European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

DOI: 10.46827/ejes.v7i12.3473 Volume 7 | Issue 12 | 2020

CONNECTING MATHEMATICS AND PRACTICE: A CASE STUDY OF TEACHING EXPONENTIAL FUNCTIONS

Nguyen Huu Loi[1i],
Tran Luong Cong Khanh[2],
Le Van Tien[3]
[1]Department of Education and Training,
Ho Chi Minh City,
Vietnam
[2]Department of Education and Training,
Binh Thuan Province,
Vietnam
[3]National College of Education,
Ho Chi Minh City,
Vietnam

Abstract:
There is a need for teaching exponential functions to show the necessity for a better match between the knowledge of exponential functions in high schools with the practical application of it in fields. In this research, a teaching process was built in association with teaching situations to show students the relationship between mathematics and real life. The research sample included 76 students in high schools in Vietnam. Additionally, two problems of compound interest and population growth were integrated and were the main research instruments. Data were collected, including student work, and they were analyzed qualitatively. The results showed that students had improved their problem-solving skills and saw the relationship between mathematics and practice. Furthermore, there were some recommendations suggested for textbook authors and teachers.

Keywords: mathematics, practice, exponential function, students' problem-solving skills

1. Introduction

Stemming from the times' needs, mainly Vietnam's education is a substantial reform of educational programs and educational methods oriented to develop students' capacity and the innovation of teaching methods. It is no longer a tendency but has become a sweeping movement in the education industry of our country to achieve the goal of training highly qualified human resources to meet the needs, tasks, and challenges of the

[i] Correspondence: email loinh999@gmail.com, loinh999@yahoo.com

current process associated with knowledge economy development, international integration to adapt to the current opportunities and challenges.

Teaching research that connects the subject's teaching with the real world is deeply interested in many domestic and foreign educators, particularly in the field of math education; researchers try to find solutions to pushes students' understanding of the relationship between math and practice, specifically the relationship of math with other subjects. One teaching strategy that meets this requirement is teaching math modeling because it develops students' math skills and connects their perceptions of mathematical and real-world relationships. This issue is also mentioned in the study of Geiger, V. et al. (2018).

Sello, M. and Percy, S. (2013) carried out a study of the advantages of an exponential and logarithmic functioning transformation approach in 12-year mathematical schools. The study followed a pre-test — intervention — post-test design with qualitative data informing quantitative information. Data collection strategies included a test (on exponential and logarithmic functions) given to learners before and after the intervention. The intervention strategy was carried out to enhance conceptual understanding by teaching exponential functions through the transformation approach. The study, reported from two different schools, took part in a convenience sample of 38 experimental and 40 12^{th} grade student comparison groups to avoid contaminating collected data. The findings indicated that the interventional strategy influenced the learners' academic performance. Furthermore, when solving exponential and logarithmic functions, learners began to understand and apply the learned strategy.

Exponential and logarithmic functions are vital mathematical concepts in advanced mathematics, according to Aranka, E. (2009). These concepts, unfortunately, also present serious problems for students. Therefore, the writer gained an overview of exponential and logarithmic functions in Hungary, Austria, and the Netherlands. This comparison provided Hungarian teachers with some realistic ideas on how to incorporate this subject into their practice.

Additionally, Nunes, L. F., Prates D. B., Silva, J. M. da (2017) analyzed mathematical content related to the teaching and learning of exponential functions in a freshman group of students registered in the first semester of the Science Bachelor and Technology. As a contextualization tool strongly mentioned in the literature, the modeling approach was used as an educational tool for contextualizing exponential functions in the teaching-learning process. Similarly, Yamamah S. (2018) researched the effectiveness of solving exponential problems to impact students' achievements and attitudes.

According to Duong, H. T. et al. (2019), a fundamental goal of teaching mathematics is to develop students' ability to apply mathematical knowledge to solve real-world problems. This purpose is also associated with evaluating specific mathematical competencies: mathematical thinking and reasoning, mathematical inference and proof, mathematical communication, modeling, problem-solving and representation, using mathematical symbols and languages, using calculation tools. Modeling is the capacity mentioned by many educators worldwide and is increasingly vital in many popular mathematics programs in many countries. This study aims to build students' mathematical modeling skills by teaching theorems about sinus and cosine. In this sample, 46 students in grade 10 were asked to solve many theoretically-related real-world concerns in Phan Thanh Giang high school, the province of Ben Tre, Vietnam. Results showed that many students have advanced in mathematical modeling skills, which motivate them to study and help them implement mathematics in practice.

