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PRE-SERVICE TEACHERS' UNDERSTANDING OF SELECTED CONCEPTS OF FRACTIONS

Korsi Kenneth Agbozo[1],
Jonathan A. Fletcher[2i]
[1]Presbyterian College of Education,
Ghana
[2]University of Ghana-Legon,
Ghana

Abstract:
The case study investigated prospective teachers' understanding of concepts of fractions following the IOE's chief examiners' reports that have raised concerns about the persistent abysmal performance of the pre-service teachers on items on fractions in mathematics examinations in the colleges of education in Ghana. The case study was conducted in one college of education in the Central Region of Ghana with a sample of 26 pre-service teachers using a mixed method of sequential explanatory design approach. The participants took an achievement test followed by interview with the view to gaining insights into their understanding of specific concepts of fractions. The results indicated that although almost all of the prospective teachers demonstrated high levels of computational competence, none of them was able to demonstrate an understanding of why the algorithm for the division of a fraction by a fraction works. With regard to their Pedagogical Content Knowledge (PCK) of fractions, almost half of the participants could not generate any approach of teaching division of fractions to demonstrate an effective PCK. It was recommended that a new approach of teaching fractions in which connections are made between topics related to fractions be adopted by mathematics tutors in the colleges of education to guide prospective teachers to acquire a deeper conceptual understanding of fractions and their applications.

Keywords: conceptual understanding, pre-service – early year practitioners, critical reflection, personal growth, professional development

1. Introduction

Mathematics is crucial to the world's development and can be considered a universal language. According to Department for Education and Science [DfES] (2006), the need to

[i] Correspondence: email kinggbozo@yahoo.co.uk, dcjonfletcher@aol.com

give every child a good background in mathematics is seen as the way to overcome economic and social disadvantage and make equality of opportunity a reality for every child. Indeed, without functional numeracy pupils would find it almost impossible to succeed because of the difficulty they would have in accessing the secondary curriculum. Also, on a personal basis, mathematics develops the intellect, self-reliance, self-confidence, attitude of discovery and the power of concentration. According to Hamilton (cited in Sidhu, 1995), "*the study of mathematics cures the vice of mental distraction and cultivates the habit of continuous attention*" (p.11). Considering its significance therefore, one cannot but bemoan the falling standards of mathematics in Ghanaian colleges of education at a time the world is moving towards science, technology, engineering and mathematics (STEM) era. Prospective teachers in these colleges are required to deliver the basic education curriculum when they complete their various diploma programmes. Yet these prospective teachers are found to be struggling with certain basic concepts in mathematics (Institute of Education, 2011). Among these basic concepts are concepts of fractions and their applications in real-life situations (Davis & Ampiah, 2009).

Elsewhere, studies on prospective teachers' knowledge and understanding of some concepts in mathematics have called for an increased investigation into teachers' conceptual understanding of different mathematical concepts. A search into literature by Ball, Lubienski, and Mewborn (2001), for example, revealed that on multiplication and place value, division, measurement and rational numbers, teachers were able to perform the algorithms but could not provide explanations for the place value concepts; or why the division algorithm works. They assumed that as perimeter increases, the area also increases directly; and were unable to find a number, one-third of the way from two given fractions. Other researchers have explored how prospective teachers think about their mathematical knowledge and how they understood or misunderstood topics like rational numbers (Tirosh, Fischein, Graeber, & Wilson, 1998); measurement (Bantura & Nason, 1996); and fractions (Lin, 2009).

Within the last decades, from the late 1990s, there has been a renewed interest not only in teacher content knowledge and pedagogical content knowledge but also in teachers' conceptual understanding of the mathematical knowledge they have (Ball, Lubienski, & Mewborn, 2001). Researchers have found a positive correlation between teachers' content knowledge and their students' gains in learning mathematics (e.g., Wilmot, 2009; Hill, Rowan & Ball, 2005) and there is a substantial positive effect of pedagogical content knowledge on students' learning gains (Baumert, Kunker, Blum, Brunner, Voss, JorCharity, Krussman, Krauss, Nembrand, & Tsai, 2010).

The implication of the positive correlation between teachers' knowledge and their students' learning is that, students are at a loss when it comes to showing understanding in the concepts learned if their teachers do not understand the concepts themselves (Ball, Lubienski, & Mewborn, 2001). Thus, the students' difficulties are the result of the fragmented mind possessed by those who teach them (Ball, 1990) and the rigid and segmented content knowledge they have in mathematics (Tirosh, Fischbein, Graeber, & Wilson, 1998). To teach effectively in basic schools, teachers are required not only to have a strong background in mathematics content but also to develop a deep conceptual understanding of the content. This type of knowledge is referred to as *knowledge of mathematics for teaching* (Hill, Rowan & Ball, 2005; Ball, 1990) and it is what Ma (1999) calls *profound understanding of fundamental mathematics* (PUFM), since it provides a vision of the ideal structure of elementary teachers' conceptual and procedural knowledge.

