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AFFECTIVE LEARNING WITH CREATIVE DRAMA? TRANSFORMATION GEOMETRY AND TEACHERS' CANDIDATES' LEARNING

Gözdegül Arık-Karamık[1i],
Murtaza Aykaç[2]
[1]Akdeniz University,
Department of Mathematics Education
Turkey
orcid.org/0000-0002-9478-6264
[2]Niğde Ömer Halis Demir University,
Department of Early Childhood Education
Turkey
orcid.org/0000-0001-7204-0835

Abstract:
Art is a special tool for mathematics education. Many artists have used mathematics and geometry in their artworks. This study aims to examine M.C. Escher's drawings with creative drama in the light of transformation geometry. In this study, the explanatory sequential was chosen from the mixed-method designs. Quantitative data of the study were collected with the Geometry Attitude Scale (GAS). Qualitative data were collected with the Workshop Evaluation Form, the Academic Achievement Measurement Form (AAMF), and semistructured interviews. The study group consisted of 21 volunteer participants. The data obtained from GAS were analyzed with SPSS and qualitative data were analyzed using descriptive and content analysis. Participants' GAS scores differed significantly after the implementation. Regarding both quantitative and qualitative analysis obtained from this study, which consisted of 8 workshops in total, an increase was observed in participants' attitudes towards geometry after participating in the workshop.

Keywords: transformation geometry, creative drama, M.C. Escher

1. Introduction

One of the famous mathematicians, Hardy (1940/2005), in his work titled "A Mathematician's Apology" emphasized the relationship between mathematics and art

[i] Correspondence: email gkaramik@akdeniz.edu.tr

with "*What a mathematician does should be as beautiful as that of a painter or poet. Thoughts should match each other harmoniously, like colors and words* (p.4)."

Many artists have used mathematics and geometry in their artworks. Leonardo Da Vinci, Luca, Picasso, Giotto, P. Uccelo and M.C. Escher are some of these artists. For example, Leonardo Da Vinci contributed to the book "About the divine proportions" written by Luca Pacioli with his drawings; he also put spheres, cones, cylinders, pyramids, and Platonic objects to the shapes he made on top of the layers. Da Vinci worked on Platonic objects from 2 different angles. The first is the planar perspective; the second is what he calls the "empty view" (vacua), which reveals the entire structure of the polyhedron and where the edges disappear. In addition, the "Mona Lisa" of Da Vinci is the first work in which the golden ratio was used. Da Vinci's painting "The Last Supper of Jesus" is considered one of the best examples of the use of geometry, that is, the rules of perspective, in art.

On the other hand, M.C. Escher used symmetry, regular division of the plane, paradoxes, and metamorphoses in his works. Using the concept of infinity, M.C. Esher has also signed many works that simultaneously include 2- and 3-dimensional elements. Having not been interested in mathematics in school, M.C. Escher expressed his thoughts as "*Although I lacked a science education, I felt closer to mathematicians than my artist friends*." His principal works are; Reptiles (1943), High and Low (1947), Drawing Hands (1948), House of Stairs (1951), Waterfall (1961), Moebius Strip (1963), Metamorphosis (1967-1968). His last work is "Snakes," exhibited in 1969, which took about six months to complete.

Escher adopted different approaches in his works. One of them is the regular division of the plane. In his paintings made with this technique, the artist explores how irregular shapes or combinations of shapes interlock entirely to cover a surface or plane. This method is similar to the plane-filling problem in mathematics. Escher performs this process fantastically using various animal figures. The Circle Limit series, in which he used a hyperbolic plane, is the most impressive among the works collected in this group (Kappraff, 1991).

Another approach is metamorphosis. The surface-figure relationship is strikingly emphasized in this series, and an impossible interdimensional journey is depicted. In metamorphosis, which means a change in nature, shapes that are constantly deformed without disturbing the regularity of the plane transform into each other, night turns into day, and fish evolves into birds. Escher's most striking works are his paintings in which he addressed the concept of paradox (contradiction) and infinity. The worlds he constructed using impossible figures lead us to contradiction. Even if one constantly moves up or down in the hierarchical orders established to create cyclical paradoxes, they still come to the starting point despite the hierarchy (Bool, 1982). In Figure 1, there are some examples of Escher drawings.

