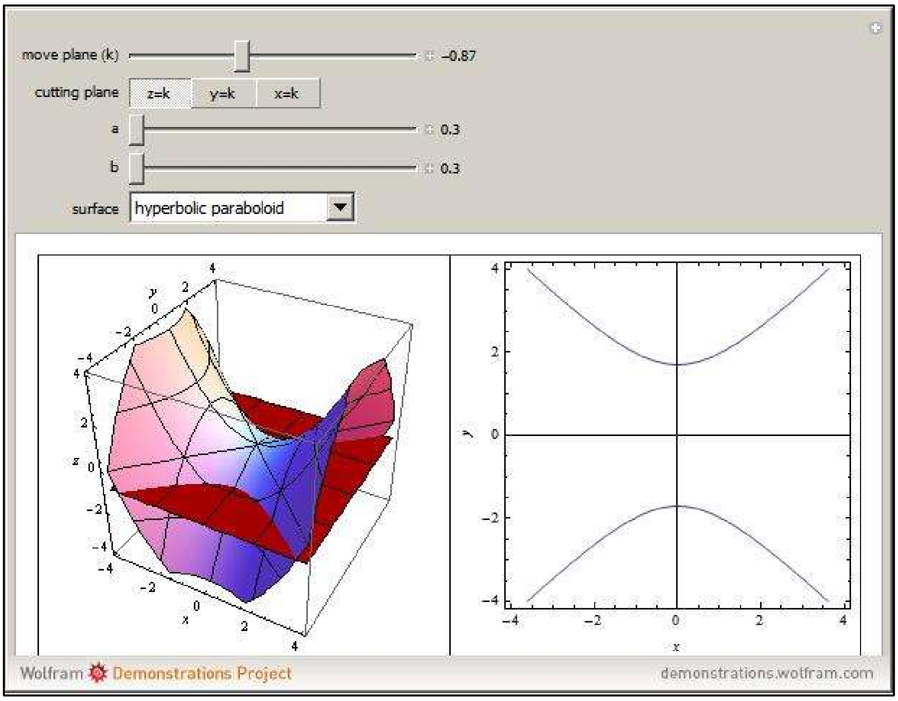


Figure 9: A screenshot displaying the cross section of hyperbolic paraboloid

As can be seen in the activity samples in Figure 8 and Figure 9, the activities allowed the participants to infer cross-sections by viewing the geometric solid in all perspectives. Also, by means of the surface graphs drawn in Mathematica, they examined the plane cross-sections on these graphs.

At the end of the study, the SBST was re-administered to both groups as the post-test.

2.4 Data analysis

The data obtained with the tests and questionnaires conducted in accordance with the pre-test and post-test experimental research design were analyzed using arithmetic mean and standard deviation for descriptive statistics. The differences between the groups were analyzed using non-parametric methods, the Mann-Whitney U test and the Wilcoxon signed-rank test, because n<30.

3. Results

For the SBST that was given before the procedure to determine the difference between the experimental and control groups in terms of their ability to infer cross-sections, the pre-test score means of the experimental and control groups were 0.46 and 0.47, respectively (see Table 2). Table 2 shows the comparison of the mean scores of the experimental and control groups.

Table 2: Differences between the mean scores of the experimental and control groups

	Experimental Group(n=10)		**Control Group (n=10)**	
	Mean	**SD**	**Mean**	**SD**
Pre-test	0.46	0.19	0.47	0.28
Post-test	0.75	0.15	0.55	0.25
Differences	0.29	0.04	0.08	0.03

As can be seen in Table 2, the pre-test mean scores of the groups were very close to each other whereas the mean score of the experimental group in the post-test was higher than that of the control group (M=0.55 < M=0.75). The net difference between the mean gains was 20.00.

Mann-Whitney U test was carried out in order to determine whether the difference between the pre-test mean scores of the groups was statistically significant or not. Table 3 shows these results.