The response to ensuring these requirements (strong background in mathematics content and deep conceptual understanding of the content) in Ghana was the Joint School Project (JSP) initiative, producing new mathematics course for secondary schools in West Africa, up to school certificate level (Lockard as cited in Mereku, 2000). This was to change the method and approach to mathematics teaching and learning from rote memorization to problem-solving and problem-posing and also from giving and receiving of information by students and teachers respectively to facilitating and constructing knowledge by teachers and students respectively (Fletcher, 2000). According to Fletcher, programmes such as the Whole School Development (WSD) were undertaken through a number of workshops organised by the Teacher Education Division (TED) of the Ghana Education Service (GES), all in a bid to shore up teacher competence and productivity. For these changes to take place, there were corresponding policy changes in the institutional direction towards the effective implementation of a new delivery mode of the mathematics curriculum. Yet these policy changes could not bring the desired change because of a lack of change in the teaching patterns of mathematics teachers (Fletcher, 2005).

Put differently, quality education can only be given through quality teacher development (Fletcher, 2001). It is in line with this that the Educational Reform Review Committee of Ghana (MOESS, 2002) acknowledged the deciding role of teachers and suggested, among other things, the upgrading of teacher education through the reform of teacher training institutions to update trainees' competences and skills to enable them to offer prime teaching and quality learning schemes in our basic schools. The colleges of education in Ghana then had and still have the responsibility of training pre-service teachers to meet such aspirations as those expressed in the set of objectives and aims in the mathematics curriculum for the basic schools, including creating critical thinking among pupils (Ministry of Education [MOE], 2001, pp. ii-iii). In spite of the awareness of the relationship between teacher knowledge and skills in mathematics and student performance in the subject, there continues to be a persistent fall in students' performance in mathematics at basic and senior high levels of education (West African Examinations Council [WAEC], 2011, 2012).

Indeed, WAEC mathematics examiners' reports since 2003 have identified problem areas such as basic number operations, division, fractions, measurement and geometry (WAEC, 2012). Further, the colleges of education (COE) chief examiners' reports indicate that prospective teachers show weakness in manipulating negative numbers and fractions and in supporting answers with reason or geometric facts (Institute of Education, 2011, 2008). If prospective teachers who are to deliver the basic education mathematics curriculum suffer from such deficiencies, then stakeholders of education in Ghana have every cause to worry because students' achievements are driven by teachers' ability to understand and use subject-matter knowledge to carry out the tasks of teaching (Ball, 1990; Shulman & Richert as cited in Hill, Rowan, & Ball, 2005). This study therefore attempted to bring to the fore, what constitutes the nature of the pre-service teacher's conceptual understanding and their pedagogical content knowledge in fractions.

The topic of *fractions* is considered a very difficult topic for teachers to teach and for pupils to learn (e.g., Davis, Bishop & Seah, 2010). It had been pointed out earlier that teaching as well as learning of fractions has traditionally been one of the most problematic areas in elementary school mathematics (Lamon, 2005; Delaney, Charalambous, Hsu & Mesa, 2007). Tirosh (1998) had earlier listed some of the reasons (given by other authors cited in Tirosh) why students have difficulty in fractions. These included children not having the same everyday experience in using fraction as they do with whole numbers and because children, more often than not, encounter their activities in the form of counting whole digits and, as a result, tend to consider fractions in the same light. Hiebert and Lefevre (1986) had also observed that children find it difficult to accept a given fraction as a number and tend to view it as two whole numbers. Most students refused to see fractions as a unit or quantity in their own right because of the idea of the divisional character that fractions assume. Further, mathematical terms such as fraction, numerator, denominator, prime numbers, and prime factors, carrying and borrowing may be problematic to students as well (Addy, 2006).

The topic of *fraction division* is particularly difficult for students including prospective teachers (Tirosh, 2000; Ball, 1990) and in-service teachers as well (Ma, 1999). The algorithmic procedure of fraction division presented to students as "invert and multiply" is easily learned. Findings of Li and Smith (2007) showed that prospective teachers' knowledge in content and pedagogy was procedurally sound and yet conceptually weak. They wrote that this gap between knowledge and concepts attest that "these prospective teachers did not know what they would be expected in order to develop effective teaching. Their confidence was built upon their limited knowledge in mathematics and pedagogy" (p.187). The majority of the participants (about 93%) were reported to have computed $\frac{7}{9} \div \frac{2}{3}$ correctly, however, none of these prospective teachers tried to explain the computations as why one can flip and multiply (e.g., why one can transform "divide $\frac{2}{3}$ by 2" to "multiply $\frac{2}{3}$ by $\frac{1}{2}$") (p.189).

The difficulty of the fraction to the Ghanaian prospective teacher just as any other topic in mathematics is even more so because according to Wilson (cited in Addy, 2006) mathematics teaching in Africa has long been different from what pertains to the Western world. This is because factors such as lack of appropriate textbooks, materials for activities as well as overcrowded classrooms have been responsible for the adoption of the traditional chalk-and-talk method by most teachers. This might contribute to increase the difficulty most students have with mathematics, which translates into their adult life in the application of the subject. Since mathematical literacy is conceptually abstract and difficult to comprehend and communicate in a meaningful way, it is incumbent upon prospective teachers to have a deep understanding of mathematical concepts generally, and acquire appropriate pedagogical content knowledge of fraction, in particular in order to effectively facilitate the learning of the requisite knowledge of fraction among their prospective students.

Other studies on fractions (Tsay & Hauk, 2007; Ball,1990, 1988) show that when teachers are presented with critical classroom situations they are unable to explain and represent those contents to students due to insufficient understanding that limits their capacity to do so, lacking what Ma (1999) describes as PUFM. Despite these findings, there are no known study of the Ghanaian prospective teacher's conceptual understanding of different aspects of fractions that has been done by mathematics education researchers in Ghana.