The advent of exponential functions has a substantial impact on science and technology, including mathematics itself. In engineering science, the specialness of exponential functions is maximized, in which modeling the growth (decline) present in nature is the characteristic topic for this special. Then, the meaning associated with the exponent in the real model will be shown. The models analyzed in the chapter are population growth, radioactive decay, and the object's heat reduction, showing that the physical meaning is exponentially associated. The modeling also shows clearly the meaning of the concept of variables and the concept of dependence in reality, which is also a particular difficulty in teaching and learning an exponential function.

Researchers have conducted knowledge analysis to teach exponential functions in textbooks of Vietnam, France, and America. Accordingly, the Vietnamese, French, and American textbooks construct exponentials in three different ways. Vietnamese textbooks define exponential functions based on expanding the concept of powers after building the concepts of real numbers and limits (Tran, V. H. et al. (2016)). In contrast, French and American textbooks have different construction methods. This approach shows that there exist many ways to build exponential concepts in teaching practice. One important thing is the inadequacy of real-world problems associated with exponential functions in Vietnamese textbooks.

For this reason, students may not be able to see the practical applicability of exponents. Therefore, the problem posed needs to reinforce such problems to promote students' understanding of exponential functions in real-world phenomena. Besides, the classification of problems, including real-world problems, is clarified in the book of Le, V. T. (2005).

2. Research Objectives

This study was conducted with the following primary purposes:

1) Develop students' problem-solving skills as they resolve real-world problems associated with the topic of exponential functions.
2) Show students the positive relationship between mathematics and practice.

3. Research Methodology

3.1 Participants

The sample included 76 students in grade 12 in high schools in Ho Chi Minh city. This experiment was done on 12th-grade students after they had finished learning about exponential functions.

3.2 Instrument and Procedure

3.2.1 Problem 1

Mr. A has deposited money in Bank B, and now Mr. A request to withdraw all the deposited amount with the interest rate. Due to the software's numerical display format error, the bank announces the total amount to be withdrawn as a formula, but the result cannot be recorded in standard form. Mr. A received notice that the amount in his account is $3^{\sqrt{2}}$ billion VND.

1) Please rely on the datasheet below to help Mr. A know the amount he will receive (exactly VND).
2) If possible, write a few lines to instruct a Grade 11 student (who has not learned powers with irrational exponents) how to determine $3^{\sqrt{2}}$ from the learned knowledge.

3.2.1.1 Purpose of Problem 1

Problem 1 is intended to help students understand the value of powers with real exponents, especially powers with irrational exponents in reality (the value is significant for the real situation). Analysis of textbooks shows that the definition of exponentiation with real exponents based on a challenging, abstract concept for students is the concept of limitation. The value of a limit is entirely determined mathematically. However, some limitations are not precisely defined or unexplained in reality, which have different meanings for each specific problem. A famous statement about the Zenon paradox is a case in point. The following problem helps students better understand the meaning of irrational exponentiation in the field of bank deposits.

3.2.1.2 Solution Strategies of Problem 1

- Strategy S1A: this is a safe option for students who do not fully understand the meaning of powers with real exponents.
- Strategy S1B: a strategy for choosing the answer is B. When it is unclear about the convergence of ranges 31.4, 31.41, 31.414, ... for this practical problem (just need to be accurate to in copper units), then this is the option that the student thinks the last number in the datasheet is the most accurate.
- Strategy S1C: the answer choice strategy is C. If students choose the answer of 4.728804387 (this is the value of the last four numbers in the table to be nine decimal places), it will show that students notice the convergence of powers to rational exponents 31.4, 31.41, 31.414, ... in this real situation (the received sums are calculated to the units of copper). If a student chooses an answer other than 4.728804387, this is either an error or an uncontrollable student's technical error. The probability of a mistake or error, in this case, is not high.
- Strategy S1D: the answer choice strategy is D. This option is to collect additional explanations from students other than A, B, or C (including the answers you do not know how to count, do not know the answer). Thereby, it further identifies students' conceptions of the concept of powers with real exponents.

