



TEACHING FRACTIONS TO BILINGUAL STUDENTS: A CASE STUDY IN SCHOOLS OF THE DODECANESE, GREECE

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Abstract:

In this study we present a method for the simultaneous interaction of many bilingual students by the use of tests. Its application was to fractions, where the difficulties were deemed to be the basis of a theoretical model. With this same theoretical model we created a teaching method for the teaching of fractions. First there was an evaluation of the situation, followed by the drawing up of a teaching scheme on themes of language to improve communication between pupil and teacher. Subsequent examination that there was an improvement in the students' comprehension of fractions.

Keywords: representations, factual Greek schools, language in teaching, bilingual

1. Introduction

A person who can speak two languages with equal fluency or someone who makes "periodic use of two languages" (de Avila and Duncan, 1981), is considered to be "bilingual". Further, we can classify as bilingual any pupil who belongs to a social group that speaks two languages, for example a family.

In Greece, and particularly in the larger Aegean islands, the last decade has seen a wave of immigrants from the Balkans and from Eastern European countries. In the main these are economic immigrants who work in hotel units and in the wider field of tourism. The majority of the immigrants come to the Dodecanese islands with their families. Many of the children from these families who come here are very young, even if they have not been born in Greece. So, they attend lessons in Greek schools, but at home they speak the language of their country of origin.

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The phenomenon of having pupils at school who speak a different language at home is very common in these regions, because, in the Aegean as a whole, and mainly on the large islands of the Dodecanese, there is great touristic activity, and many of these tourists choose to stay in the Dodecanese, working mainly in the tourist industry. We may also see a large number of mixed marriages, mostly Greek men married to women from northern Europe, such as the Scandinavian countries, Germany etc. This phenomenon started in the 1970s and 1980s, which was the “golden age” of tourism in the large islands of the Dodecanese. In these cases, the children usually speak 2 languages at home and with other individuals (in the family environment, with friends of the parents, etc.) and at school they use only the Greek language.

The above information which is relevant to the Dodecanese is not in accord with the definitions of De Avila and Duncan, to whom we referred in our introduction since the student does not have equal fluency in both languages. This is because bilingual people use their languages for different reasons in different areas of their lives and with different people. Because the needs of the bilingual student are usually quite varied, a bilingual student rarely develops the same speaking ability in both languages (Grosjean, 1999). Thus, in almost all the works that refer to bilingual students, any comparisons between bilingual and monolingual individuals are made based on the linguistic dexterity, models and rules that apply to monolingual individuals (Grosjean, 1999).

However, in order to be able to help a student, and particularly in the case of the bilingual student, we must evaluate his needs, gaps in knowledge, and the difficulties he meets if we are to advance his comprehension of new concepts etc. As regards mathematics this evaluation must cover a wide cognitive area and give a complete view of the mathematic knowledge of the student, and not evaluate only a number of individual skills. Only then, through proper evaluation, can a real teaching scheme be formulated. In particular the educator must have the ability to give clarifications, to check if a student has understood the exercise and to rephrase an unclear or badly phrased exercise so that it becomes comprehensible to all students. The following is also necessary:

- a) the teacher must recognise what ability the students have to formulate problems.
- b) the students must apply various strategies to solve the problems.
- c) the students must verify their results through their own experience.
- d) they must be able to come up with solutions.
- e) The students themselves must be able to justify their thinking, not only using classic justifications but also inductive justification, and finally.
- f) they must be able to formulate their own hypotheses (Philippou Christos, 2002).

In this present work we will be concerned with a continuous process of input and re-input in the classroom. We will use testing, which will allow us to conclude whether or not we have succeeded in our didactic objectives. At the same time, we will be able to form our teaching in such a way as to move forward at the pace required by the particular abilities of the students.

2. Theoretical Framework

The theoretical frame of our work falls substantially into two parts.

The first part focuses on the role of technology as an aid to the teacher, so that he/she may learn where his/her students have difficulties. The difficulties that the teacher might initially meet are mainly lexical. Without facing up to these the students cannot understand the teaching.

In the second part, reference is made to the most important theories of fractions, since fractions were used in this present research.

