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COGNITIVE MODELLING SKILLS FROM NOVICIATE TO EXPERTNESS

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Abstract:
The purpose of this study is to reveal the cognitive modelling skills of 6th grade students after a long term modelling implementation. The cognitive modelling skills are regarded as understanding the problem, simplifying, mathematising, working mathematically, interpreting and validating. Seven-month modelling sequences were designed and conducted, and the first and last implementations were particularly examined in the study. The participants were four students, while the data collection tools were solution papers for two different modelling problems in the implementations and transcriptions of the video records concerning the solution and solution presentation processes. When the data were analysed through a rubric and presented descriptively, it was seen that a development was revealed in cognitive modelling skills from noviciate to expertness. In other words, the students displayed richer approaches in the context of each cognitive modelling skill in the last implementation.

Keywords: mathematical modelling, cognitive modelling skill, cognitive perspective, novice modeller, expert modeller

1. Introduction

Mathematical modelling is utilised more in schools in line with the increase in the importance of mathematical modelling in curricula of different countries since 1980s. In the educational discussions, responses are sought for the question of how mathematical modelling and its applications will be integrated into daily school classes (Maaβ, 2006). The applications in which students can display their modelling skills by ensuring the

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integration in question gains importance. Modelling skills are defined as the skills and abilities of completing the modelling process purposively and properly, where the individual should be willing in this process (Kaiser & Maaß, 2007; Kaiser & Schwarz, 2006; Maaß, 2006). Besides this, modelling capability is defined as passing through the steps of the modelling process independently (Blomhoj & Jensen, 2003) and displaying different approaches at different steps (Blomhoj & Kjeldsen, 2006). When the definitions quite similar to ones above and those in the literature are examined regarding modelling skills, it is noted that the modelling process is made to come to forefront in each of them. In order to define, interpret and explain what is going on in the minds of students working on the modelling process, the cognitive viewpoint of modelling is expressed to be necessary (Blum, 2011). The modelling skills dealt with in parallel to the steps of the modelling process are thought to be considered as cognitive modelling skills.

Developing modelling skills is among the stated objectives of mathematics teaching (Blum, 2011; Kaiser, 2007). Accordingly, in the study, it is aimed to ensure the development of cognitive modelling skills of students who have no modelling experience through modelling applications. In this context, in order to find responses to the question of how such development will be achieved, firstly studies in the literature were examined and a long term modelling application was decided to be carried out. Therefore, the purpose of this study is to reveal 6th grade students' cognitive modelling skills after experiences modelling sequences. In other words, we aimed to find an answer to the question whether 6th grade students who are novices in modelling become experts through modelling applications.

Theoretical Framework and Related Studies

As the study dealt with the cognitive aspect of modelling, the contextual framework of the study was chosen as Modelling Cycle under a Cognitive Perspective (see Figure 1), which Borromeo Ferri (2006) reconstructed in cognitive sense by examining different modelling cycles. Cognitive modelling skills are considered as understanding the problem, simplifying, mathematising, working mathematically, interpreting and validating according to this framework. This framework is utilised in collecting and analysing the data, and interpreting the results of the analysis.