Predicting the probability of an S1A strategy occurring is highest because now the handheld computer is a student's always-on tool, followed by the S1B strategy, the S1C and S1D strategies are less likely to happen.

3.2.2 Problem 2

In 2018, City A had a population of 13,000,000 people. The annual natural population growth rate is 1.2%. Each year, 84,000 people come to live in city A. The number of people leaving city A to live elsewhere is negligible. By 2030, all people living in the city will receive free medical care in City A's socio-economic development plan. It is estimated that by 2030, health care funding per person per year will be 30 million VND. To perform the proposed plan well, the city needs to have forecast data in the coming years, including population data. To estimate the budget that City A will need to spend in 2030 on health care for its residents as planned, one needs to calculate the city's population by 2030. Know the formula for calculating the population of city A is: $P(t) = 13000000e^{(0,012+0,006)t}$, where 13 000 000 is the population of city A in 2018, e is the natural logarithmic base, 0.012 is the natural growth rate, 0.006 is the mechanical population growth rate, t is the period (year) from 2018. Let $r = 0.012 + 0.006 = 0.018$. We have another formula to calculate the population by year:

$$P_1(t) = 13.000.000 * (1+r)^t$$

where t is the number of years (t = 0 corresponding to 2018), the value 0.006 is the mechanical population growth rate.

The quarterly (3 months) population formula is:

$$P_4(t) = 13.000.000 * (1+\frac{r}{4})^{4t}$$

where t is the number of quarters.

The formula for population by month is:

$$P_{12}(t) = 13.000.000 * (1+\frac{r}{12})^{12t}$$

where t is the number of months.

Know that

$$\lim_{n\to\infty} P_n(t) = P(t),$$

with

$$P_n(t) = (1+\frac{r}{n})^{nt} = \left[(1+\frac{r}{n})^{\frac{n}{r}}\right]^{rt}.$$

1) Please tell us: P (t) can be used to calculate population growth over some time. Why?
(Based on the expression $\lim_{n\to\infty} P_n(t) = P(t)$).

2) Plot the graph shapes of the exponential functions (without exact calculation) representing population growth

$P_1(t) = 13.000.000 * (1+r)^t$

and

$P(t) = 13.000.000 * e^{rt}$.

3) Please comment geometrically on the graphs just drawn.
4) According to the above comment, please state each graph's meaning for the phenomenon of population growth.

3.2.2.1 Purpose of Problem 2

Problem 2 is intended to help students approach a unique property of exponential functions, which is the solution of differential equations representing the change in the state of things, phenomena in nature, science and technology, and copper the time of enhancing practical skills, creative experience activities, applying mathematical concepts into practice, combining integrated teaching according to current requirements (this experimental project problem tries to put in some terms, situations of sociology, public administration). The following situations are designed in the direction of teaching using the exponential application by modeling.

3.2.2.2 Solution Strategies of Problem 2

- Strategy S2.2A: month-quarter-year strategy. Based on the equations P1 (t), P4 (t), P12 (t) gave students possible answers: month, quarter, year, month-quarter-year.
- Strategy S2.2B: monthly strategy. Due to not relying on limit expressions, students rely on any P12 (t).
- S2.2C strategy: quarterly strategy. Due to not relying on limit expressions, students rely on any P4 (t).
- Strategy S2.2D: year strategy. Due to not relying on limit expressions, students rely on any P1 (t).
- Strategy S2.2E: every time strategy. Here is the strategy for the expected answer. The interpretation of the answer thanks to the limit expression

$\lim_{n\to\infty} P_n(t) = P(t)$.

Specifically, since

$\lim_{n\to\infty} P_n(t) = P(t)$ the larger n is $P_n(t)$,

the closer it is to *P(t)*, then we can calculate the population growth in smaller intervals (all times) using the formula *P(t)*. This way shows the practical significance of the exponential continuity P (*t*) with the real-time variable *t*.