The role of language in teaching is considered a fundamental tool through which the student can understand the various concepts presented. The successful teaching of a new concept cannot begin without the aid of language. With the aid of language, the individual can also communicate with his environment. Vygotsky (1986) maintains that all learning comes through the interaction of the child with the environment of course, as he maintains, "*social games of interaction*" also play a fundamental role in the development of language. Vygotsky (1986) devised the concept of the "*central development zone*" to summarise the possibility of cognitive development. If there is no social realization and no dialogical communication, this "*zone*", which is called the Vygotsky Zone, can never develop fully. From this point of view, it is postulated that intellectual development is determined by communication and is developed through language. It is further maintained that whatever new thing the student learns can be "*named*" and "*used*" by him/her, again with the help of language (Boudourides, 1998 Briner, 1999). It is underlined that in the case of scientific thinking, comprehension (which is achieved admittedly through language) is of great importance (Avgerinos, & Marinos, 2005).

According to Vygotsky, the development of language is connected with the comprehension of concepts through words spoken every day. A student who does not speak both languages well perform less well in school lessons than a student who speaks a single language well (Cummins, 1976). That is to say, the level of linguistic ability achieved by a bilingual child in the first and second language has repercussions on all the lessons which are taught in school and on cognitive development in general. In order to describe the above situation Cummins invented the term "*semi-lingual*" from which is derived our own translation of "*semilingualism*". Later, however, the term semilingualism was rejected by the scientific community for a variety of reasons, such as, for example, the fact that the concept confuses the degree of ability which the individual can develop, the level of linguistic ability and the level of linguistic development with the different variations a person might meet in a language. Such variations might be dialects, or regional idioms. Among those who rejected the term were Cummins, Skutnabb and Kangas (who initially had defended the concept) on the grounds that it confused the issue more that it clarified and reference to the term had no theoretical value. (Cummins, 1994). In addition, such a term complicates the issue and renders research discriminatory for bilingualism (Ekstrand, 1979, 1983). Finally, there is great difficulty in accepting the concept of semilingualism empirically (Paulston, 1982).

How, then, can we understand, listen to and help a student to participate in the didactic process? The ability of the student to explain scientific concepts and to discuss scientific ideas can become much easier if the teaching is based on the context of the students. Thus, Tytler and Peterson (Tytler and Peterson, 2000) have recorded the oral explanations of various scientific themes as given by bilingual students. These explanations comprise a fundamental element in the development of their thinking, and their ability to comprehend mathematical concepts.

The students in Secondary Education live in particular social and cultural patterns. These social and cultural patterns define in their turn their values and convictions. These patterns are powerful enough to determine, in their turn, a different approach to various mathematical concepts. Of course, there are studies which have been developed in the field of ethnomathematics (D'Ambrosio, 1985), according to which various mathematical practises are handled differently. The diversity has to do with the different social and cultural framework in which it presents itself. In any case we must examine bilingual students in a model that contains three levels:

- The framework of bilingualism
- The development of the communicative repertoire of the bilingual individual (e.g. the written or the spoken word) in order that through this the faculty of comprehension of mathematical concepts can come into being.
- Ways in which the bilingual individual communicates (e.g. forms of writing and linguistic structure) (Hornberger 1989)

According to Jim Cummins technology can play a positive and important role in education if it is accompanied by paedagogical principles. Cummins distinguishes three paedagogical directions:

- 1) Traditional paedagogy, where explicit orientation in the transmission of information and skills exists.
- 2) In Progressive paedagogy constructional teaching methods stress the importance of the interaction between teacher and student. Here the student is encouraged to build certain concepts actively and to be involved consciously with activities. In this case we have activated the students' previous knowledge, allowing them to use their own experiences and the representations they have in their minds in order to solve problems and manufacture new knowledge.
- 3) An important representative of this teaching method is the cognitive model of Vygotsky for the zone of proximal development.

In Restructured paedagogy, which shares many similarities with Progressive Paedagogy, consideration is given to the social components of learning. Thus, the students often act in such a way as to influence a social reality.