Table 3: Mann-Whitney U Test Results According to Experimental and Control Group Pre-test Scores in the SBST Indices

Indexes	**Group**	**N**	**Mean**	**Rank**	**M.W.U**	
Simple Orthogonal	Experiment	10	10.1	101	54	No difference
	Control	10	10.6	106		
Simple Oblique	Experiment	10	10.4	104	69	No difference
	Control	10	8.6	86		
Joined Orthogonal	Experiment	10	7.35	73.5	30.5	No difference
	Control	10	12.45	124.5		
Joined Oblique	Experiment	10	11.2	112	71	No difference
	Control	10	8.4	84		
Embedded Orthogonal	Experiment	10	9.3	93	62	No difference
	Control	10	11.7	117		
Embedded Oblique	Experiment	10	9.85	98.5	55.5	No difference
	Control	10	9.95	99.5		
All problem types	Experiment	10	10.6	106	49	No difference
	Control	10	10.4	104		

As can be seen in Table 3, in the pre-test, all the index scores of the experimental group were higher than those of the control group except for the SBST Simple Orthogonal, Joined Orthogonal, Embedded Orthogonal and Embedded Oblique indices. Using a table of critical U values for the Mann-Whitney test shows that For N1 = 10 and N2 = 10, the critical value of U is 27 (p>.05). Our obtained U values which all the index scores in the SBST are higher than this critical value. Then, there was no significant difference between the experimental and control group scores for the SBST indices according to the Mann-Whitney U Test results. Therefore, it can be suggested that the experimental and control groups had similar levels of achievement in terms of inferring surface cross-sections before the procedure.

Mann-Whitney U test was carried out in order to determine whether the difference between the post-test mean scores of the groups was statistically significant or not. Table 3 shows the index scores of the SBST that was administered to the experimental and control groups as the post-test and the results of the Mann-Whitney U test.

Table 4: Mann-Whitney U Test Results According to Experimental and Control Group Post-test Scores in the SBST Indices

Indexes	Group	N	Mean	Rank	M.W.U	
Simple Orthogonal	Experiment	10	12.05	120.5	65.5	No difference
	Control	10	8.95	89.5		
Simple Oblique	Experiment	10	12.2	122	67	No difference
	Control	10	8.8	88		
Joined Orthogonal	Experiment	10	10.8	108	47	No difference
	Control	10	10.2	102		
Joined Oblique	Experiment	10	13.85	138.5	16.5	**Difference**
	Control	10	7.15	71.5		
Embedded Orthogonal	Experiment	10	11.1	111	44	No difference
	Control	10	8.2	82		
Embedded Oblique	Experiment	10	12.15	121.5	34	No difference
	Control	10	8.65	86.5		
All problem types	Experiment	10	13.15	131.5	23.5	**Difference**
	Control	10	7.85	78.5		

As can be seen in Table 4, in the post-test, the scores of the experimental group in all the SBST indices and for the entire problem types were higher than the control group scores. On the other hand, there was a statistically significant difference between the experimental and control group scores for only the SBST Joined Oblique and all the problem types according to the Mann-Whitney U Test results ($p>.05$).

This part of the study presents the results about the comparisons between the pre-test and post-test scores of the experimental and control groups. Table 5 shows the SBST indices pre-test and post-test mean scores, standard deviation and Wilcoxon Signed-Rank Test results for the experimental group.

As can be seen in Table 5, there was a significant difference between the pre-test and post-test scores of the experimental group for all the indices of the SBST ($p<0.05$). Therefore, according to Table 5, there was also a statistically significant difference between the pre-test and post-test scores in all the problem types ($p<0.05$).

Table 5 shows the SBST indices pre-test and post-test mean scores, standard deviation and Wilcoxon Signed-Rank Test results for the control group.

Table 5: The SBST indices pre-test and post-test mean scores, standard deviation and Wilcoxon signed-Rank test results for the experimental group

	Experimental Group(n=10)					
Indexes	**Pre-test**		**Post-test**			
	$\overline{X}$	**S**	$\overline{X}$	**S**	**z**	**p**
Simple Orthogonal	0.44	0.29	0.68	0.25	2.52	0.0059
Simple Oblique	0.5	0.25	0.64	0.26	2.20	0.0139
Joined Orthogonal	0.4	0.18	0.76	0.26	2.42	0.0078
Joined Oblique	0.48	0.25	0.88	0.13	2.66	0.0039
Embedded Orthogonal	0.38	0.25	0.86	0.18	2.80	0.0026
Embedded Oblique	0.32	0.25	0.72	0.16	2.66	0.0039
All problem types	0.46	0.19	0.75	0.15	3,87	0.0000

According to Table 6, there was an increase in the control group in all the SBST indices. There was a significant difference between the pre-test and post-test scores in Embedded Orthogonal and Embedded Oblique indices and in all the problem types ($p<0.05$). On the other hand, there was not a statistically significant increase between the pre-test and post-scores of the control group in the Simple Orthogonal, Simple Oblique, Joined Orthogonal, and Joined Oblique indices.