- Predict the strategy S2.2A with the highest chance of occurring, followed by the strategy S2.2B, S2.2C, S2.2D, and finally, the strategy S2.2E.
- Strategy S2.3A: plotted line or reliable curve graph. In this strategy, students will plot the function P1 (t) as a fold line connecting the specified points on the coordinate axis and the graph of the function P (t) as a solid curve.
- Strategy S2.3B: the graph of the function is always a solid curve. In this strategy, you will plot two functions, P1 (t) and P (t), that are similar to a solid curve.
- Strategy S2.3C: Plotting the correct graph. With this strategy, the graph of the function P1 (t) is represented by dots on the coordinate axis, and the functional graph P (t) is the ascending solid curve of a covariance function.
- Predict the strategy S2.3A with the highest chance of happening, followed by the S2.3B strategy and finally, the S2.3C strategy.

3.3 Six phases of the teaching process

Experimental exercises in this teaching project take place continuously in 6 small phases with six answer cards:

Phase 1: (Teacher gives each student a worksheet 1 and records it after 10 minutes). The teacher did not explain after the students made an individual on the 1st sheet; this was intended for the students to exchange when making the second vote.

Phase 2: (Teacher hand out the student group number 2 and collect it after 10 minutes). The goal of this phase is for students to answer the problem in groups to check how students improve their work after exchanging. Students explain to each other during the grouping process to help them absorb the teacher's explanation about the meaning of the problem more efficiently. It also helps students understand the value of powers with real exponents, especially powers with irrational exponents in practice.

Phase 3: (Teacher distributes to each answer questionnaire number 3 and records them after 6 minutes) This question explores students' ability to handle real situations (corresponding to step 4 (in 4 steps) of mathematical modeling). The teacher will guide how to handle this later.

Phase 4: (Teacher passes out the worksheet 4 to groups of students and records them after 8 minutes). This phase is intended for students to answer the problem in groups to check how students improve their work after exchanging. After talking to each other, students will better absorb the teacher's explanation. The problem is printed with a data sheet that includes the approximate values of the expressions and the exponentiation expression to help save time on experimenting in class and as a guide to learn. Students performed consistently in the answer questions and are being used as illustrations for the teacher's explanation.

Phase 5: (The teacher works in this phase with the worksheet 5 for individual students to answer in 5 minutes). This phase aims to help students better understand the formulas for population growth, where formula P (t) can calculate population growth at any given time, corresponding to the function P (t).) is continuous at all its defined points.

Phase 6: (Teacher performs this phase with the worksheet 6 individually for 10 minutes). The teacher explains sentences 2 and 3 of lesson 2 in 10 minutes. Question 3 is done independently from question 2 not to let students go back to correct answer 2, because when the answer is finished, students may have more information to correct their answer 2. The purpose of this question understands students' notions (through functional graphs) about the continuum of exponential functions for population growth. Thereby, the teacher reinforces the continuity of the exponential function for population growth.

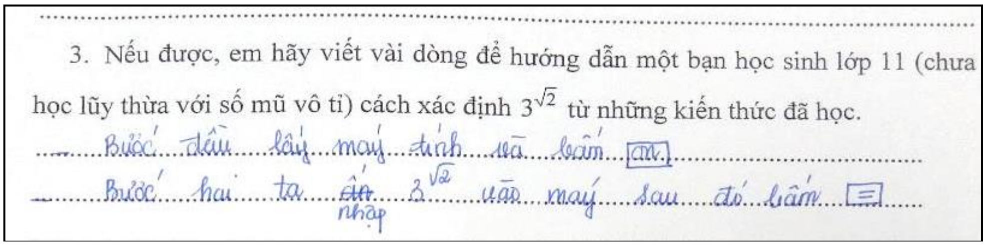
4. Results and Discussion

The teacher handed out worksheet 1 and asked students to do the exam in 10 minutes to test their understanding or understanding of real exponents' powers. The results achieved were shown in the table below.

