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USING ANALOGIES TO OVERCOME DIFFICULTIES IN TEACHING OF THE INTEGERS IN THE MIDDLE SCHOOLS

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Abstract:
This study aims to determine the difficulties that middle school mathematics teachers have in teaching integers, the analogies they use to solve these difficulties, and the efficiency of these analogies. The study was conducted with 10 mathematics teachers, 6 females and 4 males, who were working in middle schools affiliated with the Ministry of National Education (MNE), selected using the convenience sampling method. The study used the case study design among the qualitative research methods. The data were collected using open-ended questions and analyzed using descriptive and content analysis. The findings were presented under three categories which are *the difficulties they have in teaching integers, the curriculum on the subject of integers,* and *the analogies found in textbooks or used by teachers and their efficiency.* Recommendations were made based on these findings. Centralized Early Childhood Development Policy of the early childhood education.

Keywords: analogy, difficulty, teaching integers, curriculum, mathematics text books

1. Introduction

The new world of constant and rapidly increasing information combined with technological developments produced a social order based on knowledge and information, and this social order expects different skills from individuals today (MEB, 2009). The increased necessity of using and understanding mathematics in daily life and the negative results obtained on national and international exams (Altun, 2009) has necessitated Turkey's education system to make changes to the framework of its

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expectations from students; and accordingly, curriculum have been reorganized based on this constructive approach. The constructive approach defines learning as an active process where individuals construct their own interpretation and explanation of new information they learned from their environment based on their previous knowledge and experience (Driver and Bell, 1986). According to constructivism, learning new information depends on prior information, because new information is constructed upon the previously acquired information; and the prior information is accepted as a starting point for teaching new information (Bıyıklı, Veznedaroğlu, Öztepe and Onur, 2008). In this regard, the necessity of connecting the new and prior information for a more meaningful and permanent learning experience becomes prominent. A way to facilitate these connections is analogies based on the constructive approach (Glynn and Takahashi, 1998; Pittman, 1999; Şahin, Gürdal and Berkem, 2000; Kaya and Durmuş, 2011).

An analogy is the process of understanding the unknown by comparing and connecting the known and unknown information based on the conditions of the known (Günay Bilaloğlu, 2005). According to another source, an analogy can be defined as a mapping mechanism that helps students construct the new information based on their previous knowledge (Parida and Goswami, 2000). In analogies, which are the bridges between the known (prior information) and unknown (new information) (Kesercioğlu, Yılmaz, Huyugüzel Çavaş and Çavaş, 2004), the known concept is called the source (analog), and the concept being taught (unknown) is called the target concept (Akkuş, 2006). The aim of the analogy is to facilitate the understanding of the target by finding appropriate similarities between the source and the target (Gülçiçek, Bağı and Moğol, 2003). Glynn (2007) highlights that the target should be identified, the source should be organized according to the target, the similarities between the target and the source should be determined, these similarities should be compared, the circumstances under which the analogy does not work should be identified, and conclusions should be drawn about the target concept.

Studies show that analogies contribute to solidifying abstract subjects (Günay Bilaloğlu, 2005; Aykutlu and Şen, 2011; Heywood, 2002), constructing information (Akkuş, 2006), eliminating misconceptions (Atav, Erdem, Yılmaz and Gücüm., 2004), drawing students' attention to the course (Duit, 1991), learning and developing scientific ideas and concepts, making comparisons with the real world, increasing students' motivation and taking students' previous knowledge into consideration (Dagher, 1995), and improving students' scientific thinking and creativity (Aykutlu and Şen; 2011). Günay Bilaloğlu (2005) stated that analogies also enable students to grasp certain information and summarize the subjects comprehensibly. However, although analogies are useful in the learning and teaching processes as mentioned above, they lead to negative results and inefficiency when used incorrectly.

Palmquist (1996) stated that a good analogy should include the following features:

- ***Structural Richness:*** An analogy should show a variety of relationships between itself and the other ideas or concepts being compared to in terms of meaning.
- ***Applicability:*** An analogy should include the structure of the relationships and be applicable for the concept and not cause misunderstandings.
- ***Appropriateness:*** An analogy should be appropriate for the target group.
- ***Comprehensiveness:*** An analogy should be understood by the target group in the same way.