The concept is a piece of knowledge which, by its existence contributes to the recall of other knowledge. Theorists maintain that a concept never exists in isolation in our thinking, since a) in order for us to think of it we are obliged to call upon other concepts and b) when it appears in our thinking, associations are sparked off which lead us to other concepts related to it in a variety of ways. We could say that every concept

exists only by reason of its relationship to other concepts. It is like a net, where no part of the mesh has an autonomous existence but only exists because of the other parts of mesh with which it is connected. Thus, the net comprises a whole. In general terms this is the structure of conceptual thinking. The elements of it are concepts. Thanks to the possibilities of language the mesh of an unbreakable net is formed which includes the total of our knowledge. Our thinking, then, is mainly an immense organization of concepts.

3. Educational communication with bilingual students

Studies of bilingual and multilingual mathematics classes have described how communication between the students and the teachers occurs. For example, the students may use different languages in order to better approach the various concepts (Adler, 1998), since in this way the students better recognize the mathematical ideas (Moschkovich, 2002 Avgerinos, & Marinos, 2009) during the Mathematics lesson.

3.1 What is the role of discussion between students and teachers in the classroom?

In the classroom, discussion and cooperation are important elements. The parallel use of technology and teaching helps the students to defend the mathematical ideas and hypotheses that they make, while at the same time they can experiment by trying out different viewpoints (solutions).

The analytical curriculum for each subject gives increased emphasis to the processes of problem solving, to reasoning and to communication. Additionally, it promotes the discussion of mathematical concepts between students (ERO 2000). A similar trend is also observed in analytical curricula in Greece.

Much research has been centred on the fact that cooperation and discussion between the teacher and the students has a beneficial effect on learning (Cobb, 1995). This is due to the fact that the teacher can detect and then immediately cover any gaps in the students' knowledge. Of course, classroom discussion of the problems which one student faces exercises a beneficial effect on the other students too. Examining the question of immediacy of the communication between teacher and students, however, we observe that there is often no possibility of the teacher being able to hear from each student. Thus, in the classroom the opinions of a few students only are heard. Techniques such as the organization of the students into groups of two or three or more individuals is not an invariable solution, since the representative of a group may not be able to fully help or notify the teacher of the weakness of each student. Sfard and Kieran (2001) refer to a study of Lavy which revealed that students who worked in cooperation with others made notable progress which did not manifest itself if each student worked separately and alone. For this reason, they maintain that group work is not a panacea and that teachers must check the learning method of each student in accordance with the student's own needs and guide the learning process (Cobb, 1995). Only then can we have better results for the whole group.

3.2 The same instructive profit to all students

An important question in the international bibliography is that of how the students can all progress at the same rate. Even though this is largely determined by the socio-cultural environment, research has shown that in the classroom students who perform poorly are not able to participate in the learning process, since this ability is taken away from them by the better students. This means that for various reasons students who are performing poorly have been deprived of previous knowledge. These students are not included in the work process of learning, they learn with more difficulty and, finally, they have less instructive interaction with the teacher. The research proposes that the teacher should intervene in order to strengthen the confidence of the whole class in the teaching process under discussion.

Of course, many objectives set by teachers may not be the answer for the majority of a class. These objectives may, for example require more skills than the students have, skills they have been deprived of. They may also be based upon former knowledge, knowledge, however, that is not there for all the students. (Cohen, 1994).

Thissen believes that traditional methods of teaching do not produce adequate results and speaks of a didactic application of the Constructivist model of learning, which has been developed based on the older model of cognitive psychology and which includes all those influencing factors which characterize the supplying of knowledge. And so, he concludes that learning is an active process designed to build knowledge and is always successful when connected with pre-existing knowledge. The student must play an active role in this building process and must ask questions and occupy himself with the material in his own particular way (Thissen).

Mager (Mager, R., 1962) refers to the use of a suitable evaluation process with which to ascertain the student's degree of understanding. The teacher may then proceed to offer new pieces of knowledge. We understand that this process should be continuous and facilitate the making of decisions (immediately) in the classroom.

In other words, we can say that the above is the interdependence of learning and the learning process.

