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USING ANALOGY IN SOLVING PROBLEMS: A CASE STUDY OF TEACHING THE RADICAL INEQUALITIES

Bui Phuong Uyen[i]
School of Education,
Can Tho University
Can Tho City,
Vietnam

Abstract:
In mathematics education, teachers can use several reasoning methods to find solutions such as inductive, deductive and analogy. This study was intended to guide students to find solutions to problems of radical inequalities through analogical reasoning. The experiment was conducted on 36 grade 10 students at a high school in Can Tho city of Vietnam. The instrument used was a problem of radical inequalities. A three-phase teaching process had been organized with this class comprising individual work phase, group work phase and institutionalization phase. The data collected included student worksheets and was qualitatively analyzed. As a result, many students discovered how to solve the above inequality by using the analogy, and they had a considerable improvement in their problem-solving skills. Additionally, a few ideas were discussed about the use of analogy in mathematics education.

Keywords: analogy, radical inequalities, students' problem-solving skills

1. Introduction

An analogy is especially useful in mathematics education, where it serves as a way to teach students some new concepts and may techniques while also making discoveries. When utilizing the analogy, students should learn the old material in order to be able to discover new concepts on their own. Thus, students have a good opportunity to discover new theories and conduct experiments to refine their understanding of a hypothesis. This process promotes thinking development because it requires learners to consider, analyze, compare, compare, generalize knowledge; from there, it encourages the passion for learning and is the driving force to promote students' independent thinking, critical thinking and creative thinking.

Moreover, students will often make many errors in acquiring knowledge of mathematical facts when they are learning a subject such as this. Sometimes they are

[i] Correspondence: email bpuyen@ctu.edu.vn

mistakes in mechanical calculation, but sometimes they are mistakes in reasoning, mistakes due to lack of knowledge. The process of researching students' mistakes is critical to advancement in learning. It is necessary to consider student error analysis as an effective method to expand and perfect knowledge, analyze and recognize mistakes to identify the mistake, cause the mistake, and prevent it.

When encountering a new situation, students tend to compare and compare it with previous similar problems, thereby finding out how to address it. These same inferences can help find new knowledge, helping students practice independence and creativity in problem-solving. Only one inference does, but all of that variety yields incorrect conclusions. Students make errors and interpret due to failing to account for details when they use an analogy.

1.1 The role of analogy in teaching and studying student errors

Even though math learning and solving problems utilize different concepts, the vast majority of examples use analogies in their examples. Much human inference is related to the same and is made using schemas from everyday life (Sarina & Namukasa, 2010). This approach also means that people perceive relationships by analogies because the process is natural and common to human understanding. The authors have studied analogies in mathematics and have shown that analogies can provide new problems and improve efficiency, problem-solving ideas. Recent studies pay more attention to the role of analogy in learning, in scientific research, particularly in children's learning of mathematical concepts. Mathematics textbooks often use analogies (Kepceoğlu & Karadeniz, 2017). Likewise, the Vietnamese Mathematics education program discusses this issue (Ministry of Education and Training, 2018).

Besides, teachers often use analogies to make concepts easier to understand. Many analogies are regarded as the earliest and straightforward models or a way to grasp theoretical concepts. Teachers often find analogies useful and do not realize that it is the effect they have on their words have on others. Dwirahayu et al. (2017) demonstrated instruction's success with an analogy for derivative concepts on his students' mathematical representation capabilities. Similarly, it can play an important role in helping students build their knowledge, consistent with the constructivist learning perspective. Similarly, it can help students build bridges between concepts, what is familiar, with what is new. And then provide students with illustrations that can help them visualize novel, complex, or confusing concepts. In the research of Richland et al. (2004), eighth-graders effectiveness was shown about analogical thinking.

Furthermore, in teaching mathematical theorems, we can use attribute analogy or relationship similarity between objects to make hypotheses and then prove or reject them. The first step in forming scientific theories is analogies, which begin by proposing abstract comparisons to existing conditions. However, just like the hypothesis, the analogy's conclusion is not inevitable; it may be true, it may also be false. Nevertheless, the analogy is not proof; it helps us to broaden our understanding, build hypotheses; Its conclusions must rely on deductive evidence or positive practice to know whether true or false.

Additionally, there are many kinds of methods at the disposal for solving problems when dealing with math applications that are equal in effectiveness to or better than the standard algorithms. As students will never know everything they need to know in a class, there will always be errors in the learning process. Pange et al. (2009) conducted a study about the errors related to analogical reasoning in mathematics. Similarly, Loc & Uyen (2016) surveyed students' errors when they coped with analytic geometry problems. In the process of learning mathematics, students may make mistakes due to many different reasons such as miscalculation, unintentional, not mastering concepts, properties. Since students are mindful of all learners try to acquire knowledge by making systematic mistakes, all seek to expand their understanding, according to a constructivist theory. Identifying and correcting students' mistakes plays an important role because the awareness of the characteristics of the mistakes is the factor that constitutes the meaning of the knowledge that teachers want to build for students.