Regarding procedural knowledge, it is incumbent upon a mathematics teacher to possess the capability to solve mathematical problems using a procedure aside having the understanding of the concepts to be able to explain such concepts to students. This knowledge of mathematics procedures is referred to as procedural knowledge (e.g., Lin, 2009; Ball, 1990) or procedural competence (Star, 2002). According to Eisenhart et al (as cited in Zakaria & Zaini, 2009), "*procedural knowledge is the mastery of computational skills and familiarity with procedures for identifying mathematical components, algorithms and definitions*" (p.202). This form of knowledge aids the student or teacher to work mathematics methodically following a known procedure. Thus, procedural knowledge refers to a teacher's or student's ability to obtain an answer using a valid method without *necessarily* (italics ours) knowing why a certain step, method, operation, or formula is used or works in the problem-solving process (Forrester & Chinnappan, 2010).

On the other hand, possessing conceptual knowledge or procedural understanding requires that a student working on mathematical topic has the knowledge to link pieces of fundamental ideas about the topic to arrive at the desired solution (e.g. Haung, Liu, & Lin, 2009: Zakaria & Zaini, 2009; Rayner, Pitsolantis & Osana, 2009) or procedural understanding (e.g. Jansen & Spitzer, 2009; Li & Smith, 2007; Nillas, 2003). Conceptual knowledge (CK) or procedural understanding (PU), according to Hiebert and Lefevre (1986) and re-echoed by Lin (2009), is a connected web of knowledge, a network in which the linking of relationships is as prominent as the discrete pieces of information. Eisenhart et al (cited in Haung, Liu & Lin, 2009) refer to CK as the underlying structural relationships of mathematics and the interconnections of ideas that explain and give meaning to mathematical procedures. This means that relationships pervade individual facts and propositions so that all pieces of information are linked to some network. CK can, therefore, be seen as characterized by well-knit relationships. Thus, CK provides the basis for why a formulae works, why that idea is deemed justified and how the idea is connected or related to other mathematical ideas; and facilitating understanding of abstract principles (Schneider & Stern, 2010).

A conceptually knowledgeable prospective teacher is confident and ready to handle students' misconceptions in diverse ways. Li and Smith (2007) reported that prospective teachers mostly revealed confidence and knowledge in preparedness to teach and showed that their knowledge in mathematics and pedagogy for teaching fraction division was procedurally sound but yet conceptually weak. This is enough to suggest that their confidence was built not on their procedural understanding but on their limited knowledge in content. Historically, knowledge bases of teacher education programmes have focused on the content knowledge of the teacher (Shulman cited in Piccolo, 2008). From the mid-1980s, teacher education shifted its focus primarily to general pedagogy, often at the expense of pedagogical content knowledge (Ball & McDiarmid, cited in Veal & MaKinster, 1999). Research on pedagogy focused on the application of general pedagogical practices in the classroom, isolated from any relevant subject matter. However, several researchers (e.g. Hauk, Jackson & Noblet, 2010; Roche & Clarke, 2009; Turnuklu & Yesildere, 2007) have rekindled the discussion about the importance of teachers' pedagogical content knowledge in learning to teach.

The term pedagogical content knowledge (PCK) as introduced by Shulman refers to the ability to represent important ideas in a way that makes them understandable to students. According to Zulbiye (2010) and Enfield (1998), PCK includes the actions and strategies of teaching, organization of classroom experiences, and knowledge of students' cognition (preconceptions, misconception, and conceptions), and evaluation and implementation of learners' prior notions and transformation of these ideas into understandable pieces. Thus, PCK represents an effort to capture the 'instruction strategies' teachers use when they teach specific subject matter content (Lee, 2002). Lee explains further that in order to present PCK as a special province of knowledge for teachers, Shulman presented a model for pedagogical reasoning and action. The model includes six components, namely, comprehension, transformation, instruction, evaluation, reflection and new comprehension. Accordingly, pedagogical reasoning begins with the comprehension of the subject matter and continues with new comprehension after reflection on instruction in that continuum.

2. Purpose of the Study

The purpose of this study was to investigate pre-service teachers' conceptual understanding of the concepts of fractions in mathematics. The study was therefore aimed at filling the void regarding research on Ghanaian prospective teachers' conceptual understanding of different aspects of fractions. In exploring the Ghanaian prospective teacher's understanding of fractions and basic operations on same, the following research questions were considered:

1) What procedural knowledge do prospective teachers possess in fraction manipulations?
2) What conceptual knowledge do prospective teachers possess in fraction?
3) How does prospective teachers' knowledge of fractions affect their pedagogical content knowledge of fractions?

3. Methods and Materials

3.1 Research Design

The research was intended to investigate (prospective teachers' understanding, meaning, explanation and content knowledge in fractions). The method of the case study was used.

This enabled the researchers to describe the phenomena of interest in detail, in the original language of the respondents. As Sidhu (2001) rightly points out, a case study research is used to study the state of how and what the nature of things are. Case studies are characterized by subjectivity and intuition and are generally oriented towards the solution of a problem at an individual or group level rather than towards the derivation of generalizations that have scientific validity. In this instance, mainly non-numeric data was collected to describe phenomena as they pertained to the situation. The interest here was in the meanings that prospective teachers make of the experiences, processes, and understandings gained through words or pictures of their 'world of fractions'.