In the second part of the bibliographical framework, we refer to the difficulties of fractions. Fractions are numbers which take the form $\frac{\alpha}{\beta}$ (where α and β are whole numbers while $\beta \neq 0$) (Exarchakos 1991, Megas, 1982). α is called the numerator, while the natural number β below the horizontal line is called the denominator. Both of these are called the terms of fractions (Grand Larousse, 2001, Microsoft Encarta 1996). The above fraction is read as " α over β " or " α by β ". Gagatsis (Gagatsis Evangelidou Ekias Spyrou, 2004) mentions concisely a method which is connected with the view of a number or researchers in which to facilitate the teaching of fractions (Kieren, 1976). All these researchers arrive at the creation of a theoretical model, which is related to the teaching of the various dimensions of fractions. Such a theoretical model is:

- a) The fraction as part of the whole. In this case the fraction may be presented as a part of the surface area of a geometric figure which is divided into uniform parts or as part of a total of objects.
- b) The fraction as a word. The fraction as a word is conveyed by the concept of a comparison between two quantities. The students must understand the concept of relative amounts (Lamon,1993) in order to fully understand the concept of fractions as proportion. They must understand that the two quantities which have a proportional relationship change together – multiplied or divided – so that their relationship remains steady.
- c) The fraction as a measure. The numerical line expresses the concept of the number as a measure, which is to say as distance. (Lamon, 1999). The fraction may be presented as a point above the numerical line between two integers. This is a very useful skill for the conceptual understanding of fractions, despite being rather abstract for the students Ni (Ni, 2002) points out that above the numerical line fundamental concepts of fractions may be depicted, such as, for example, density, sequence, uniqueness and the infinity of fractional numbers. The unit of measure of the numerical line can be continually divided into smaller units, presenting different names of fractions. When different units cover the same distance, then we have equivalent fractions. That is to say the numerical line solves the problem of intermediary numbers, since it represents the fraction which is found between the two original fractions.
- d) The fraction as a Divider. As regards the dimension of fractions as a quotient, the fraction may be considered as the result of the division of the numerator by the denominator. Students understand the role of the dividend and of the divisor and understand that the dividend refers to the number of objects to be divided while the divisor refers to the number of equal parts which each object will be divided into.
- e) The fraction as a Multiplier. In this case the product is $5 \times \frac{4}{3}$ the multiplication 4×3 goes before the division by 3. The product in fractions as opposed to the product with whole numbers can give results smaller than the factors which are to be multiplied (Gagatsis et al., 2006).

4. Research Questions

The results following such an intervention suggest that the following questions should be answered:

- Could finding the linguistic difficulties each student faces help with the subsequent success of the objectives of the unit which we want to teach?
- What are the greatest linguistic difficulties that the students face?
- Was there an improvement in the understanding of the concept of fractions?

- Does connection and correlation of the concept of fractions with decimals and whole numbers help? Does the presentation of fractions through examples from daily life help?

4.1 Experimental Groups

There were two (2) experimental groups. They were made up of students in Secondary Education, in the 2nd class of Junior Secondary School.

These students had been deprived of mathematical knowledge and in particular that of fractions. This, however, had created problems also in other lessons, for example in Technology, Physics and Chemistry. The research took place in the Technology laboratory and many examples were from the field of Technology. This was intentional since all their parents' occupations were in manual work and the Technology lesson was the most familiar to them.

A test had preceded the experiment where it was ascertained that both the bilingual students and Greek mother-tongue only pupils had no knowledge of fractions. The exercises which were evaluated were similar to those which were later set by the teaching plan with discussion and facing up to the linguistic difficulties and developing a theoretical teaching model.

There were 5 students, of whom 2 were Bulgarian. There was another student from South Africa and 2 from Finland. All these students had finished their three last school years in Greece. The two from Finland had been born in Greece and had Greek fathers and Finnish mothers.

We must point out that all these students had been taught fractions and had done general sums with fractions in the last classes of Primary School.

The research was done in two phases:

In the first phase questions were given to the second group in order for us to get to know what linguistic difficulties the children were dealing with. Then there was help through the teaching formation which had been set from the pre-evaluation so that they could handle these difficulties.

In the second phase exercises were given, in accordance with the theoretical model to which we refer above, for the forms that we meet in fractions, such as a) the fraction as a part of a whole b) the fraction as a word c) the fraction as a measure d) the fraction as a divider and e) the fraction as a multiplier.