Akkuş (2006) expressed that teachers should be sure that they make appropriate and similar connections between the target and analog concepts, and the analog should help teachers map the similarities and differences of these connections for analogies to be useful. Kaptan and Arslan (2002) stated the issues that teachers should consider when using analogies in learning and teaching activities as follows:

- Teachers should identify how and for which subject they will use an analogy and be able to draw students' attention to the analogy by making a plan according to this identification.
- Teachers should direct and give opportunity to students to create their own analogies and use visual materials when required.
- Teachers should ensure that the analogy they use is closely related to the subject and connected to the students' prior information from their daily lives.
- Teachers should ensure that the analogy they use is appropriate for the students' cognitive and comprehension level.

These issues emphasize the importance of using analogies correctly and appropriately by teachers to be efficient in teaching.

Analogies are strong strategies in mathematical research (Krieger, 2003). They play a significant role in mathematical thinking (Saygılı, 2008). One of the subjects in which students are known to have problems of understanding is the concept of integers and the mathematical operations with them (Bahadır and Özdemir; 2013). Studies show that the natural numbers that previously exist in students' minds can help with learning positive integers, but negative numbers cannot be learned by observing the physical world since nonpositive objects or object groups do not exist (Davidson, 1992; Mc Corkle, 2001; Şengül and Dereli, 2013). Işıksal Bostan (2010) stated that the biggest problem with negative numbers is the inability to interpret and comprehend these numbers. Although students do not have difficulty in placing negative numbers on the number line, they have difficulty in comparing their superiority. It can be said that these difficulties and mistakes in comparing negative numbers are based on the misconception that the features of positive numbers can also be generalized for negative numbers. Another difficulty is the calculation errors due to incorrect use of the addition and minus signs since students cannot understand whether these signs reflect the numbers' direction or the operation itself. In multiplication and division, students try to use the generalizations in natural numbers and cannot grasp the conceptual information behind these operations, which leads them to make calculation errors. Teachers have an important role in the elimination of these misconceptions that students have in conceptualizing negative numbers, interpreting the operations made with these numbers, and the calculation errors they make due to this misconception (Işıksal Bostan, 2010). Based on this, teachers' use of analogies in eliminating the misconceptions about integers and concretizing the relevant concepts is considered to be effective.

This study aims to determine the difficulties that middle school mathematics teachers have in teaching integers, the analogies they use to solve these difficulties, and the efficiency of these analogies. The answers of the following questions were sought in line with the aim of this study:

- What difficulties do mathematics teachers have in teaching integers?
- Which analogies are used in the mathematics curriculum and textbooks in the subject of integers?
- Which analogies do mathematics teachers use in teaching integers?
- How efficient are the analogies used by the teachers in the subject of integers?

2. Method

This study used the case study design among the qualitative research methods to answer the study questions and obtain in-depth information (Yıldırım and Şimşek, 2008).

2.1 Sample of the study

The study was conducted with 10 mathematics teachers, 6 females and 4 males, who were graduated from elementary mathematics teaching and working in middle schools affiliated with the Ministry of National Education (MNE), selected using the convenience sampling method. Three teachers are older than 30, the others' age is changed between 25-30 years. Furthermore, two teachers of the participations have an experience between 5 and 10 years, the others have an experience between 2 and 5 years.

2.2 Instruments and Data Analysis

The data were collected using a questionnaire that included 8 open-ended questions questioning the difficulties that teachers have when teaching the learning objectives related to the integers topic which is included in the mathematics curriculum applied in the 2013-2014 academic year and the textbooks approved by the MNE for middle schools. Additionally, the questionnaire asked for the analogies that the teachers use to resolve these difficulties. This draft questionnaire prepared by the researchers was finalized according to the corrections recommended by three field experts. In this questionnaire, firstly, the concept of analogy was defined, then the following examples of the analogies used in mathematics were shown and finally teachers were asked to specify the analogies they use in teaching the topic of integers.