3.2 Participants

The target population for the research comprised 847 second year prospective teachers in the colleges of education in the Central Region of Ghana. The researchers' focus was on prospective teachers in the second year because they were deemed to have sufficiently completed the required course (on fractions) in mathematics for the teacher training programme. More so, they were considered to have been prepared well enough to go on an internship at the end of the (second) year and so were the appropriate respondents in this instance. The accessible population was made up of 299 prospective teachers in the second year of one college selected at random from the two mixed colleges of education in the Central Region of Ghana.

A sample size of 26 pre-service teachers was selected from the accessible population through stratified random sampling, considering the programmes the participants were studying, the levels of their achievement and their gender. The sample was made up of two main different groups: 14 prospective teachers studying science programmes and 12 studying non-science programmes. The overall sample size was against the backdrop that this was a case study and qualitative in approach. There was, therefore, a need especially for typical cases, "*since case studies cover many facts of the total picture and extend over a long period of time and …. it is common practice to restrict the study to the thorough investigation of a few cases*" (Sidhu, 2001, p. 226).

In selecting the sample for the study, an attempt was made to select randomly, both high and low performers from each of the five classes. The high-low criterion was based on a cut-off criterion of a 'C' (C or lower grade depicting a low grade) to determine high and low performers. With the assistance of both the head of department and the assessment officer, each of the five class lists was sorted according to their grade in mathematics in the end-of first semester examination. Computer-generated random numbers were then used to select five each of high and low performers from the five classes to get a total sample size of fifty. After the intentions and processes through which the study was to be conducted were explained to the sampled respondents, forty-three agreed to participate in the study but the researchers ended up working with a sample size of twenty-six (eighteen males and eight females) instead, as all the low performers except one dropped out of the study. Table 1 shows the programme and gender distribution of the participants:

Table 1: Programme and Gender Distribution of Participants in the Study

Programme Category		**Actual Programme of Study**						**Total**
		Pure Sci.	**Visual / Gen. Arts**	**Business**	**Technical**	**Voc. Skills**	**Agric**	
Science	Male	6			2		3	11
	Female	3			0		0	3
	Total	9			2		3	14
Non-Science	Male		5	2		0		7
	Female		3	1		1		5
	Total		8	3		1		12

The science programmes (SP) were made up of technical or agricultural or pure science subjects. Elective mathematics was studied by all science students. The non-science programmes (NSP) comprised all other programmes not requiring the study of elective mathematics. The participants designated as 'science students' (SS) were students (i.e prospective teachers) considered to have gained additional mathematical knowledge and skills over and above those acquired by students who studied only core mathematics as part of their programme. Indeed, all the students were required to have passed core mathematics as part of the entry requirements for further studies in Ghana. Furthermore, in the colleges, the science students studied both core and elective mathematics while the 'non-science students' (NSS) studied only the required core mathematics.

3.3 Instrumentation

The groups' grade point averages (GPA) in mathematics from the external examinations at the time of the study showed that: the mean GPA for the sciences was 2.91 as against the 2.96 of the non-sciences with ranges of 1.625 and 2.25 respectively. The sample median GPA was 3.125 with 50% of the respondents from both groups above it. For the sake of confidentiality, all participants were assigned pseudonyms. The main tools that were used in this research work were an achievement test and a semi-structured interview schedule. The achievement test was used to collect information on content knowledge for analysis and interviews were conducted on an individual basis to get in-depth information on respondents' substantive knowledge on procedural knowledge, procedural understanding and hence on their articulateness of pedagogical content knowledge. The face validity as well as the content validity of the test was ascertained by three senior mathematics Educators at the University of Cape Coast. The reliability of the instrument was ensured by verifying in a pilot test that the Cronbach's alpha for each of the various sections was well over .70 (Cohen, 2006). The reliability of the test items of the study was found to be 0.72. This value was repeated in the data collected in the main study.

To ensure the validity of the semi-structured interview schedule, a pilot test was conducted using 11 respondents with similar backgrounds and characteristics as those of the sample to determine how well respondents performed on the instrument. From the results of the pilot test, some of the interview items were eliminated; some were reconstructed and others merged to properly capture the intended responses from respondents. The validity of the semi-structured interview schedule for the main study was based on the four proposed criteria proposed by Guba and Lincoln as cited in Trochim (2007). These were credibility, transferability, dependability and confirmability, the equivalent of internal and external validity, reliability and objectivity respectively for the quantitative instrument. To increase credibility, we used intra-personal cross-checking of the coded results. Transferability is the degree to which results can be generalized and it is primarily the responsibility of the one doing the generalization. The person who wishes to 'transfer' the results to a different context is then responsible for making the judgment of how sensible the transfer is. To make transferability possible to the reader, the interview protocol was used to provide complete background knowledge of the respondents together with their experiences with fractions. To increase dependability, the results were collected using a twofold approach: written response and interview. Secondly, efforts were made to maintain the same strategies of questioning throughout the interviewing process as much as possible. For confirmability, the respondents were contacted again after the data collection exercise to revisit issues raised that were not clearly explained during the interview session for confirmation.