This was given to both groups.

The teacher questioned the students who had lexical difficulties and who did not understand the exercises which they had been set.

A. First Phase (concerns the second group)

Hasapis mentions a distinction which was adopted by Morris (1955) within the framework of mathematics lessons in order to deal with the linguistic difficulties faced by the children. According to Morris in the mathematics lessons an analysis of the difficulties can distinguish:

a. Difficulties of syntax

These are located in the relationships of linguistic points and the symbols between them. They appear in comparative adjectives and expressions. Such as :

- Greater than/ smaller than/ more than/ less than/ most and least, as much..... as, equal to, twice as much as/ three times as much as
- Prepositions such as: by, for, plus, to, per.
- Passive voice verbs: Added, subtracted, multiplied, divided.
- Conjunctions: like, and, or.

The following table was presented to the students asking them to say whether or not the terms correspond correctly to them.

Table 1: Symbols that students should choose

	Selected answer
>	Greater
<	Equal
=	Smaller

The semantic difficulties which are located in the relationships of the linguistic signs or symbols with the text or the concepts to which they refer (declarations), or with the meanings which are given to them.

b. We discern

Vocabulary difficulties, such as single-word Mathematical terms for example: numerator, denominator, quotient.

An example of an application given to the students:

The students were resented with a fraction and asked to say which number was the denominator and which the numerator:

Table 2

$\frac{3}{2}$

- Periphrastic mathematical terms: Least common multiple, greatest common divider.
- Words in everyday speech which have a different meaning in the context of the language of mathematics, such as whole, equitable angle, power.
- Synonyms such as: to add, increase.
- Lexical expressions of symbols such as = , > , < , () .

c. Difficulties of reference

- Articles and determiners such as a/the number (the square of the sides of a tetragon is equal to its area)

d. Factual Difficulties

Here, factual analysis is located in the use and interpretation of linguistic signs or symbols η from various people in varying contexts of activity. We can locate them in lack of knowledge and experience, and ignorance of the terms of the problem.

For example the students have difficulty in understanding problems that speak of Municipal Taxⁱⁱ. This is a tax which is levied only in the Dodecanese and nowhere else in Greece.

We used the text in which Hornberger (1989) maintains we can determine the narrative forms a student may encounter also in his/her first language. The use of narrative is present in many cultural practices and in the daily life of the bilingual student. Access to scholastic problems is through school textbooks which are written in Greek. There are various concepts in the narratives of which, with the help of questions, we ask for the students' comprehension. Of course, when the students again do not understand what is said in the text, we proceed to demonstration with pictures or schematic representations. The optical aspect of communication can supplement the lexical aspect to an important degree.

This more or less agrees also with Kress and Van Leeuwen (1996) who point out that the optical communication is easier to understand in different cultural contexts. For example, students were given two pictures and then the teacher read out one of four responses. The students selected one answer and justified their choice.

B. Second Phase

Since, with the help of a test, it was ascertained that the students had made very few mistakes, we went on to fractions. Again, with the help of a test (for the second group), we attempted to understand the level of the students' knowledge.

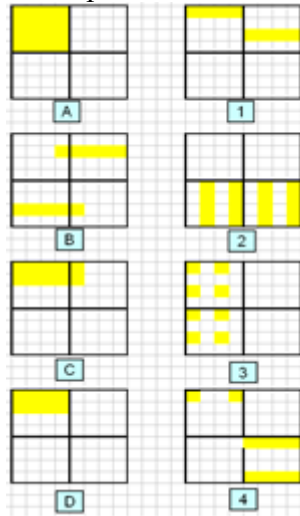
The same exercises were set also for the first group.

The students were given the following exercises which refer to different forms that fractions can take.

ⁱⁱ Municipal Tax is applied only in the region of the Dodecanese in Greece and is collected by the Municipalities of the region, with the object of using the income for development.

a. The fraction as part of a whole

Table 3: Representations of fractions



In this case, as we said in the bibliographical reference, the fraction can be presented as part of the area of a geometrical figure, which is separated into sections of uniform size, or as part of a whole object.