Table 6: The SBST indices pre-test and post-test mean scores, standard deviation and Wilcoxon signed-rank test results for the control group

	Control Group(n=10)					
İndexes	**Pre-test**		**Post-test**			
	$\overline{X}$	**S**	$\overline{X}$	**S**	**z**	**p**
Simple Orthogonal	0.46	0.28	0.5	0.32	1.00	0.1587
Simple Oblique	0.4	0.32	0.48	0.23	1.60	0.0548
Joined Orthogonal	0.68	0.34	0.72	0.28	1.60	0.0548
Joined Oblique	0.36	0.27	0.44	0.36	1.60	0.0548
Embedded Orthogonal	0.54	0.37	0.66	0.25	2.36	0.0091
Embedded Oblique	0. 4	0.33	0.5	0.36	2.02	0.0217
All problem types	0.47	0.28	0.55	0.25	2.20	0.0139

3.1 Students' opinions and researchers' observations

After the procedure, the students in the experimental group were asked about their opinions about the computer aided instruction that they experienced during this study. There were 10 students in the experimental group and the opinions of three of them were quoted below. The following is the opinion of a student (Caner) who correctly answered eight questions in the pre-test and 22 questions in the post-test regarding his experience of computer-aided instruction:

> *"We studied this subject with Mathematica and Wolfram demonstrations because they made it possible for us to see and mentally visualize three-dimensional solids. They also made it possible for us to visualize drawings we would normally prepare in a two-dimensional environment. I think they improved our three-dimensional perspective."*

The following is the opinion of a student (Merve) who correctly answered three questions in the pre-test and 18 questions in the post-test:

> *"What we see and draw on the classroom board is limited. But the computer environment let me view and mentally visualize a solid from all perspectives. I think we need computer support and this software to infer surface cross-sections and to draw a graph of a solid. I believe that it is really useful in terms of using both time and the course efficiently."*

The following is the opinion of a student (Pınar) who correctly answered five questions in the pre-test and 10 questions in the post-test:

> *"Because of the computer practice, it was much easier for us to get involved in three-dimensional learning. When I first took the test, I found it really difficult. But what we saw in this software was different from what I drew. So I don't have any difficulty preparing a drawing."*

In addition, the researchers observed that, during the activities requiring them to draw the graph of the given surface equation in the first stage of the procedure, the student had difficulty in identifying the type of the conics she found when she inferred the cross-section of the surface with planes. The student had difficulty in drawing the surface graph because she did not know which curve the conic equations that she found represented. That is why she stated that her drawing on the paper and her drawing on the computer were different from each other. Students normally need to infer various plane cross-sections accurately so that they can mentally visualize a surface. The experimental group students had difficulty in inferring cross-sections on paper. However, when they became engaged in Mathematica and Wolfram demonstrations, they had the opportunity to see these cross-sections on computer through these dynamic demonstrations. In this way, they were able to realize and correct their errors. The students in the control group, on the other hand, did the activities in the second stage of the procedure on paper and the classroom board to draw graphs of geometric solids by inferring cross-sections. The drawings that the control group students made during these activities and those they made to correct their errors were sometimes confusing for them. Also, the paper-and-pencil drawing activities were time-consuming. Figure 10 shows a graph drawing that one of the students made by inferring the cross-section of an ellipsoid with equation

$$\frac{x^2}{12}+\frac{y^2}{5}+\frac{z^2}{7}=1$$

with $x=0, y=0, z=0$ planes.