Two days were used to collect data for the research. First, participants took an achievement test on fractions and were required to show all the details of their work with or without the use of calculators (even though the questions did not lend themselves to the use of calculators) and simplify their answers. The test was conducted in a friendly and tension-free atmosphere and the participants confirmed that they did not feel that they were taking a test. The time allowed for the test was one hour thirty minutes, but many students submitted their scripts well before time. All the scripts were collected on the same day after the test. The data so collected from the achievement test was used as the basis upon which the interview was conducted the following day. The interview was on one-on-one basis and audio recorded. A participant was taken through a series of questions, with each session lasting between five and ten minutes. The interviews were completed on the second day and were done in such a way that those who had been interviewed did not have the opportunity to discuss the questions they were asked with their colleagues. After each interview, the participant went straight into a class where they were engaged in different activities on fractions by a research assistant. Moreover, each participant was interviewed on the answers he or she provided in the written test.

Thematic and descriptive statistics were used to analyse the interview data and this, in conjunction with the quantitative data helped to answer all the research questions. The variables were grouped according to the emerging themes. Descriptive statistics was were used to describe some aspects of the participants' work in the achievement test and the outcome of the interviews.

4. Results

4.1 Procedural knowledge of prospective teachers in fraction

Research question one was aimed at finding out about pre-service teachers' procedural knowledge in fractions in order to understand whether the acquisition of this knowledge was conceptually based or rule-bound. Respondents were interviewed on fractional items in the division, concepts (fraction concept of diagrams), equivalence, ordering, addition, subtraction, multiplication, and transfer of fraction concepts, based on their responses to the written test.

4.2 Division and Addition of Fractions

Respondents were asked to explain how they arrived at their answers on the item '$1\frac{3}{4} \div \frac{1}{2}$'. The question was meant to find out about the accuracy of the solutions and hence the appropriateness of the procedures used to get to the result of $3\frac{1}{2}$ or $\frac{7}{2}$ or 3.5

For the explanation, if a respondent said, for example, "…for division, you have to reciprocate the half given so that will be $\frac{2}{1}$ and the division sign too will change to multiplication", then the respondent was asked 'why did you change the sign and reciprocate the half?' Similarly, on addition, respondents were asked 'how do you solve a problem such as this $1\frac{2}{3}+\frac{3}{4}=$?' Whatever answer that a respondent got, he or she was made to justify it by answering the question, 'what made you believe your work is correct?'

Table 2: Distribution of Responses to Items on Division and Addition Processes

Operations	Sciences		Non-Sciences	
	Appro n (%)	**Inappro** n (%)	**Appro** n (%)	**Inappro** n (%)
Division (Computation)	12 (85.7)	2 (14.3)	9 (75)	3 (25)
Division (Explanation)	11 (70.6)	3 (21.4)	11 (91.7)	1 (8.3)
Addition (Computation)	14 (100)	0 (0.0)	7 (58.3)	5 (41.7)
Addition (Explanation)	12 (85.7)	2 (14.3)	9 (75)	3 (25)

Note: Appro = Appropriate; Inappro = Inappropriate explanations

It can be inferred from Table 2 that even though the science students seemed to have performed better on computation in division and addition than the 'non-sciences', the latter demonstrated knowing how, in explanation for division of fraction. As indicated in Table 1, while nearly 92% of the non-science students could give appropriate explanations to their answers in the division, about 79% of the science students could do same. This was due to the fact that during the interview the non-science students exhibited what Agbenyega (2010) referred to as the attitude of 'reflective thinking' about their answers which they had provided. This posture enabled these respondents to reflect on their previous actions and therefore had realized their errors and retracted to make amends. The only non-science students' error during the explanation of division computation was replicated by two of the science students who committed the same error: that is, finding LCM for numbers that were being divided.

Though the approach used by those who found the LCM and adopted the procedure they described led to the correct answer of $3\frac{1}{2}$ in this particular instance, this could only be treated as one of the accidental cases since it does not apply to all sets of fractions. To change the dividend and divisor into like fractions before working was also an alternative procedure. That is, changing $1\frac{3}{4}$ divided by $\frac{1}{2}$ into $\frac{7}{4}$ divided by $\frac{2}{4}$ means the same as dividing 7 by 2 and dividing 4 by 4 (i.e., dividing a fraction by a whole). This must have been the procedure for the pre-service teachers who found the LCM though none of them used it appropriately.

One of the science students' difficulty was in expressing a mixed number as an improper fraction but he did not appreciate a helpful suggestion during the interview because he believed he was relying on the appropriate approach to solve the problem. Generally, the pre-service teachers knew that to solve the division problem, one must convert mixed fraction to improper fraction, reciprocate the divisor and multiply and get an answer of $3\frac{1}{2}$ or $\frac{7}{2}$ or 3.5 . The computational procedures were clear and unambiguous. Four (2 each of science and non-science students) correctly computed the algorithm but failed to simplify their final results. Therefore, the results were considered incomplete.

Though respondents could do the division arithmetic and explained their approaches, they did not understand the real concept behind the 'invert and multiply' rule of division by fraction they so used hence their varied responses as illustrated in Table 3.

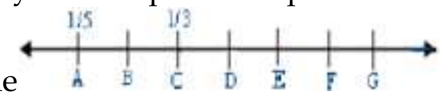
Table 3: Reasons for which Respondents Change Division Sign to Multiplication and Reciprocate the Divisor

Reasons	SS	NSS
It is a concept or rule or principle that must be used	6	3
You can't divide fraction by fraction	2	3
That is how I was taught to know it	3	3
Multiplication is the reciprocal of division	1	-
Could not give any reason	2	3

For instance, those who think it was not possible to divide a fraction by a fraction were thinking only of division by whole numbers. Computation of addition of fractions seemed to be the easiest for the science students as compared with the division. However, not all of them could comfortably explain how they got their answers. Table 3 shows fewer of the science students compared to the non-science students were able to explain their approaches. All of them followed the same approach, finding the LCM to getting their results: a common error that ran through the work of all who had the computation wrong was their inability to convert the mixed fraction to improper fraction.