Table 1: Statistics of Students' Strategies in Phase 1

	Strategy S1A	Strategy S1B	Strategy S1C	Strategy S1D
Numbers	49	10	11	6
Percentages	64.5%	13%	14.5%	8%

From statistics showed that, as expected, the majority of students chose the S1A strategy, accounting for 64.5%. Related to the correct strategy S1C, only 11 students (14.5%) chose. This problem had shown that students computed power values well mathematically, but the judgment about the actual meaning of powers had not been shown well. Student explanations also showed that the S1A strategy was dominant.


3. Nếu được, em hãy viết vài dòng để hướng dẫn một bạn học sinh lớp 11 (chưa học lũy thừa với số mũ vô tỉ) cách xác định $3^{\sqrt{2}}$ từ những kiến thức đã học.

Bước đầu lấy máy tính và bấm [ON]

Bước hai ta nhập $3^{\sqrt{2}}$ vào máy sau đó bấm [=]

Figure 1: An Illustration of the Student's Solution According to Strategy S1A

During phase 2, the researchers distributed worksheet 2 to groups of students and asked them to do it for 10 minutes. The strategies students chose were as follows.

Table 2: Statistics of Student Groups' Strategies in Phase 2

	Strategy **S1A**	Strategy **S1B**	Strategy **S1C**	Strategy **S1D**
Numbers of group	5	0	1	1
Percentages	71.4%	0%	14.3%	13.3%

Although they had done the individual homework, when they were exchanging groups to repeat the above question, did not choose the best strategy, which showed that they were interested in pure math power values rather than what it meant in practice. After these two operational phases, the teacher consolidated the concept of power, explained the meaning of the problem, and provided more sense of the concept of powers to students.

In Phase 3, students were given worksheet 3 and worked on their assignment for 6 minutes. The students answered with the following strategies:

Table 3: Statistics of Students' Strategies in Phase 3

	Strategy S2.1A	**Strategy S2.1B**	**Strategy S2.1C**
Numbers	17	4	55
Percentages	22.3%	5.3%	72.4%

Students chose the most S2.1C strategy, where the number of students choosing line 7 was 28 and explain because the lines from here onwards were stable in the outcome value. Thus, these students were here affected by problem 1. The result was an unexpected situation. The number of students who chose other lines in the S2.1C strategy was probably selected at random as analyzed. Strategy S2.1B had four students chosen. It was revealed that students were not familiar with problems applying mathematical knowledge to real-world situations.

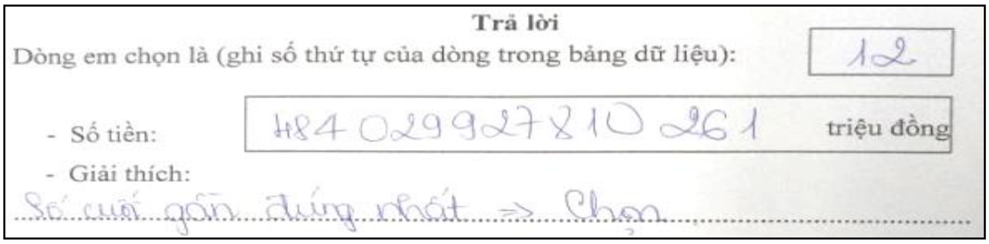

Trả lời
Dòng em chọn là (ghi số thứ tự của dòng trong bảng dữ liệu): 12
- Số tiền: 484029927810261 triệu đồng
- Giải thích: Số cuối gần đúng nhất => Chọn

Figure 2: An Illustration of the Student's Solution According to Strategy S2.1C

In Phase 4, seven groups of students all chose the S2.1C strategy, which included explanations such as: "for the 10th line of the sum of money", "the convergence of the maximum value (10)", "Press the calculator according to the given formula, based on the results of the data sheet lookup. Choose number 7 because line 7 numbers began to converge. Results of phases 3 and 4 revealed that students were not familiar with the form of problems applying mathematical knowledge to real situations, then they only focused on mathematical calculations. On the other hand, it was seen that students were somewhat affected by problem 1. Therefore, teachers' explanations for these problems were necessary.