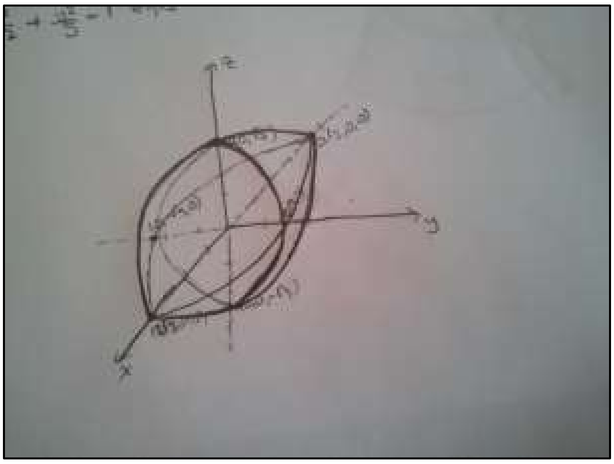


Figure 10: Drawing of an ellipsoid made by using plane cross-sections

4. Conclusion

This study investigated the effect of computer aided teaching method using Mathematica, a computer algebra system, and Wolfram demonstrations on pre-service teachers' performance in inferring cross-sections. The results revealed that, although the two groups in the study had similar levels of knowledge before the procedure, the achievement score of the group that learned the subject with computer aided teaching method increased from 0.46 to 0.75 whereas the achievement score of the group that learned the subject through paper-and-pencil and lecture method increased from 0.47 to 0.55. The results showed a statistically significant increase in average achievement for both groups. On the other hand, this increase in the averages was significant for all the indices, and therefore for all problem types, of the experimental group SBST whereas it was significant for the embedded orthogonal and embedded oblique items and for all problem types of the control group ($p<0.05$). In this regard, in both the experimental group, who were taught with computer aided instruction, and the control group, who were taught through paper-and-pencil drawing activities, achievement of the pre-service teachers was affected significantly. On the other hand, the significant increase in the control group was observed only in the embedded orthogonal and embedded oblique plane cross-sections. Although embedded solids seemed to have a more complicated structure than simple and joined solids (see Figure 1), the increase in the achievement score of the control group, who were taught through paper-and-pencil drawing activities, can be regarded as a research topic that needs examining qualitatively. In addition, the scores of the experimental group, who were assisted with computer aided instruction, than those of the control group in all the indices and problem types of the SBST, there was a significant difference between the experimental and control group scores for only SBST Joined Oblique and for all the problem types in favor of the control group ($p>.05$). The researchers' observations showed that the drawing activities performed in only paper-and-pencil format during the activities about inferring cross-sections led to a slight increase in achievement, but the increase was substantial when these activities were accompanied by computer aided activities. Restricting three-dimensional activities to paper-and-pencil activities may cause students to fail to notice some views and, therefore, to come up with misconceptions. The use of dynamic computer software in three-dimensional activities, however, allows students to view a surface from all perspectives by rotating it 360 degrees. We could suggest that viewing solids from all perspectives makes it easy to mentally visualize the cross-sections of these solids with different planes and, therefore, to identify the curves resulting from these cross-sections. Computer aided learning activities facilitates students' three-dimensional thinking. Because the static image obtained by the rotation operation on a computer screen is two-dimensional, it corresponds to the drawing in paper format (Christou, Pittalis, Mousuulides, & Jones, 2007). In this way, computer aided learning activities provide students with the opportunity to view the images that they cannot normally see when they study directly in paper format, which increases their achievement. The classroom observations also showed that, because the students engaged in computer aided learning activities acquired the skills to view a solid as a whole, they performed better in identifying the cross-sections inferred with different planes by mentally rotating that solid.

The researchers' observations during the procedure and the students' opinions about computer assisted learning activities provided some implications as well. To begin with, the students stated that computer software made three-dimensional visualization easy for them in making two-dimensional drawings of three-dimensional solids. In addition, they stated that their drawings in paper-and-pencil environment or on the classroom board caused them to think in a restricted way whereas the dynamic Wolfram demonstrations on computer made it possible for them to view solids from all perspectives and, therefore, they were able to better visualize both solids and their plane cross-sections. They stated that they found these demonstrations useful in terms of using both time and the course efficiently. Moreover, one of the students stated that the graph of the geometric solid that she prepared on paper was not similar to the graph drawing on the computer screen and she had difficulty in drawing graphs. On the other hand, being able to infer different plane sections of a solid when drawing its graph, to identify the type of the curves resulting from these cross-sections, and to accurately draw its two-dimensional graph would help her draw an accurate graph of that solid. For this reason, making sure that pre-service teachers acquire these skills as a part of their pedagogical content knowledge. These results suggest that providing pre-service teachers with graph drawing activities performed solely in paper-and-pencil format may prove insufficient in limited durations of time. Therefore, using dynamic software activities in teaching-learning environments can enhance the efficacy of teaching.

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