4.3 Other operations on fractions

On subtraction of fraction, respondents were asked to work out the following difference $4\frac{3}{8} - 3\frac{3}{4}$; this question sought to find out the procedural knowledge that respondents have in the subtraction of fraction and only one 'science student and three non-science students could not produce a satisfactory result.

In the cases of order of fraction and equivalent fractions, respondents were expected to indicate the largest fraction in the set of numbers: $\frac{5}{9}, \frac{6}{11}$ and $\frac{4}{7}$ and to write down three equivalent fractions with respect to the diagram  respectively. These questions were meant for participants to differentiate among a set of fractions as greater than or less than; and to successfully form equivalent fractions from a set of objects (pictorially). Students were given sufficient information on the items.
For fraction concept (using diagram), respondents were asked to determine what fraction of a circle is represented by half of this piece , the correct answer to which is $\frac{1}{8}$. This was to test for procedural knowledge of proficiency and representation of fraction and therefore respondents were expected to come up with a representation as $\frac{1}{8}$.
In multiplication, participants were asked, 'suppose you are given the problem $\frac{3}{4} \times \frac{2}{3} =$?, how would you solve it?' In the transfer of fraction, they were expected to provide the value for the position of the letter 'G' on the number line 

, where 'A' and 'C' are at the points $\frac{1}{5}$ and $\frac{1}{3}$ respectively. Table 4 shows computational competences exhibited by respondents on a fractional item in the various operations.

Table 4: Responses to Fractional Items

Computation of	**Science Students (n=14)**		**Non-science Students (n=12)**	
	Correct	**Incorrect**	**Correct**	**Incorrect**
Fraction concept (diagram)	7	7	6	6
Equivalent fraction	9	5	6	6
Order of fraction	10	4	9	3
Subtraction of fraction	13	1	9	3
Multiplication of fraction	13	1	10	2
Trans. of fraction concept	2	12	-	12

As displayed in Table 4, 50% of each group could not generate the correct response to the fraction concept as many as did for subtraction and multiplication. Respondents showed higher competences in fraction multiplication, subtraction and order of fraction than that exhibited on fraction concept and fraction equivalence.

Generally, though, there was a strong showing of procedural knowledge on all five fractional domains except the transfer of fraction for which only two respondents (sciences) gave correct answers. However, one of them could not explain how he came by his answer. As many as thirteen varied answers were provided as incorrect responses to the transfer of fraction item and most of the participants assumed the numbers on the number line as decreasing from 'A' to 'G' instead. This assumption was based on the visual perception of '5' is greater than '3' instead of perceiving $\frac{1}{5}$ as a number that is less than the number $\frac{1}{3}$. In other words, the '5' was considered a separate entity from the '1' so was the '3' and '1'. The most frequent error response was negative one as in: "What I can say about this number line is, the number is decreasing so from A, $\frac{1}{5}$; B, $\frac{1}{4}$; C, $\frac{1}{3}$; for the D it will be $\frac{1}{2}$; for that of E it will be one-over-one; and for this one (*referring to F*) I will get zero…..so I will get minus-one-over-two", Jacob (non-science).

4.4 Pre-service teachers' conceptual understanding of fraction

Research question two sought to find out if respondents can demonstrate procedural understanding of the mathematical meaning or establish logical link between the answer, dividend and the divisor in the case for division and also to know their reasoning behind what was written. It was further to find out if respondents know of any other approach or approaches and what posed difficulties in determining responses to the number line item. Respondents were asked, for instance in division, 'What does your answer to $1\frac{3}{4} \div \frac{1}{2}$ mean? Or what is the mathematical meaning of this result- $\frac{7}{2}$ or $3\frac{1}{2}$?' To this, none of the respondents could give appropriate meaning to the result and three could not offer any at all.

4.5 Equivalent fraction

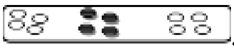
Respondents were asked to explain with reasons how they determined answers to this diagram . For an appropriate response, it must be consistent with any well known, tried and tested procedures of solving mathematical problems lazed with some degree of innovative approach otherwise it was inappropriate.

Table 5 shows the number of participants responding to an explanation of fractional items. It also shows the number of respondents who appropriately demonstrated a procedure different from what had been used on their worksheet.