Table 4: Statistics of Students' Strategies in Phase 5

	Strategy S2.2A	**Strategy S2.2B**	**Strategy S2.2C**	**Strategy S2.2D**	**Strategy S2.2E**	**No answer**
Numbers	13	5	3	12	7	36
Percentages	17%	7%	4%	16%	9%	47%

The unexpected situation happening in Phase 5 was that the number of blank votes or written answers did not mean 36 votes. Although there was an extra guide for students to rely on the expression

$$\lim_{n\to\infty} P_n(t) = P(t)$$

to answer, 36 (48%) of the students who did not have the answers indicated that they were perplexed about the concept of limits and the application of functions learned to solve real-world problems. Additionally, this result showed that students had not made fair use of the exponential function's continuity.

The remainder chose to focus on strategy S2.2A, and strategy S2.2D showed random or emotional selection. The correct S2.2E strategy had only 7 (9%) options, showing very few students exploiting the exponential function's continuity.

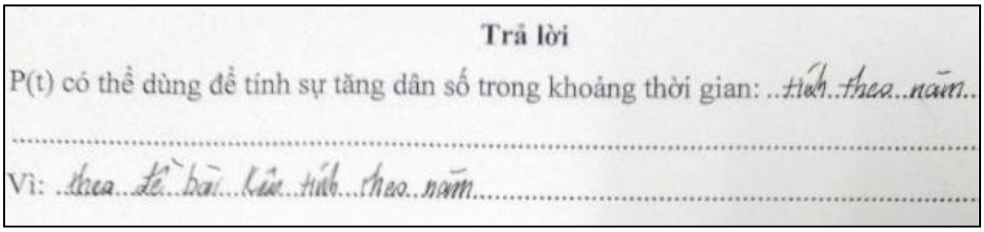


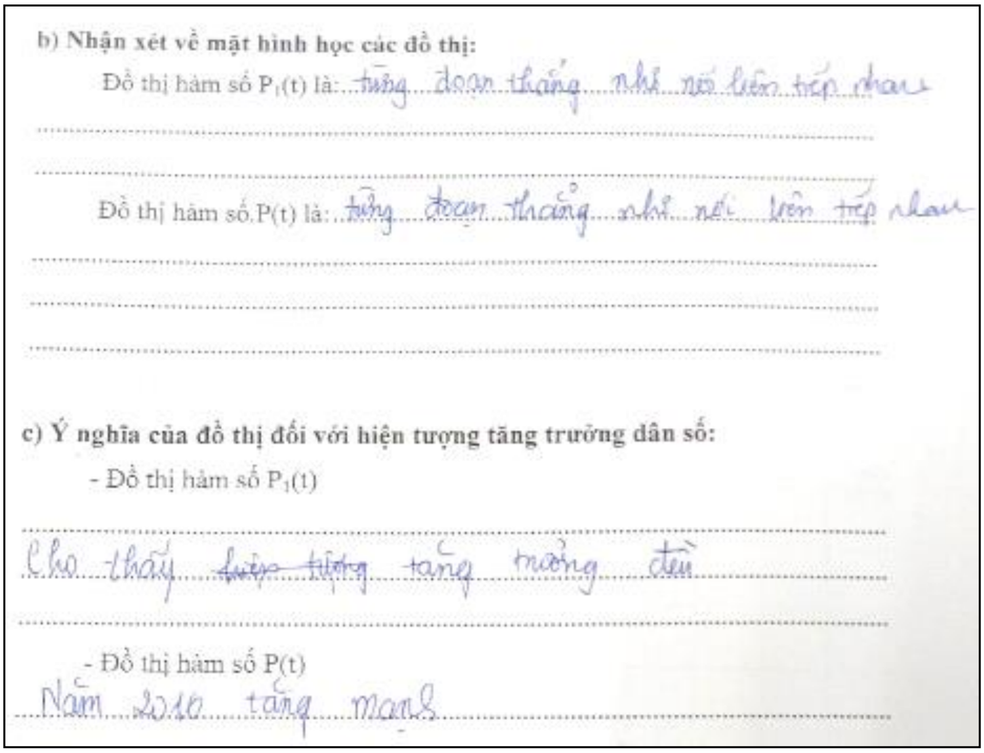
Figure 3: An Illustration of the Student's Solution According to Strategy S2.2A