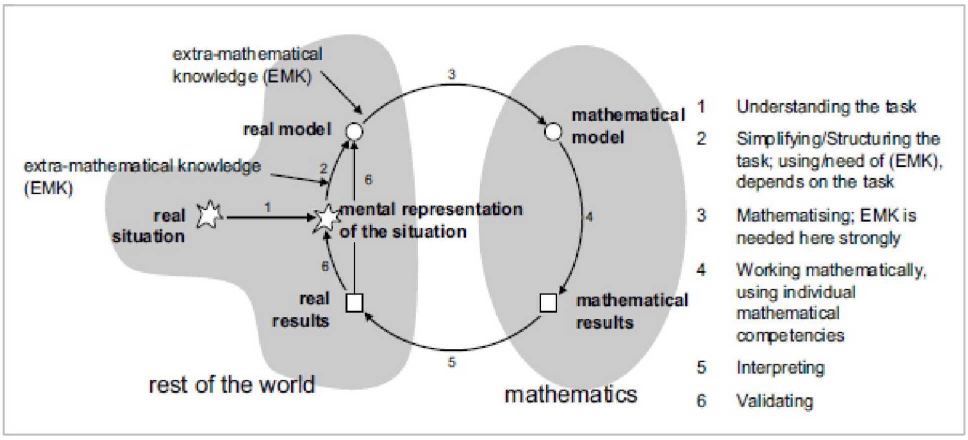


Figure 1: Modelling Cycle under a Cognitive Perspective (Borromeo Ferri, 2006)

When studies in the literature are examined, it is evident that having modelling experience can influence students' modelling behaviours. The studies particularly including long term modelling applications enabled development in modelling. In the first of the studies mentioned above, Maaβ (2005; 2006) developed modelling units and integrated them into classes, and examined the students' progress in modelling skills. In another study, Biccard (2010) revealed 7th graders' modelling skills at the end of 12 weeks of modelling applications. In the KOM project considering mathematical skills as a tool for improving mathematics education, the researchers used students' project work directly to achieve improvement of mathematical modelling skills (Blomhoj & Hojgaard Jensen, 2010). Bracke and Geiger (2011) integrated mathematical modelling into mathematics classes on a regular basis and revealed that integration had a positive effect on students' modelling behaviours. In another study, Ji (2012) compared the modelling abilities of students who were experienced or inexperienced in modelling. Grünewald (2012; 2013) investigated promotion of modelling skills in 9th grade students in her studies in a 5-month modelling project.

Although the studies with students on different levels are observed to have been carried out regarding mathematical modelling practices since 2010 in Turkey, most of these cannot go beyond implementation of a few modelling tasks. There are rarely any studies in the national literature featuring long term modelling applications. For example, in one of the studies, Bukova Güzel (2011) examined pre-service mathematics teachers' behaviours in constructing and solving mathematical modelling problems during a college course in the faculty of education for a semester.

Considering this study's purpose, it is thought to contribute to both the national and international literature. In the national perspective, it differs from others because of the terms of enabling novice students to be experts in modelling by a long term modelling application. Considering it in its international aspect, students' levels, socio cultural situations, implemented modelling tasks and their effects on their modelling behaviours will present a novel point of view.

Method

This study concentrates on the first and last implementations of a long term modelling implementation to reveal the development of students' cognitive modelling skills and it is conducted as a qualitative study.

Process

Because the volunteer mathematics teacher did not have any information about mathematical modelling and its instruction, she was given a seminar featuring mathematical modelling, modelling tasks and possible solution approaches. It was decided to conduct the study in Mathematics Applications, an elective course. This course was chosen because it is an elective course which partly includes real life problems in the textbooks (MNE, 2012a; 2012b).

The 7-month implementation comprised twelve modelling sequences developed by the teacher and the researcher. After the implementation of each sequence, they held an assessment and planning meeting. At these meetings, transcripts of the video records of the previous implementation were examined and cognitive modelling skills of the students were evaluated in general terms. In the evaluations of issues, the skills where the students made progress, ones where they had problems and the general problems encountered were determined, and the content of the next sequence was decided. When the purposes of the sequences were enabling engagement in different modelling tasks in initial implementations, they concentrated on the definite cognitive modelling skill in the following ones. In the implementations, the researcher and the teacher acted as cognitive coaches (Blum & Leiβ, 2007; Chan, 2010; Chan & Foong, 2013) when the students were working on the problems, and they asked the questions considered to be revealing the thought processes of the students. This study investigated the first and the last part of the modelling sequences. In both implementations, the groups presented their solution approaches to their classmates after the solution process was done. All groups explained their solutions in a couple of minutes and other groups asked questions if any.