Table 5: Distribution of Explanation of Responses to Fractional items

Explanation	**Sciences**			**Non-Sciences**		
	Appro n (%)	**Inappro** n (%)	**DK** n (%)	**Appro** n (%)	**Inappro** n (%)	**DK** n (%)
Division (Meaning)	-	12 (86)	2 (14)	-	11 (92)	1 (8)
Equivalent fraction	10 (71)	4 (29)	-	6 (50)	6 (50)	-
Order of fraction	9 (64)	4 (29)	1 (7)	7 (58)	2 (17)	3 (25)
Multiplication of fraction	11 (79)	3 (21)	-	8 (67)	4 (33)	-
Subtraction fraction (other procedures)	4 (29)	2 (14)	8 (57)	3 (25)	3 (25)	6(50)
Multiplication of fraction (other procedures)	7 (50)	2 (14)	5 (36)	5 (42)	-	7 (58)

Note: Appro = Appropriate; Inappro = Inappropriate explanations; DK = Don't know

With the exception of one science respondent, (see Table 5) all others who could give explanation to their procedures used the idea of multiplication of factor or direct equivalence to get at their correct set of equivalent fractions of $\frac{4}{12}, \frac{3}{9}, \frac{2}{6}$ or $\frac{1}{3}$. Five of the non-science students could use the diagram to explain two of the fractions while nine of the science students depended on factor equivalence for their results.
The procedure demanded that respondents use the diagram to determine equivalent fractions. This was to find out how they conceived the concept of fractions and equivalent fractions. And for this only one of the science students demonstrated that.

4.6 Order of fraction

Though as many as ten science students and nine non-science students successfully computed for the order of fraction, only nine of the former and seven of the latter could appropriately explain how they got their answers. Most of those who gave an inappropriate explanation to their answers, confused fractions with large denominators with whole numbers with large numbers and thought the larger the denominator, the bigger the fraction irrespective of the numerators involved. In the cases of two science students, instead of dividing using the denominators, one used the numerators and the other believed that answer can be obtained only by the use of a special formula so he said: "I guessed because I didn't use any formula and I know there is a formula for it".

4.7 Multiplication of fraction

As indicated earlier, 13 science students could compute but only 11 could explain their answers. Similarly, 10 of the non-science students could compute but only 8 could explain how they got their results. This was indicative of the fact that most respondents were competent in computation, but it was difficult for them to demonstrate 'how and why' their methods worked. For those who had a problem explaining their procedures, some could not differentiate between the procedures of fraction addition or subtraction from that of fraction multiplication.

4.8 Pedagogical Content knowledge in fractions

Research question three was about prospective teachers' ability to teach fractions with understanding. Participant's pedagogical knowledge in fractions was examined in both the written test and the interviews. The items were on classroom scenarios involving the teaching of fractions. The items were mainly on addition, subtraction, division and multiplication of fraction. Items here were discussed based on the themes that came up in the written presentations by the respondents. Table 6 shows the emerging approaches that respondents knew to apply to the teaching of the concepts in fraction on the various fractional topics. The responses in Table 6 have been discussed in the following paragraphs.

Table 6: Instructional Approaches as used by Respondents

Instructional procedure	**Science Students (n=14)**				**Non-science Students (n=12)**			
	Div	**Add**	**Sub**	**Mul**	**Div**	**Add**	**Sub**	**Mul**
Material with Appropriate Explanation	1	1	1	1	0	1	3	3
Material with Inappropriate Explanation	2	6	5	2	5	6	3	4
No material with Appropriate Explanation	7	3	5	4	2	0	1	1
No material with Inappropriate Explanation	2	4	0	6	4	4	4	2
Don't know	2	0	3	1	1	1	1	2
Total	14	14	14	14	12	12	12	12

In using material with appropriate explanation, participants gave explanations that well described how they would approach the teaching or representation of the fractional items in question. Also, the participants demonstrated how the material so chosen could be manipulated so as to achieve this aim appropriately. Coming from Table 6, only one science student could appropriately demonstrate this in division.

In using material with inappropriate explanation, the responses given were appropriate in the participants' choice of material for representation of the fractional items, but the explanation did not lead to getting the required results. In division, 26.9% (i.e., 7 out of 26) could appropriately demonstrate the use of materials in this wise, 46.1% (i.e., 12 out of 26) could do same in addition, 30.8% (i.e., 8 out of 26) and 23.1% (i.e. 6 out of 26) did so in subtraction and multiplication, respectively. All these participants attempted using the area model of division for instance to get the answer to the division of a fraction by half but failed.

Some participants provided no examples of the use of any material in their teaching of fractions but offered appropriate explanations. Thus, in this situation, although there was no material mentioned in the demonstration of the representation of fractional item, the instructional directions provided were explicit enough to lead to an answer. In any case, this approach was just a comeback of the traditional 'reciprocate and multiply' or 'show and tell' posture that seem to have characterized teaching over the years. A total number of 9 respondents (7 science students and 2 non-science students) approached the fractional items in division in this manner, 3 in addition, 6 in subtraction, and 5 in multiplication.

In the case of using no material with inappropriate explanation some participants did not apply any form of representing the fractional items and explanations of the instructional approaches were sometimes difficult to follow. Eventually, the respondents in this category unsurprisingly stalled! These respondents were among those that expressed difficulty in giving any representation on fractional items earlier. For division, 6 respondents (2 science students and 4 non-science students) representing 23.1% were found to be in this category, while 30.8% made up the number for addition, with 15.4% in subtraction and 30.8% in multiplication.

The last category was made up of prospective teachers who had no idea of how to go about the task. Of the 26 respondents, 3 did not know what to do in the division task and the figures for the addition, subtraction and multiplication tasks were 1, 4 and 3, respectively. Generally, the results from the PCK tasks suggested that pre-service teachers have difficulty in demonstrating the teaching of division of fraction and that of fractions in general. In fact, it was observed that that only in 11 out of the 104 cases (i.e., 4 x 26) involving addition, subtraction, multiplication and division, only that students demonstrated using appropriate material for the representation of fractional items with good explanation!