- *As we take from one side of the scales, we take the same quantity of load from the other side so as not to tip the scales. Similarly, we divide both of the sides of the equation by -60.*
- *A sphere is like an orange.*
- *An acute angle is similar to the angle that occurs when you open a pair of scissors.*
- *15x2: (10x2)+ (5x2):: 14x3 : ______*

The coding system developed by Serin-Ergin (2009) to analyze the source-target match was used to determine the efficiency of the analogies that teachers developed different from the curriculum and textbooks. Accordingly,

- ***Full Association (FA):*** This means fully and correctly expressing the similarities and differences of the source-target relationship.
- ***Partial Association (PA):*** This means correctly but incompletely expressing the similarities and differences of the source-target relationship.
- ***Incorrect Association (IA):*** This means associating the source with a different concept other than the target.
- ***No Association (NA):*** This means that the source-target relationship is not associated with the analogy, i.e. the answer includes statements such as "not related; I couldn't associate." or the question was not answered at all.
- ***Association Including Misconception (AIM):*** This means expressing the source-target relationship in a way that causes misconception.

In this study, *the valid, partially valid* will be used instead of full association and partial association. Also, invalid will be used for the incorrect association, no association and association including misconception.

The data were analyzed upon adding the new themes generated after an in-depth analysis of the themes and categories determined by reviewing the relevant literature, and the results were comprehensibly interpreted, including direct quotations as a way to reflect the themes. The teachers' names were not given in these direct statements to protect their identity; instead, they were coded with numbers in parentheses.

The mathematics curriculum and textbooks for the sixth and seventh grades were the other sources of data. The documents were analyzed using the content and descriptive analysis methods (Yıldırım and Şimşek, 2008).

3. Findings

3.1 The Mathematics Teachers' Opinions on the Difficulties They Have in Teaching Integers

Table 1 shows the themes and subthemes generated from the middle school mathematics teachers' opinions on the difficulties they have in teaching integers.

Table 1: Themes and Subthemes Regarding the Middle School Mathematics Teachers' Opinions on the Difficulties They Have in Teaching Integers

Themes	Subthemes
Interpreting integers and showing them on the number line	Not having difficulty Inability to comprehend negative integers Inability to show negative integers on the number line
Determining and explaining the absolute value of a integer number	Not having difficulty Inability to extract negative integers from absolute value
Comparing and putting integers in order	Inability to comprehend that -1 is the biggest negative integer number Not paying attention to the signs before integers Inability to put negative integers in order
Adding and subtracting integers	Not having difficulty Having difficulty in adding integers with different signs
Comprehending that subtraction in integers means to add the opposite sign of the minuend	Memorizing the rule Inability to understand the reasons Inability to understand the concept of the negative of negative integers
Using the features of addition as strategies for a fluent operation	Having difficulty in understanding the order of operations Having difficulty in distributing the negative sign before the parentheses into them
Multiplying and dividing integers	Not having difficulty Inability to comprehend that the division and multiplication of numbers with the same sign will always result in a positive sign Miscomprehending multiplication
Indicating the repeated multiplication of integers as exponential numbers	Not having difficulty Perceiving repetitive multiplication as addition

	Having difficulty in exponentiating according to the location of the parentheses when exponentiating the numbers
	Inability to identify the sign of the result of the even and odd exponents of negative numbers

Table 1 indicates, based on the teachers' opinions on the curriculum addition of "*Interpreting integers and showing them on the number line*", that students have difficulty in comprehending negative numbers and showing them on the number line. A teacher considered that he did not have difficulty on this issue and said: "*I don't have difficulty in the comprehension of integers and showing them on the number line. Because students start the sixth grade as already having learned the natural numbers, they can transfer their information on natural numbers into integers.*" (T3)

The statements of some teachers who consider that they have difficulty in teaching integers are presented below:

> *"Negative numbers are difficult to understand. Students ignore the minus sign before the number as they think they are positive numbers."* (T1)
> *"They place negative numbers on the number line as -1, -2, -3, -4... from left to right towards zero. Namely, they cannot show negative numbers on the number line."* (T2)
> *"I don't generally have difficulty in positive integers, but students have difficulty in placing negative numbers on the number line."* (T6)

These statements reveal that students can use their knowledge about natural numbers on positive integers but have difficulty in comprehending negative numbers and showing them on the number line.