Participants

The study was conducted with twenty three sixth grade students who registered for the course. The studies in the literature show that working collaboratively in modelling makes positive contribution to the development of the modelling skills (Maaß, 2006; Biccard & Wessels, 2011; Maaß & Gurlitt, 2011; Maaß & Mischo, 2011). For this reason, five study groups were formed as three groups of five people and two groups of four people. While forming the groups, the solution approaches of the students solving the Apple Pie problem (Schukajlow, Leiß, Pekrun, Blum, Müller & Messner, 2012) were examined by the rubric to be presented in the following sections. When presenting the results, it was seen impossible to provide space for the solution approaches of all groups because of limitations. In this case, only one group was randomly chosen and their modelling approaches were examined in detail. The participants were given code names Ender, Ege, Mehmet and Batuhan.

Data Collection

The data collecting tools are the group's solution paper to the tasks in the first and last implementations and their video records of both the solution and the solution presentation processes.

The students solved the Bridge Problem (Jahnke, 1997 cited in Maaß, 2006) in the first implementation and were asked to answer some questions in order to enable them to work in parallel with modelling stages as they had no experience in modelling. The problem and probing questions are given in Figure 2.

BRIDGE PROBLEM

The biggest bridge of the world is the one constructed over the Gulf of Hangzhou, west of China and it is 36 km long. Consider there is a traffic jam along the bridge. How many vehicles will be stuck in there? Please write your thoughts in detail.

1. What information do you need to solve the problem?
2. How do you solve the problem?
3. Is the result comprehensible? If yes, explain the reason. If no, revise your solution.
4. Is your solution correct? If yes, explain the reason. If no, revise your solution.

Figure 2: Bridge Problem (Jahnke, 1997 cited in Maaß, 2006)

In the last implementation, the Ancient Theatre problem (Tekin, Hıdıroğlu & Bukova Güzel, 2010) was solved (see in Figure 3).

ASPENDOS ANCIENT THEATRE

A group of tourists went to Aspendos Ancient Theatre in the trip they had to Antalya. You can see a photograph taken during this trip.

1. What can be the real distance between the marked people?
2. What can be the real height of the ancient theatre?

Figure 3: Ancient Theatre Problem (Tekin, Hıdıroğlu & Bukova Güzel, 2010)

Data Analysis

There were two different types of analysis conducted in the study as rubric assessment and descriptive analysis. In both analyses, the video records of the solution and presentation processes were examined along with the solution papers. To be able to present the group's cognitive modelling skills quantitatively, the Rubric for Assessment of the Modelling Skills [RAMS] (Tekin Dede & Bukova Güzel, 2014) was used. Dimensions, levels and the detailed explanations of these are given in Table 1.

Table 1: Rubric for Assessment of the Modelling Skills [RAMS] (Tekin Dede & Bukova Güzel, 2014)

	Levels	Definition
Understanding the Problem	Level 1	Includes the expressions showing that s/he did not understand the problem, not determining the givens and goals, and not forming or mistakenly forming a relationship between them.
	Level 2	Includes the expressions showing that s/he understood the problem to some extent, determining the givens and goals to some extent but not forming or mistakenly forming a relationship between them.
	Level 3	Includes the expressions showing that s/he understood the problem completely, determining the givens and goals but not forming or mistakenly forming a relationship between them.