We used a video projector to show the figures.

Thus, the students could mark on a piece of paper correlating the area which was covered by the colored squares in the column with the letters with the coloured areas (in the bigger rectangle) in the column with the numbers (table 3)

The teacher asked the students to show their choices, attempting to understand the reasons for which the students selected a particular answer. Then giving out another example he left them to think for a little while about their answer.

b. The fraction as a word, as we said before (in the bibliographical reference). Here the fraction as a word is attributed with the concept of a comparison between two quantities.

Thus, the number $\frac{2}{3}$ was given to the students. They were asked to choose one of the following correct answers so that $\frac{2}{3}$ would remain constant:

On the projector the following table appeared and the students marked an answer in their notebooks. At the same time each student noted in which way he/she wanted an explanation. This explanation would then be referred also to the teacher.

Figure 4: Suggested answers from students

Suggested Answers
Both the numerator and the denominator to be multiplied by a different number. E.g. the numerator by 3 and the denominator by 2
Both the numerator and the denominator to be multiplied by one number e.g. 3
The numerator to be divided by the denominator
In order to remain steady neither the numerator nor the denominator should be multiplied by any number.

If one of the above students could not choose an answer or chose a wrong answer, then the teacher gave further clarification.

c. The fraction as a measure

A fraction can be presented as a mark on a numerical line between two whole numbers. This is a very useful skill for the conceptual understanding of fractions, despite being rather abstract for the students. Ni (Ni 2002) points out that above the numerical line fundamental concepts of fractions may be depicted, such as, for example, density, sequence, uniqueness and the infinity of fractional numbers. The unit of measure of the numerical line can be continually divided into smaller units, presenting different names of fractions. When different units cover the same distance, then we have equivalent fractions. That is to say the numerical line solves the problem of intermediary numbers, since it represents the fraction which is found between the two original fractions.

Table 5

$\frac{2}{3}$	$\frac{1}{2}$
The fraction $\frac{2}{3}$ is greater than $\frac{1}{2}$	

Table 6

$\frac{1}{2}$	$\frac{1}{4}$
The fraction $\frac{1}{2}$ is greater than $\frac{1}{4}$	

d. The fraction as a divider

Here we used the example of Marshall (1993) where he asked the students share three pizzas between 4 children. The students need to understand that the pizzas are cut into quarters and each child given three pieces.

Table 7


















	1 st child	2 nd child	3 rd child	4 th child
A				
B				
Γ				
Δ	 	 		

Figure		
U	I	R
$\frac{160}{100}$	2	$\frac{80}{100}$
$\frac{80}{100}$	1	$\frac{80}{100}$
$\frac{240}{100}$	3	$\frac{80}{100}$
$\frac{800}{100}$	10	$\frac{80}{100}$

e. The fraction as a multiplier

The fraction as a multiplier means that for the product $4 \cdot \frac{3}{5}$ the multiplication 4×3 precedes the division by 5. The product in fractions as opposed to the product in whole numbers can give results smaller than the factors of the product.

Below we use the rule of Ohm (Ω) $U = i \cdot R$ as well a continuous current electrical source We explain the problem to the students, which is that if we simultaneously hit a current switch and a resistance switch, what will be the value of the voltage?

Although the problems were written in Greek they were closely connected with everyday life. Their objective is to illustrate daily situations in which mathematics are present.

5. Conclusions

In the present work we have presented the role of a teaching formation and examples of its application in the classroom in order initially to handle the students' linguistic difficulties and then to re-teach fractions with the help of a theoretical model. To begin with there was an evaluation, and then we formed our teaching strategy based on this evaluation. It is observed that there was a marked improvement in the comprehension of the concepts that a student can encounter in fractions because we dealt with theory linguistic difficulties (according to the distinction we made above). Thus, the students managed to handle linguistic difficulties which arose in both the symbolic formulations that have developed in the mathematics framework and in the equivalent lexical phrases which appear in the mathematical text. They were faced with difficulties of interrelation of syntax, semantics, and the pragmatic characteristics of the language. These appeared when the children attempted to give interpretations of mathematical concepts and rules using their own daily language. Evaluating the students' level with the help of tests revealed that the greatest part of their difficulties was semantic. This is probably because the problems contain concepts that require a full comprehension of the Greek language. Significant difficulty, too, was faced by the students when they tried to cope with their deficiencies in the examples of the Greek passive voice in the exercises. Such verbs are: *added to, subtracted from, multiplied by, divided by*.