T5, who thought that he did not have difficulty with the curriculum addition of "*Determining and explaining the absolute value of an integer number,*" said: "*There is no problem when the absolute value of integers are associated with problems in daily life and are well-exemplified. I did not have difficulty in this subject."* The statements of some teachers who emphasized the difficulty in extracting negative integer number from the absolute value are presented below:

> "*They cannot comprehend that the absolute value changes the sign; they extract negative numbers as negative.*" (T7)
> "*They learn to extract negative numbers from the absolute value as positive, but they begin to extract positive numbers as negative*." (T10)

It can be concluded from the teachers' statements in Table 1 that all teachers have difficulty with the curriculum addition of "*Comparing and putting integers in order.*" It is seen, according to the subthemes, that the difficulties mostly focus on the inability to comprehend that -1 is the biggest negative integer number, ignoring the signs before the numbers, and the inability to put negative numbers in order. Some statements of teachers on the difficulties teachers have are as follows:

"It takes time for them to understand that -1 is the biggest negative integer number and the others are smaller. They make the operations with the information they memorized." (T9)
"Students generally do not pay attention to the sign before integers when comparing them." (T5)
"Students think that negative integers are getting bigger as they get away from zero." (T2)

The subthemes generated from the opinions on the addition and subtraction of integers are not having difficulty and having difficulty in the addition of integers with different signs. The teachers who did not have difficulty stated that this is due to the information that students acquired about addition and subtraction in natural numbers.
The statement of one teacher who did not have difficulty is presented below:

"Actually, the students use their knowledge about addition and subtraction in natural numbers when adding and subtracting integers, only the number needs to be highlighted when adding and subtracting two negative numbers." (T3)

Some statements of the teachers who have difficulty with the addition of integers with different signs are as follows:

"They do addition without seeing the negative number when adding positive and negative numbers at once." (T4)
"There is no problem when adding two positive integers. But they have difficulty in adding negative and positive numbers or two negative numbers." (T8)

According to the Table 1 for the curriculum addition of *"Comprehending that subtraction in integers means to add the opposite sign of the minuend,"* students memorize the rule and cannot understand the reasons of the addition or the concept of the negative of negative integer number. The difficulty with this addition is the inability to add the integers with different signs. The statements of the teachers who have difficulty in the said curriculum addition are as follows:

"Students can only memorize this addition as a rule." (T1)

"Comprehending subtraction is an important addition, but they cannot understand why they are doing it." (T6)
"They cannot make the connection of negative of the negative. They make calculation errors when they distribute the minus before a negative number." (T7)

The subthemes generated from the opinions of the teachers on the difficulties they have with the curriculum addition of "*Using the features of addition as strategies for a fluent operation*" are having difficulty in understanding the order of operations and distributing the negative sign before the parentheses into the parentheses. All teachers had difficulty with this addition, and some of their statements are as follows:

"They often do multiplication, division, and even addition or subtraction before doing the operations in the parentheses." (T4)
"*Students have difficulty in giving priority to the operations in parentheses when solving the problems.*" (T9)
"They forget to distribute the minus sign before the parentheses into the parentheses." (T3)

Another problem that teachers have in teaching integers concerns multiplication and division. While T2, who stated that he did not have difficulty, said "*No problems occur when they understand multiplication*," the subthemes generated from the opinions of the teachers who had difficulties are the inability to comprehend that the division and multiplication of numbers with the same sign will always result in a positive sign and miscomprehending multiplication. Some of the teachers' statements under the said subthemes are presented below:

"They make incorrect generalizations about the fact that the multiplication and division of numbers with the same signs will be positive." (T10)
"*They have difficulty in understanding multiplication. They even do addition sometimes."* (T5)

The last learning objective indicated in the mathematics curriculum about integers is *"Indicating the repetitive multiplication of integers with themselves as exponential numbers."* The difficulties with this addition are perceiving repetitive multiplications as additions, having difficulty in exponentiating according to the location of the parentheses when exponentiating the numbers, and the inability to identify the sign of the result of the even and odd powers of negative numbers. Some statements of the teachers on this learning objective are presented below:

"Students perceive repetitive multiplication of exponential numbers as repetitive additions." (T8)

"They are confused whether to take the power in or out of parentheses when the exponent is in parentheses. Especially for negative numbers, this confusion causes calculation errors since the exponent's being in or out of parentheses affect positivity or negativity." (T2)

"They can exponentiate positive numbers. They have difficulty in identifying whether the result is negative or positive." (T6)

3.2 The Analogies Used for Integers in the Middle School Mathematics Curriculum and Textbooks

Table 2 shows the analogies used for integers in the mathematics curriculum applied from the 5th to 8th grades and in the mathematics textbooks advised by the MNE for the 2013-2014 educational year.