2. Research Objectives

These studies were primarily intended to be done for these goals:

1) Investigate students' errors when solving math problems because of their use of analogy.
2) Instruct students to use an analogy when addressing radical inequalities.

3. Research Methodology

3.1 Participants

The study sample includes 36 students in grade 10A1 of the High School for Pedagogy Practice, Can Tho City, Vietnam. Student participation was limited to free students to take on the day that we offered the class, offering suitable, interesting contributions. Researchers examined whether or not prejudice or disrespect toward students had any effect on their ability to enroll children successfully but found nothing conclusive.

3.2 Instrument and Procedure

3.2.1 The Problem

Solve inequalities:

$$\sqrt{x^2 - 3x} < x + 1$$

This mathematical problem is one of the basic math forms of problems presented in the 10th math textbook of Vietnam (Tran et al., 2016).

3.2.2 Solution Strategies of the Problem

- Strategy S1: Students will make consequences change:

$$\sqrt{x^2-3x} < x+1 \Rightarrow x^2-3x < (x+1)^2 \Rightarrow 5x+1>0 \Rightarrow x > -\frac{1}{5}.$$

There is an impossibility of attempting the solution again because doing so would lead to an error.

- Strategy S2: Students will square the two sides.

$$\sqrt{x^2-3x} < x+1 \Leftrightarrow \begin{cases} x+1 \ge 0 \\ x^2-3x < \left(x+1\right)^2 \end{cases} \Leftrightarrow \begin{cases} x \ge -1 \\ 5x+1>0 \end{cases} \Leftrightarrow x > -\frac{1}{5}.$$

This solution does not meet the condition $x^2-3x \ge 0$; it is unsuitable.

- Strategy S3: Students will do equivalent transformation.

$$\sqrt{x^2-3x} < x+1 \Leftrightarrow \begin{cases} x^2-3x \ge 0 \\ x+1 \ge 0 \\ x^2-3x < \left(x+1\right)^2 \end{cases} \Leftrightarrow \begin{cases} x \le 0 \vee x \ge 3 \\ x \ge -1 \\ 5x+1>0 \end{cases} \Leftrightarrow \left[\begin{array}{l} x \ge 3 \\ -\frac{1}{5} < x \le 0. \end{array}\right.$$

3.2.3 Three phases of the teaching process

The situation is posed to grade 10 students when studying radical inequalities.

- Phase 1 (students work individually - 10 minutes): Students resolve the problem individually.
- Phase 2 (students working in groups - 5 minutes): Students discuss in groups of 4 students to agree on overcoming the given problem.
- Phase 3 (validation - 5 minutes): The teacher collects students' worksheets, asks questions and answers them.

a) What is the solution you used to address the math problem? From where do you deduce this solution?
b) What are the complete solutions to the given problem?
c) Comment on the solution to this problem?

3.3 Pedagogical intentions

A method that challenges students to search for new solutions and apply and review what they know in light of previous successes.

- Phase 1: Students are expected to explore ideas on their own to resolve problems. They need to look for relevant knowledge and known problems to deduce how to deal with a new problem.
- Phase 2: Students work together to express ideas, then choose the most effective way to use them to create a solution. This stage is also an opportunity for students to receive feedback through the exchange process and discuss it with other classmates.

- Phase 3: When teachers ask their students challenging questions, new problems expand their problem-solving ability.

4. Results and Discussion

- **Phase 1**

Students who completed several worksheets about the results of their investigations were listed in the following table.

Table 1: Statistics of Students' Strategies in Phase 1

	Strategy S1	Strategy S2	Strategy S3	Other Strategy
Numbers	2	5	27	2
Percentages	5.56%	13.88%	75%	5.56%

Concerning the results in Table 1, the most prevalent strategy was found to be S3. This outcome proves that most of them chose the right strategy to solve the given problem. Two students used the consequence transformation similar to the math problem solving, but they did not try again. Besides, five students used the strategy of squaring two sides of the equation but did not set conditions for the radical sign's expression. This mistake was caused by the students using the same solution as the problem-solving equations ($\sqrt{A} = B \Leftrightarrow \begin{cases} B \geq 0 \\ A = B^2 \end{cases}$); in there, the condition $A \geq 0$ could be dropped. Some other students still used the strategy of squaring the two sides of the inequality but only set conditions for the radical sign's expression.