From another point of view ignorance of concepts and terms that have to do with Greek reality did not cause difficulty to the students, since, as it seems, they have an immediate relationship with their own economic environment. Such discussions as emerged would also take place in their own homes and with their friends.

As far as the attempt by the teacher to teach fractions is concerned, the evaluation and the teaching of the concept of fractions took place in such a way as to avoid the use of procedural knowledge (i.e the use of rules, algorithms and procedures). This was achieved by using diagrams and pictures which helped there to be no mechanical learning and assisted the students in making a connection between concepts and knowledge of rules when a relationship is applied. Moreover, during the teaching it was attempted to make a conscious connection and correlation between the concept of fractions, decimals and whole numbers. Again, we avoided presenting representations for fraction with a particular, stereotyped and often completely guided manner since we ourselves constructed questions based on the students. The questions for each student separately created an environment within which they felt free to express their personal perceptions and to use subjective criteria, and to take initiatives without being tied to the formal diction which is used for the scholastic problems which students have to solve.

With the help of evaluation tests, we created such questions as to maximize our ability to decode the children's answers. We discussed things in such a way that each wrong answer was easily interpreted. That is to say we could qualitatively investigate those mistakes that were considered an "unavoidable and essential part of learning". Again, we used them as a "reliable source of information round the learning process".

After we had confirmation that the student had a grasp of the series of concepts that were essential for the understanding of other concepts, we proceeded to the presentation of new material, once more beginning with the simplest concepts of fractions. Many of these concepts also contained pictures from daily life in order for us to facilitate the narration of everyday situations in fractions.

In essence, tests were used and then followed a dialogue with all the students with the assistance of a parent.

The above was done in order to make things easier for bilingual students in mathematical knowledge at the level of basic education.

5.1 Future investigation

The transfer of the examples and methodology suggested by the above work to a technical environment.

Conflict of Interest Statement

The authors declare no conflicts of interests.

About the Author

Andreas Marinos, son of Christos, was born in Rhodes-Greece in 1966. He has a PhD in the Didactics of Mathematics and Evaluation Post-PhD in the Administrator with

mathematics tools in Education with and post graduate qualification in School Administration. Has studied Electrical Engineering teacher and a graduate of the School of Education of the University of the Aegean. He has over 100 scientific published, all with the critique system. He has presented his own work at 16 International conferences. In 2000- 2001 and 2003-2007 he taught in the School of Education of the University of the Aegean (Gov. Gazette) From 2012-2017 with the Higher School of Pedagogical and Technological Teaching Placement Contract he taught the following lessons: Organisation, Management and Sociology of Education, Educational Assessment, the Teaching of Speciality Courses, Didactic Methodology, and the Practice of Teaching. In 2016-SUMMER 2018 worked with a placement contract for he taught of the Neapolis University of Paphos. In 2020-today with a placement contract for he taught the modules: Issues of educational policy and educational change ". Evaluation and Quality Assurance in Education, and Theories and Principles of Educational Administrator and Leadership. in the postgraduate programme of the Frederick University. In 2019-20 with a placement contract for he taught the modules: Measurement and Evaluation in Special Education of the University of Nicosia. He has worked in dozens of training programmes for civil servants, such as Municipal – Regional – School of Education Teachers, and also private employees (traders, hotel employees, business executives). He has been the Headmaster of the 3rd General Lyceum, Rhodes since 2015. He was Director of the Vocational School of Rhodes from 2011 to 2014 and has been teaching in secondary education for 30 years. He is involved in public affairs and has been president of the Union of Middle School Teachers Association of the Dodecanese, (a body of 1200 members) an elected member of the Dodecanese District Council for Secondary Education. He is also an elected member of the Senior Executive Council of the South Aegean and an elected member of the Dodecanese Disciplinary Board of Secondary Education Teaching Staff.

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