Table 2: Analogies Used for Integers in the Mathematics Curriculum and Textbooks

Objectives	Analogies in the curriculum	Analogies in the textbooks
Interpreting integers and showing them on the number line	Stating the floors in an elevator Air temperatures below and above zero	Sea level Thermometer Elevator Steps forwards and backwards Profit-loss relationship
Determining and explaining the absolute value of a integer number	Associating with real situations using elevators, thermometers, or bank accounts	The distance between ants moving in opposite directions from a starting point The relationship between the object and the image in a mirror Comparing the depths that a diver dives
Comparing and putting integers in order	-	Air temperature Debit-credit relationship Profit-loss relationship
Adding and subtracting integers	Associating the tools such as an elevator or thermometer with vertical and horizontal number line.	The backwards and forwards moves of a seismic search and rescue ship produced in Turkey for a Norwegian firm is associated with addition and subtraction in integers Addition and subtraction are also associated with debit and credit relationship by modeling with counting pieces Addition and subtraction in integers are explained through air temperatures in Ankara and the foothills of a mountain
Comprehending that subtraction in integers	Modeling with counting pieces	---

means to add the opposite sign of the minuend		
Using the features of addition as strategies for a fluent operation	-	---
Multiplying and dividing integers	-	Multiplication in integers is associated with finding the depth a diver dives in 3 minutes by giving the depth that diver dives in two minutes Determining the scores they obtain from a test (multiplication) The profits and losses of a company is used (multiplication and division in integers) The change occurring in the temperature of a substance (multiplication) Likening division to multiplication and transferring the features of multiplication into division (finding that the answer of the operation 18:6=? is three through the operation of 6x?=18)
Indicating the repeated multiplication of integers as exponential numbers	-	Likening to the number pattern of marbles being put in boxes for a competition in an electronic appliances store Likening to finding the number pattern through modeling of a rectangular prism consisting of unit cubes Associating the reproduction of bacteria with the exponents of integers

Integers are addressed in two sections in different grades in the curriculum prepared based on the constructive approach. The comprehension, comparison, addition, and subtraction of integers are addressed in the 6th grade while the multiplication and division of integers are addressed in the 7th grade. Thus, students should understand the concept of integers in the 7th grade. While it is recommended in the mathematics curriculum that in the 6th grade integers be associated with elevators, air temperature, thermometers, and bank accounts, no analogies were found in the addition of multiplication, division and exponentiation of integers in the 7th grade. This may be to prevent miscomprehension about the concept of integers and make students meaningfully construct the concept of integers.

According to the Table 2 the textbooks included analogies created by associating integers with sea level, thermometers, elevators, steps forwards and backwards, profit-loss or debit-credit relationships, air temperature, and number patterns. These analogies are similar to the analogies recommended in the curriculum, and differently from the curriculum, textbooks include analogies for multiplication, division, and exponentiation of integers. For the curriculum learning objectives above, the analogies of number patterns and the field are different from the other learning objectives. Another highlight is that the curriculum and the textbooks do not include analogies for the learning objectives of subtraction and addition with the opposite sign of the minuend in integers.

3.3 The Analogies Used by Mathematics Teachers in Teaching Integers

Table 3 shows the analogies used by mathematics teachers in teaching integers and the efficiency of these analogies. In the Table 3, the analogies used by the teachers but similar in the curriculum and textbooks are showed as T-C. Also, the coding system developed by Serin-Ergin (2009) to analyze the source-target match was used to determine the efficiency of the analogies that teachers developed different from the curriculum and textbooks. According to the this coding system, Full Association is called as a valid and partially association is called a partially association.