	Level 4	Includes the expressions showing that s/he understood the problem completely, determining the givens and goals, and forming a relationship between them.
Simplifying	Level 1	Not simplifying the problem, not determining the necessary/unnecessary variables and making wrong assumptions.
	Level 2	Simplifying the problem to some extent, determining the necessary/unnecessary variables to some extent but making wrong assumptions.
	Level 3	Simplifying the problem, determining the necessary/unnecessary variables and making partly-acceptable assumptions
	Level 4	Simplifying the problem, determining the necessary/unnecessary variables and making realistic assumptions.
Mathematising	Level 1	Not constructing or mistakenly constructing mathematical model/s.
	Level 2	Constructing correct mathematical model/s based on partly-acceptable assumptions.
	Level 3	Constructing incomplete/wrong mathematical model/s based on realistic assumptions and relating them to one another.
	Level 4	Correctly constructing the needed mathematical model/s according to realistic assumptions, explaining model/s and relating them to one another.
Working Mathematically	Level 1	Not presenting a mathematical solution, solving the constructed models wrongly or trying to solve the wrong mathematical model.
	Level 2	Solving correctly the mathematical models constructed incompletely/wrongly.
	Level 3	Including deficiencies/mistakes in the solution of the correctly constructed mathematical models.
	Level 4	Achieving correct mathematical solution by solving the correctly constructed mathematical models.
Interpreting	Level 1	Misinterpreting or not interpreting the obtained mathematical solution in real life context.
	Level 2	Correctly interpreting the erroneous/incomplete mathematical solution in real life context.
	Level 3	Incompletely interpreting the obtained correct mathematical solution in real life context.
	Level 4	Correctly interpreting the obtained correct mathematical solution in real life context.
Validating	Level 1	Not validating or making invalid validation.
	Level 2	Validating completely, not correcting the determined mistakes.
	Level 3	Validating completely, correcting the determined mistakes to some extent.
	Level 4	Validating completely, correcting the determined mistakes.

In addition to the quantitative analysis, the groups' solution approaches were presented descriptively to support the rubric assessment.

The video record transcripts of solutions and presentations, and the solution papers were independently examined by the two researchers with the rubric assessment. Comparisons were made after the mentioned examinations by coming together and the percentage of agreement (Miles & Huberman, 1994) between the evaluations of the researchers were determined to be over 70% for the solution process. Besides, in order to increase the reliability in data analysis, all data were subject to a second analysis a certain period after the first one by the first author according to the stability method (Krippendorff, 1980; Weber, 1985). The percentage of the agreement between the analyses performed at different times was found to be over 70%.

Results

The First Implementation

After reading the Highway problem, the group members complained about the absence of numerical values out of the length of the highway and stated they could not solve the problem. The teacher made a statement about how they could solve the problem as follows:

Batuhan: *"We cannot solve it, it is impossible. There isn't any numerical value apart from 36 km. I think there is a mistake in the problem. I have never seen a problem like this."*

Teacher: *"If you think there is not enough information to be able to solve the problem, you should make assumptions about the givens. I mean, you should identify the values by considering real life knowledge. Please be careful about taking values realistically."*

After this explanation, they stated they needed to find the length and width of the cars and the width of the highway. They made partly-acceptable assumptions about taking the length of a car 2 meters, the width 1 meter and considered the highway as 10 meter width. When their statements regarding their assumptions were examined, it was seen they only estimated numerical values for the car dimensions and never investigated the values in the frame of reality.

Ege: *"I think, a car should be 2 meter long."*

Mehmet: "*Reasonable*."

Ege: *"Let's take the width as 1 m and the bridge as 10 m."*

Ender: *"Deal. Let's calculate then."*

Then they constructed mathematical models and found 180000 cars as a result (see in Figure 4).