5. Discussion

Regarding the findings on procedural knowledge, there were five different patterns of errors observed in the test from the twenty-six respondents, with four incomplete solutions in computing one of the fraction division items. In equivalent fractions, altogether nine different errors were found to have been made by eleven respondents; seven respondents failed to properly respond to the order of fractions item, while four each failed to provide answers in addition and subtraction of fractions. Only three failed in the multiplication of fractions, and all except one failed to find a solution to a transfer of fraction 'problem'. It was obvious from these findings that while working addition and multiplication in fractions was easier for the pre-service teachers, which finding was also established by Tsay and Hauk (2009), all except one and thirteen were uncomfortable with the transfer of fraction concepts and division, respectively.

Even though Rizvi (2004) and Ball (1990) found that pre-service teachers were uncomfortable with division and subtraction, the respondents in the present study showed and expressed their mastery in subtraction. The respondents did not pretend to have had control over the fractional items with these results though, since some of them voiced their frustrations over the difficulties they had due to the differences in denominators as a cause for possible errors committed. In all, they possessed what Ball, Bass and Hill (2004) called 'common' knowledge for fractional computation and yet did not understand why the principle they used worked in the division for instance.

Another difficulty experienced by the respondents was in reading the correct position on the number line. This problem could be attributed to what Roth and McGinn cited in Mitchell and Horne (2008) referred to as semiotic misreading of mathematical diagrams or inscriptions. An action that could result in semiotic misconceptions in this case, was for respondents to consider the numbers as decreasing on the number line from 'A' to 'G'. Secondly, they misconstrued the zero-mark as within and counted the strokes marking the positions of the numbers, and not counting spaces on the number line. These misconceptions caused some to have negative results and mixed numbers as answers. Even, the only successful pre-service teacher on that task could not read off the fraction by itself, but rather converted all the fraction values on the line into fractions with a common denominator; and read the fractions as though they were on a natural number line.

Regarding conceptual understanding, it was observed from the results that the pre-service teachers had no interconnectedness in the domains of content and procedure of fraction manipulations to facilitate their choice of procedures and conceptual understandings of concepts in fraction. Thus, out of the twenty-six respondents, none attempted an appropriate response to the meaning of the division of fractions. Most of the explanations were in the form of restatement of the answers in a different form, for instance, "*It means 2 whole number of two events were given to 7 people, to some people to share and they got 7 out of the 2*" or restatement of a procedure, "*It means that when you divide one whole number three over four by one-half, you are going to get three whole number one over two which is half of one…..*"

It was found in the study that the pre-service teachers lacked the ability to conceptualize or contrive an idea about fraction dividing fraction because "*you can't divide fraction by fraction so I decided to multiply through by the reciprocal of ½….*"; which was similar to Tirosh (1998) and Ball (1990)'s findings that pre-service teachers saw ½ dividing another fraction as though it were 2 doing the division. While some perceived no possibilities of going round the question but simply thought it was a rule or principle bound. Others put their inability to give meanings to the way they were taught - as in "*that is how I was taught to know it*" though these pre-service teachers could compute for correct results in division using the appropriate procedure. This just re-established Haung, Liu, and Lin's (2009) statement that pre-service teachers continue to demonstrate greater procedural knowledge rather than conceptual knowledge in fraction.

Regarding the respondents' pedagogical knowledge in fractions, the pre-service teachers were prone to teaching fraction by applying rules, principles and procedures since fraction appears "*abstract and confusing*" to them. For example, "*I became a little bit confused by not understanding the concept and the teacher who takes us through was not able to express herself very well*". If the teacher who taught this pre-service teacher could not express herself well and she could not represent or use alternative means to teach, then more pupils would be denied the opportunity to experience conceptual meaning, understanding and interconnectedness of fraction. Pupils are therefore most likely to suffer similar fate as their teachers. Generally, the respondents demonstrated very weak pedagogical content knowledge in fractions, a finding which confirms the findings made by Davis and Ampiah (2009).

6. Conclusion

From the findings of the study, it was clear that the pre-service teachers' knowledge in relevant concepts of fraction that they would bring to bear on teaching was primarily procedural in nature, conceptually weak, rigid and not sufficient in the concepts and teaching of fractions. These results also suggested that the conceptual knowledge of teachers might be a factor limiting students' achievement, particularly in the domain of the topic of fraction. There is no gainsaying that a few months after the study, the pre-service teachers who took part in the study were going to be at the school gates where they would be required to teach these very things they seemed to be struggling with.

7. Recommendations

- Pre-service teachers have had their procedural knowledge in fraction established from so many years of learning (primary, junior high school and senior high school levels). It is therefore recommended, based on the findings of the present study that a new approach of teaching, such as the use of rich collaborative tasks, manipulative and information and communication technology (ICT) be adopted by the mathematics tutors at least in the colleges whose students formed the target population of the study. This is to re-orient pre-service teachers in these colleges to acquire conceptual knowledge for conceptual understanding of fractions. An example of such approaches would be to make connections between topics in fraction and real-life situations.
- Again, tutors in the colleges should be guided by the conceptual understanding that pre-service teachers need in the relevant concepts of fraction rather than the would-be teachers' computational knowledge in fractions, which they have developed from years of schooling.
- There are other areas of fractions that could not covered in this study such as decimal fractions. Studies needed to be conducted in these areas to further the understanding of pre-service teachers exiting the college system.

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