Table 3: The Analogies Used by Mathematics Teachers in Teaching Integers and the Efficiency of These Analogies

Curriculum Learning objective	The Analogies Used	Efficiency
Interpreting integers and showing them on the number line	Floors in a shopping center (T1)	T-C
	Temperature (T9)	T-C
	Depth or altitude according to the sea level (T7)	T-C
	Credit (+) and debit (-) relationships (T8)	T-C
	Thermometers (T2, T6)	T-C
	The minus reflect the contrasts, therefore the number gets smaller as it gets bigger (T3)	**Partially Valid**
	Elevators (T4, T6)	T-C
	I choose a certain point in the class and state that it is ahead of or behind where I stand (T5)	T-C
Determining and explaining the absolute value of a integer number	The distance of fish at various depths under the sea level (T1, T7)	T-C
	I liken the absolute value to a washing machine. If we put dirty clothes (-), we take them as clean (+). If we put clean clothes (-), we take them as clean again (+). (T2, T3, T4, T10)	**Valid**
	I determine the starting point on the number line as home and associate the absolute values of integers with the distance between home and any negative and positive numbers (T6)	T-C
Comparing and ordering integers	Credit>debit (T1).	T-C
	I associate integers with air temperature, asking if we chill more at -1°C or at -10°C and which one is colder (T2, T4, T10)	T-C
	The minus reflect the contrasts, therefore the number gets smaller as it gets bigger (T3)	**Partially Valid**

	I categorize integers as negative- and positive-signed and then associate them with debit and credit relationships (T5, T8, T9)	T-C
Adding integers (this curriculum addition was analyzed in two sections)	Going up from one floor to another in a building (T1)	**Valid**
	The + sign represents the credits. I have a credit of +5 TL from a friend and +10 TL from another friend; then I have a total credit of +15 TL. And I explain negative numbers as debts. (T2, T5, T7)	T-C
	In the operation of 4+ (-8), the minus and plus cannot get along, so the sign in the middle will be minus. In the operation of 4+ (+8), the same signs get along, so the sign in the middle will be plus (those who get along will be + and who do not get along will be -) (T3)	**Valid**
	Blowing up the + and - balloons (T4)	**Valid**
	Using counting pieces for + and - integers (T6)	T-C
	I associate negative numbers with holes in the soil and positive numbers with filling these holes (T10)	**Partially Valid**
Subtracting integers (this curriculum addition was analyzed in two sections)	I use the concept of zero pair (T2, T9)	**Valid**
	In the operation of +4-(-8), the minus and minus get along, so the sign in the middle will be + (Two signs cannot be side by side) (T3)	**Valid**
	I use the method of blowing up the + and - balloons (T4)	**Valid**
	I ask the students to think of subtraction as if it was addition and put the sign of the integer number with bigger number value to the beginning of the operation (T6, T7)	**Valid**
Comprehending that subtraction in integers means to add the opposite sign of the minuend	I explain by modeling with integers (T2)	T-C
	I use the example of a car moving (T6)	**Valid**
Using the features of addition as strategies for a fluent operation	The parentheses are our apartment, and the exponent concerns everything in the building as if it is out of the apartment. But it concerns only the flat with that number if it is in the apartment. It does not intervene the sign. (T2)	**Valid**
	I tell the students to consider themselves as doctors to make them pay attention to the signs' distribution. I tell them to pay attention to the parentheses just as they should be careful not to hurt the patient's heart in a surgery if they were doctors.	**Valid**
	Associating the operation of (+10)-(-2)=+10+2=+12 with the operation of (+5)-(-1)=+5+1=+6 (T6)	**Valid**
Multiplying and dividing integers	I use the example of "my friend's friend is my friend or my enemy's enemy is my friend" by associating friends with positive numbers and enemies with negative numbers (T1, T2, T4)	**Valid**

Indicating the repeated multiplication of integers as exponential numbers	Each number is as powerful as its exponent and multiplied as much (T3)	**Valid**
	Associating with a unicellular's reproduction (T4)	T-C
	I try to draw their attention to which number is multiplied and how many times it is writted (T5, T7)	T-C

** For example T5 means teachers.

It was observed that the teachers mostly used the analogies in the textbooks and the curriculum when teaching integers. The analogies used were focused on temperature, credit-debit relationships, thermometers, elevators, moves forwards or backwards, counting pieces, modeling with zero pairs, and using mathematical patterns. The majority of the analogies created and used by the teachers except for the analogies in the curriculum and textbooks were found to be sufficient. However, the analogy of "the minus reflect the contrasts, therefore the number gets smaller as it gets bigger" used by teachers to show integers on the number line and compare them was partially valid. The target and source relationship for the analogy was expressed correctly but incompletely, which led to a partial association. Students may have difficulty in perceiving the biggest negative number, -1, if they cannot comprehend the number value in this analogy. In addition, an unclear teaching method can lead to alternative concepts and misconception when comparing positive and negative numbers.