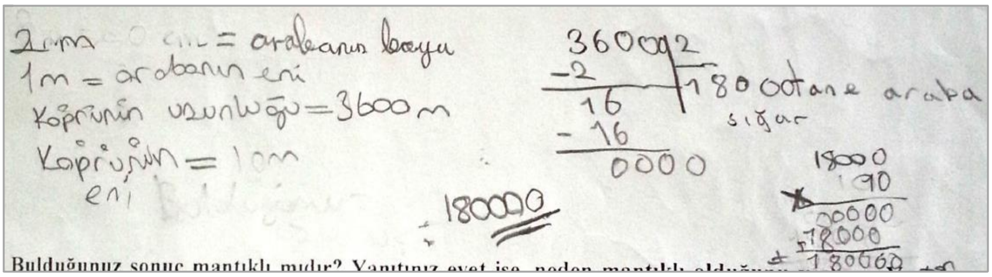


Figure 4: Mathematical models and their solutions

When their assumptions were examined, it was seen that they did not consider whether all vehicles were the same or not or whether there were safe spaces between the vehicles or not. They were unable to go beyond estimating car dimensions without any explanations. In addition to this, they did not pay regard to the existence of the lanes and how many lanes could exist. Since they did not give regard to real life while making assumptions, their assumptions were evaluated as partly-appropriate for reality. They wrote their solution was reasonable due to the correctness of the calculations on the paper. These statements indicated that they did not interpret the solutions in the problem context. When dealing with validation approaches, they just corrected a calculation mistake in the solution process and regarded it as validation. The levels of the group's cognitive modelling skills are seen in Table 2.

Table 2: The Levels of the Cognitive Modelling Skills in the First Implementation

Skills	Bridge Problem
Understanding the problem	4
Simplifying	2
Mathematising	2
Working mathematically	4
Interpreting	0
Validating	2
Total	**14**

The Last Implementation

After the students understood the problem, they decided to solve the problem by measuring with a ruler and stated that one person in the picture corresponded to 1 cm by ruler. In the meantime, Ender realised the height of the theatre was equal to the distance between the marked people and measured them as 12.5 cm. Ender made a totally unrealistic assumption on equality of 1 cm to 1 m and the height of the theatre was found as 12.5 m based on this assumption. Ege, Batuhan and Mehmet noticed the unrealistic approach and tried to interpret and validate the situation by explaining that a person becomes 1 m tall in real life if 1 cm is equal to 1 m.

Ender: *"…each 1 cm is equal to 1 m. So it is 12.5 m long."*

…

Ege: *"There is a problem here; the height of a person becomes 1 m in this situation."*

Batuhan: *"It mustn't be 1 m."*

Mehmet: *"I think it is not."*

Ender: *"I don't mean the height of a man."*

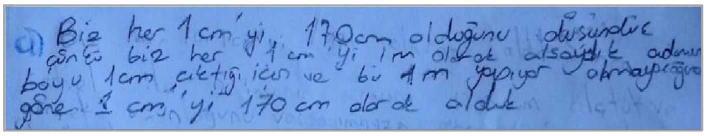
Batuhan: *"Look! If you take 1 cm as 1 m tall, the man will be 1 m tall."*

Ege: *"It is totally unreasonable."*

Then the group members made an assumption about the average man's height as 1.7 m by discussing how many centimetres in real life could be equivalent to 1 cm in the image. Batuhan put forward an idea to calculate the height of the theatre. He explained a man could correspond to three seats in the theatre and others confirmed his assumption. In this context, it was seen that the students debated on an additional solution approach.

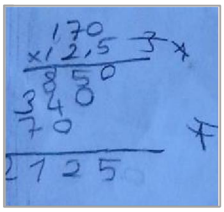
After a while, they put this assumption away and multiplied 1.7 with 12.5 by transitioning into the phase of constructing a mathematical model. They decided they completed the solution process since the resulting 2125 cm was equal both to the height of the theatre and the distance between people.

Ege: *"Look, I wrote everything. We considered each 1 cm as equal to 170 cm. If we had taken 1 cm as 1 m, we would have found the height of a man as 1 m. As this is not correct, we take it as 170 cm for providing the reality factor."*

Paper shot: 

Ege: *"Now, are we going to multiply 170 by 12.5?"*

Ender: *"Yes. [Ege multiplied them and found 2125.]"*

Paper shot: 

Meanwhile the researcher reminded them not to forget doing validation. They thought that they could use Batuhan's assumption about a person's height corresponding to three seats to validate the solution. When Batuhan indicated they should count the seats three by three, Ender asked how they could find the distance between the seat areas. Ege suggested not counting there. Since this neglect caused errors about the assumption and the solution, they stated they could not ignore this distance.