In Phase 6, the teacher gave out worksheet 6, and the students worked in 10 minutes. The results of students' homework were shown in the following strategies:

Table 5: Statistics of Students' Strategies in Phase 6

	Strategy S2.3A	**Strategy S2.3B**	**Strategy S2.3C**	**2 twisted lines**	**2 straight lines**	**No answer**
Numbers	2	20	0	5	4	4
Percentages	5.7%	57%	0%	14.5%	11.4%	11.4%

With the above results, no students were choosing the correct S2.3C strategy, which was not much of a surprise because Phase 5 showed that students were still confused about concepts, continuity and had not yet answered the real problem related to exponential functions. This difficulty also resulted in 20 students (57%) choosing the S2.3B strategy, suggesting that they did not recognize the definite set of exponentials in this real situation.



b) Nhận xét về mặt hình học các đồ thị:

Đồ thị hàm số $P_1(t)$ là: từng đoạn thẳng nhỏ nối liền tiếp nhau

Đồ thị hàm số $P(t)$ là: từng đoạn thẳng nhỏ nối liền tiếp nhau

c) Ý nghĩa của đồ thị đối với hiện tượng tăng trưởng dân số:

- Đồ thị hàm số $P_1(t)$

Cho thấy ~~hiện tượng~~ tăng trưởng đều

- Đồ thị hàm số $P(t)$

Năm 2010 tăng mạnh

Figure 4: An Illustration of the Student's Solution According to Strategy S2.3B

5. Conclusion

The experimental findings were reported that the students had good skills in mathematically processing power values, but they were still limited in applying mathematical knowledge to solving real situations in which they showed real-world meanings. Specifically, the skill of mathematically processing power values was better than identifying the actual meaning of powers. Another discovery was that some students also made errors when solving mathematical problems for some reasons such as carelessness, miscalculation, and misunderstanding.

Additionally, some suggestions are for textbook authors and teachers. The textbook authors need to present the exponential function content following its appearance in history and strengthen real-world problems in some interdisciplinary subjects such as social science, economics, physics, and chemistry. Meanwhile, teachers need to add practical problems associated with exponential functions to help students see the role, meaning, the relationship between mathematics and everyday life. From here, students will develop the necessary mathematical competencies, most notably the ability to model mathematical problems.

References

Aranka, E. (2009). Comparison of Teaching Exponential and Logarithmic Functions Based on Mathematics Textbook Analysis. *Teaching Mathematics and Computer Science,* Vol. 7, No. 2, 297-318.

Duong, H. T., Nguyen, P. L., Bui, P. U., Le, T. G. (2019). Developing the Competency of Mathematical Modelling: A Case Study of Teaching the Cosine and Sine Theorems. *International Journal of Learning, Teaching and Educational Research,* Vol. 18, No. 11, 18-37.

Geiger, V., Stillman, G., Brown, J., Galbraith, P., Niss, M. (2018). Using Mathematics to Solve Real World Problems: The Role of Enablers. *Math Ed Res* J, Vol. 30, 7-19.

Le, V. T. (2005). Methods of Teaching Mathematics in High Schools, *Ho Chi Minh City University of Education: Publishing House*.

Ministry of Education and Training. Mathematics Education Curriculum, Vietnam, 2018. Retrieved from https://data.moet.gov.vn/index.php/s/LETzPhj5sGGnDii#pdfviewer.

Nunes, L. F., Prates, D. B., Silva, J. M. da (2017). The Use of Modeling Approach for Teaching Exponential Functions. *Journal of Physics: Conf. Series*, 936 (2017) 012064, 1–9.

Yamamah, S. (2018). The Effects of Teaching Exponential Functions Using Authentic Problem Solving on Students' Achievement and Attitude. *Wayne State University Dissertations*. 1959, 1-234.

Sello, M., Percy, S. (2013). Teaching and Learning the Mathematical Exponential and Logarithmic Functions: A Transformation Approach. *Mediterranean Journal of Social Sciences*, Vol. 4, No. 13, 177-185.

Tran, V. H., Vu, T., Le, T. T. H., Nguyen, T. T. (2016). Calculus Textbook 12, *Vietnam Education Publishing House*.