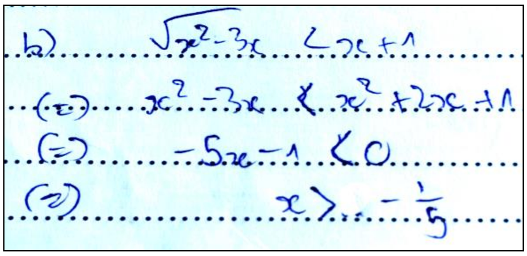

Figure 1: An Illustration of the Student's Solution According to Strategy S2

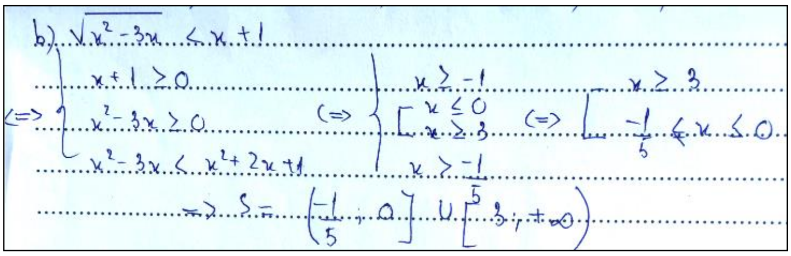

Figure 2: An Illustration of the Student's Solution According to Strategy S3

- **Phase 2**

Phase 2 involved assigning the class members into nine separate groups. The discussion and cooperation were equally distributed amongst the four groups, with everyone agreeing on how to resolve the problem. The results showed that the S3 strategy was still dominant, with seven selected groups. Especially, no group chose strategy S1 (significant change). They were able to find that they could not re-test their shortcomings when solving inequation, so they did not choose this strategy. Also, two different groups arrived at the same conclusion when using the S2 strategy. This result proved that some students still used analogical reasoning to cope with the equation $\sqrt{A} = B$ and did not realize the need to set additional conditions $A \geq 0$.

- **Phase 3**

The participants were asked a few additional questions in phase 3 to establish their expertise. For question 1 (Please state the solution that they used to overcome the inequalities ($\sqrt{A} < B \Leftrightarrow \begin{cases} B \geq 0 \\ A < B^2 \end{cases}$), a student gave the non-negative two-sided squaring strategy because it was similar to solving the equation $\sqrt{A} = B \Leftrightarrow \begin{cases} B \geq 0 \\ A = B^2 \end{cases}$. There was, however, an additional piece of information that demanded multiple conditions to evaluate. At last, they described the extremely complicated conditions, and they were able to describe the solution because of their extensive knowledge. The following was a complete solution to the inequality on question 2: a student approached the podium to present a complete solution strategy. Finally, when we asked the third question (Draw your comment on how to solve this problem), they made the following statement:

When solving an equation $\sqrt{A} = B$, it was possible to use the consequence transformation to find the solution and then try again. If students used the square of two sides, they only needed to set conditions $B \geq 0$, and the condition $A \geq 0$ could be dropped because words could be deduced. When solving inequalities $\sqrt{A} < B$, the students needed to square both sides and set both conditions $A \geq 0$ and $B \geq 0$.

Thereby, it shows that the students received feedback from the environment to correct their mistakes due to using the same knowledge they learned and finding the correct way to address new problems.

5. Conclusion

This outcome is demonstrated in some instances that experimental results that some students discovered new answers to previously unsolved problems using previously known methods. Indeed, thanks to the environment's feedback through the interaction between students in the group, between students and teachers, they recognized and corrected the errors made. Another interesting fact is that many students had remarkable progress regarding the task of resolving radical inequalities in their problem-solving abilities.

The most relevant work in this research has shown that students have difficulty in problem-solving because they keep using the same errors. Thus, teachers need to have practical measures to guide students to detect and correct mistakes due to using the same in solving math problems. Instead of offering guidance, it advocates a new pedagogical strategy grounded in students' errors. Concepts of the mistakes based on learning theories, mistakes due to analogical use in solving math problems can be deployed to train high school math teachers and train prospective teachers in mathematics education at other pedagogical schools.

Conflict of Interest Statement

The authors declare no conflicts of interests.

About the Author

Bui Phuong Uyen is a senior lecturer in the Department of Mathematics, School of Education, Can Tho University, Vietnam. As part of her innovative approach to teaching mathematics, she teaches undergraduate and postgraduate math programs. Furthermore, she is pursuing the most significant creative directions in scientific research, such as General Educational Science, teaching mathematics, testing and evaluation in mathematics education, and applying information technology in teaching mathematics.

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