Ege: *"Batuhan had a good idea. Let's use it."*

Batuhan: *"Let's count the seats three by three."*

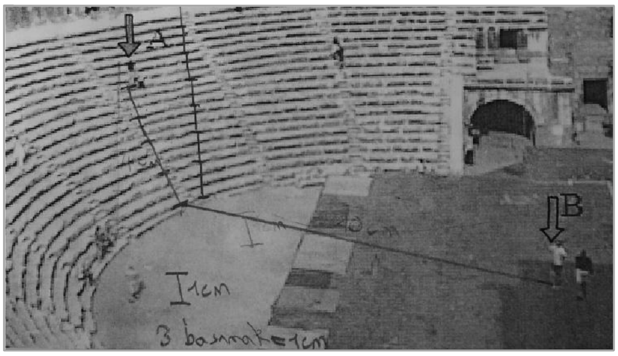
Ender: *"Ok. Then what will this [the distance between two seat areas] be?"*

Ege: *"Don't count there."*

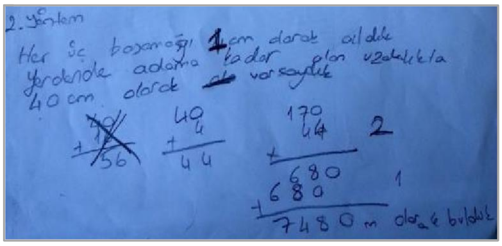
Ender: *"No way! It would be completely wrong then."*

Ender suggested another solution approach by considering the walking distance instead of aerial distance between people. The students decided to apply this suggestion by looking at Batuhan's idea. They counted the seats three by three to find the distance of person A from the floor and they indicated this as corresponding to 4 cm. They reach the end of the result as 40 cm with reference to wrong assumptions by calculating the distance of person B to the steps.

Ender: *"Eureka! We count the seats three by three. Then we measure here with the ruler."* [He sketched the so-called distances.]

Paper shot: 

Ege: [Ender gave the paper to Ege for him to write the explanations.] *"How many steps we took as 4 cm? [When they took the distance of person A from the floor as 4 cm, they took the horizontal distance as 40 cm as seen in the paper shot above.] Is it true?"*

Paper shot: 

They finished the solution by finding the distance between people as 7480 cm, solving the mathematical models constructed with wrong assumptions. After they gave the solution paper to the teacher, they went on discussing about the solution and asked back the paper realizing their mistakes. Then they decided to take the distance between people directly and confirmed their first solution. In addition to this, it was seen that they reconsidered the solution approach regarding the total height of the theatre. The students, concluding they should validate the solution by applying a different strategy, found the height 14 m by using the assumption that three seats correspond to 1 cm. The students presented their solutions as follows:

Ender: "*We found the distance between people as 12.5 m by measuring with ruler. 1 cm corresponds to a man and the length is approximately 1.7 m. The distance is 2125 cm. We measured each three seats as 1 cm. When we add them, there are 14 seat groups.*"

Researcher: "*I cannot understand how you find the total height?*"

Ender: "*Theatre's*"?

Researcher: "*Yes*."

Ender: "*14 cm.*"

Researcher: "*Cm? I'm asking you the real height. It shouldn't be 14 cm in reality.*

Ender: I'm sorry, I misspoke. We take three steps as 1 m. So it is 14 m."

When examining the solution approaches of the group, it was seen that they made realistic assumptions. They solved the correct mathematical models based on the relevant assumptions. Additionally, it was understood that they were able to conduct interpretations while making assumptions and questioning the reasonableness of the solution. The students, while validating the assumptions, mathematical models and solutions by frequently going back to initial stages of the modelling process, corrected the identified mistakes. The levels of the group's cognitive modelling skills are given in Table 3 based on the result of the related evaluations.

Table 3: Levels of the Cognitive Modelling Skills in the Last Implementation

Skills		**Ancient Theatre Problem**
Understanding the problem	4	4
Simplifying	3	3
Mathematising	4	4
Working mathematically	4	4
Interpreting	4	4
Validating	6	6
Total	**25**	**25**

Comparison of the First and Last Implementations

The levels of the cognitive modelling skills are presented in Table 4 with reference to the solution approaches in the first and last implementations.