Another partially valid analogy created by the teachers is associating the holes in the soil with negative numbers and filling these holes with positive numbers when adding integers, which was used by the teacher coded as T10. This analogy was fully associated with the addition of integers with different signs (positive and negative), however, it is considered to lead to misconception and calculation errors in the addition of integers with the same sign (positive and positive or negative and negative) since students will always be focused on filling the holes. For example, in the addition of two negative integers such as the operation of (-2)+(-3)=?, students can perceive addition of 3 holes to 2 holes as filling the holes, and cannot find the correct result if they do not pay attention to the second number. Therefore, explaining the similarities and differences in the target and source relationship in analogies considering all aspects of the subject will prevent misunderstanding. In this regard, it is important that teachers have sufficient field information and knowledge in creating analogies, and that they use these analogies correctly and appropriately.

4. Conclusion and Discussion

This study analyzed the difficulties that middle school mathematics teachers have in teaching integers, the analogies they use to eliminate these difficulties, and the efficiency of these analogies under three titles which are *the difficulties they have in teaching integers*, *the analogies used for integers in the curriculum and textbooks*, and *the analogies used by the teachers and their efficiency*.

Although the teachers stated that students have no difficulties in learning positive integers, they state that students have difficulty in comprehending and comparing negative integers and showing them on the number line. Students also have difficulty in the operations with negative integers; they have problems in the addition of numbers with different signs and the multiplication and division of negative numbers. In addition, they cannot understand the concept of the negative of a negative integer number and make calculation errors in the result being positive or negative according to the parentheses and the exponent when exponentiating negative integers. These errors may mainly be because students have difficulty in interpreting and solidifying the concept of negative integers. If the concept of negative integers is not sufficiently comprehended in the 6th grade, then it negatively affects the subject of operations with integers and becomes a serious problem for future learning.

The literature indicates that the subject of integers, taught in middle school for the first time, is one of the subjects that is difficult to learn due to its abstract structure (Şengül and Körükçü, 2012). Işıksal Bostan (2010) stated that while the students have no difficulty with positive integers, they have difficulty in interpreting and comprehending negative numbers. They can associate positive integers with real objects, but some features of negative numbers conflict with the perception of counting numbers (Linchevski and Williams, 1999). This is because natural numbers that previously exist in students' minds can help with learning positive integers, but negative numbers cannot be learned by observing the physical world since nonpositive objects or object groups do not exist (Davidson, 1992; Mc Corkle, 2001; Şengül and Dereli, 2013).

According to Piaget's (1952) cognitive developmental periods, concrete operations correspond to primary school and the abstract operations correspond to middle school periods; and problems occur in comprehending and carrying out operations with integers, an abstract subject, when passing from concrete operations to abstract operations (Dereli, 2008). Studies show that even if the students do not have difficulty in placing negative numbers in the number line, they have difficulty in comparing their superiority, which is similar to the findings of this study. In addition, students cannot fully comprehend whether the addition and minus signs reflect the numbers' direction or the operation itself and make calculation errors due to incorrect use of these signs; they tend to subtract the number with the smaller number value from the number with the bigger absolute value (Işıksal Bostan, 2010; Avcu and Durmaz, 2011). The teachers that participated in this study stated that students have problems in identifying the sign of the result in the multiplication or division of integers. Avcu and Durmaz (2011) found similar results in their study with students in the 6th and 7th grades. Another finding of the present study is that students cannot fully understand the order of operations and make calculations ignoring the fact that the sign before the numbers change the sign of the result. The study of Avcu and Durmaz (2011) supports the findings of the present study.

Regarding the sixth grade curriculum learning objective of "Using the features of addition as strategies for a fluent operation" teachers stated that students have difficulty in understanding the order of operations and make errors in distributing the negative sign before the parentheses.