Table 4: Levels of the Cognitive Modelling Skills in the First and Last Implementations

	Understanding the problem (…/4)	Simplifying (…/3)	Mathematising (…/4)	Working mathematically (…/4)	Interpreting (…/4)	Validating (…/6)	Total (…/25)
First Implementation							
Bridge P.	4	2	2	4	0	2	**14**
Last Implementation							
Ancient	4	3	4	4	4	6	**25**
Theatre P.	4	3	4	4	4	6	**25**

When Table 4 is analysed, an improvement enabling the transition from noviciate to expertness in modelling is revealed. While the students had no difficulty in understanding the problem, they made better assumptions in accordance with real life, constructed correct and more comprehensive mathematical models based on the assumptions, solved those accurately, interpreted the mathematical results in a real life context, and validated not only the solution of the models but also the assumptions, constructed models and the whole process.

Discussion and Conclusion

In this section, each cognitive modelling skill is discussed by comparing the first and last implementations and supported with the studies in literature.

Although the students seemed to make more or less realistic assumptions in the first implementation, it was understood that they just estimated some values instead of making assumptions. Similarly Maaß (2006) stated that some students could have misconceptions like the idea that simplifying is the same as guessing. Even their partly-appropriate assumptions were a little useful in the solution process. Similarly, Kaiser (2007) also stated that students who are beginners in modelling formed assumptions that were not fully appropriate for the problem situation. On the other hand, novice modellers are reported to have difficulties in representing real world situations in mathematics, in other words, making realistic assumptions (Ji, 2012). On the other hand, in the last implementation, they continuously controlled their assumptions by going back in the process and were able to decide the realism and appropriateness of the assumptions in the group discussion. As Biccard (2010) stated, constructed mathematical models vary in term of the simplifying skills of students. In parallel with his conclusion, the participants' mathematical models were affected by their assumptions directly. Although they were able to form correct mathematical models in the first implementation, their model construction approaches were evaluated as competent only to a limited extent because they built those on partly-appropriate assumptions. The students were able to solve the mathematical models both in the first and last implementations. As they were asked the correctness of the solution in the first implementation, they just checked their calculations and corrected the identified mistakes. In the last one, they similarly went through the checking and correction processes. They did not display any approaches of interpretation even if they were asked to check the solution's comprehensibility. Similarly in some studies (Biccard & Wessels, 2011; Blum, 2011; Ji, 2012; Maaß, 2006), it was emphasized that students have trouble to the largest extent in interpretation. The reason of the absence of interpretation was thought as that they have no idea about considering mathematical results in a real context. It was understood that the last implementation showed rich interpretation behaviours both in deciding on the assumptions and evaluating the results.

Ji (2012) stated that novice modellers were not able to validate the results in a real life context. However, in this study, the students were able to display validation approaches, even if those were rare. The reason why they were partly able to display validation approaches in the first implementation may be one of the probing questions including the process of reviewing the solution. However, they regarded validation as just checking the calculations and correcting the mistakes at first. As Blum (2011), Borromeo Ferri (2006), and Maaβ (2006) stated, this situation is in parallel with the finding that students regarded validation only as checking for operational mistakes. The students who had experiences in the modelling process throughout the study were successful in validation because they considered the validity of assumptions, mathematical models and their solutions as an entirety. This conclusion was seen to be contrasting Ji's (2012) conclusion about the weaknesses of experienced modellers in validating the results.

When all modelling approaches of the students from simplifying the problem to validating the results were examined, the improvement from noviciate to expertness could easily be seen. Thus, long term modelling applications had a positive effect on this improvement as stated in other studies (Biccard & Wessels, 2011; Grünewald, 2012; 2013; Ji, 2012; Kaiser, 2007; Maaβ, 2005; 2006).

This study is considered to reveal that novices can display richer approaches in modelling when they gain experience from suitable implementations developed in a goal-oriented way. It is suggested that the factors effective in the progress of this process can be studied on different levels and with different content.

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