Integers are addressed in two sections in different grades in the curriculum prepared based on the constructive approach; the comprehension, comparison, addition, and subtraction of integers are addressed in the 6th grade while the multiplication and division of integers are addressed in the 7th grade. Thus, it was aimed that students acquire the concept of integers in the 7th grade. While it is recommended in the mathematics curriculum that in the 6th grade, integers be associated with elevators, air temperature, thermometers, and bank accounts, any analogies were found in the additions about the multiplication, division, and exponentiation of integers in the 7th grade.

When the mathematics textbooks approved by the MNE for the 6th and 7th grades in the 2013-2014 academic year were analyzed it is observed that, the textbooks address the comprehension, comparison, and expression of the absolute values of integers in the 6th grade and the operations with integers in the 7th grade, while the curriculum recommend giving only the objectives that "*multiplying an dividing integers*" and "*indicating the repeated multiplication of integers as exponential numbers*". This conflict between the curriculum and the textbooks can negatively affect teaching integers. The textbooks included analogies created by associating integers with sea level, thermometers, elevators, steps forwards and backwards, profit-loss or debit-credit relationships, air temperature, and number patterns. These analogies are similar to the analogies recommended in the curriculum, and differently from the curriculum, the textbooks include analogies for multiplication, division, and exponentiation of integers. It can be highlighted that particularly for the curriculum additions above, number pattern and the field analogies have been focused, differently from the other additions. Another highlight is that the curriculum and textbooks do not include analogies for the additions of subtraction and addition with the opposite sign of the minuend in integers.

Considering the difficulties that teachers have in teaching integers, it can be said that these analogies stipulated in the curriculum will be insufficient. Duit (1991) stated that the analogies are used too infrequently and generally at a very simple level in textbooks. The common and non-common points between the target and the source is not clearly specified in the analogies used in the curriculum and textbooks. However, analogies are strong tools if how and why the common and non-common features and similarities are formed is explained (mapping) (Saygılı, 2008; Kanalmaz, 2010). Explanation of the concept of integers and the operations with integers, which students have problems with interpreting by associating with something concrete in daily life, guides students in the formation of mathematical operations (Kilhamn, 2008; Hayes and Stacey, 1990; Şengül and Körükçü, 2012; Şengül and Dereli, 2013). Therefore; it is important to enrich the curriculum and textbooks with more analogies and to specify the common and non-common features so as not to cause misunderstanding when using the given analogies in teaching to help teachers who are accustomed to the traditional approach adapt to the new education and training environment.

It was observed that the teachers who participated in this study mostly used the analogies in the textbooks and the curriculum when teaching integers. The analogies used were focused on temperature, credit-debit relationships, thermometers, elevators, moves forwards or backwards, counting pieces, modeling with zero pairs, and using mathematical patterns. The teachers mostly used the given analogies rather than creating analogies. This may be because teachers are incompetent in creating analogies.

The findings show that the majority of the teachers used the given analogies rather than creating analogies themselves. Treagust et al. (1990) indicated that teachers do not have sufficient knowledge in analogies and thus use the analogies in textbooks; however, this situation conflicts with constructive teaching. Yet, they may have to create analogies for the subject to counter against the difficulties not predicted in the curriculum or textbooks. Therefore, teachers should be provided with in-service training to create correct analogies. An incorrect analogy made by teachers, who have a significant place in the teaching and learning process, may lead to misconceptions that are hard to get rid of (Treagust, Harrison and Venville, 1996). Teachers have important roles in eliminating misconceptions and the calculation errors students make due to such a misconception (Işıl and Bostan, 2010). Based on this information, the awareness of teachers should be raised on creating, using, and adapting analogies into their teaching to enable them to use efficiently the correct analogies at the right times.

The teachers mostly used the given analogies rather than creating analogies. This may be because teachers are incompetent in creating analogies. In this regard, teachers should be provided with the required training and the in-service training should be expanded. In addition, the faculties of education have great responsibilities in training the pre-service teachers, who will raise the next generations, during their undergraduate education in terms of creating analogies and how to adapt them into their teaching. The training on analogies should not be neglected considering their benefits in education and training.

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