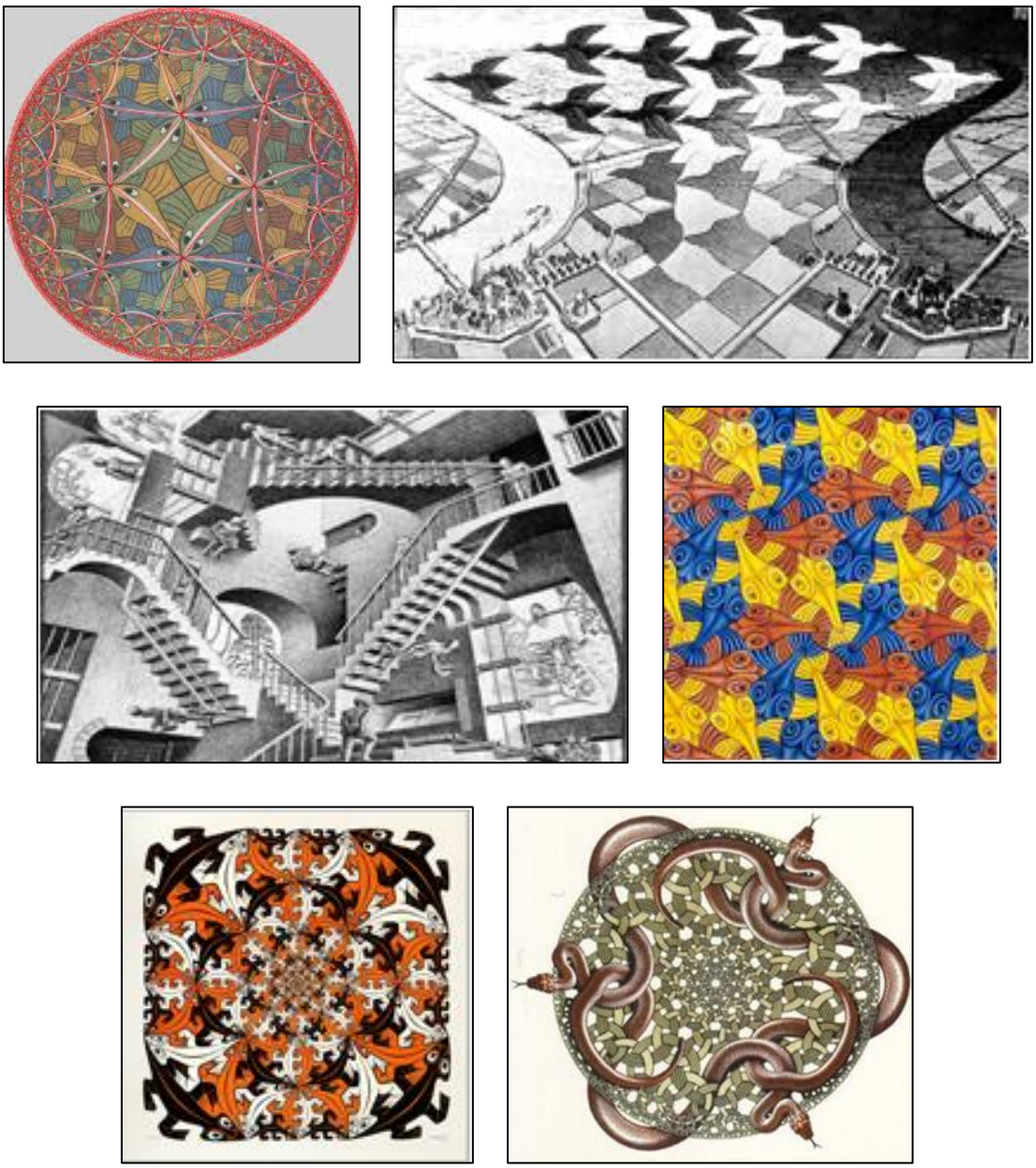

Figure 1: Drawings by Escher (Bool, 1982)

It can be said that Esher's drawings contain many concepts of transformation geometry (symmetry, reflection, translation, rotation). Symmetry is the projection, rotation, and translation of a shape or object around a particular axis (point, line, or plane) as a transformation and translation action (Lesh, 1976; Knuchel, 2004). Reflection is expressed by Zembat (2007) as a transformation that transforms all points in the plane into other points, keeping the distance between them. He underlined that it is a bijective function from R^2 to R^2, which transforms the points on the plane into other points, rather than being a movement on the plane (e.g., shifting a triangle from one position to another symmetrically).

Zembat (2013a) points out that translation mathematically implies a function rather than a physical movement, and the distance is kept in this function. The parameter of the translation is the vector. The vector is vital in translation because each component

finds its counterpart, its image, using the vector. The transformation preserves the geometric object or plane segment's internal dynamics (side lengths, angle measures, direction). As a geometric transformation, translation is different from the translational movement in daily life; one is a material movement while the other is a function that models this movement.

According to Zembat (2013b), rotation is a function that preserves distance/property, and it is called isometry, as translation and reflection. The rotation parameters are the center of rotation and the angle of rotation. Since the entire plane is subject to transformation, the definition and value sets are hidden in the plane. The given origin should be taken as the center of a circle; the process is finding the corresponding point by turning it clockwise (or counterclockwise) a certain number of degrees around it. Since the rotation is a function, the point selected from the domain is not physically rotated but corresponds to a point in the value set.

Concerning the Ministry of National Education (MoNE, 2018a, 2018b), these concepts appear in the 1st-8th Grades Elementary Mathematics Curriculum (EMC) 2018) in the relationship with "art" and in the 1st-8th Grades Visual Arts Curriculum (VAC, 2018) in the relationship with "mathematics." In EMC, "*Students get aware of the ornaments in works of art belonging to different civilizations*" (p.34) to achieve the gain of "Creates structures using shape models and draws the structures they create"; in VAC, "*Within the scope of the interdisciplinary approach, gains can be associated with the geometric shapes (p.14), ratio (p.16), symmetry (p.18) and perspective (p.30) subjects of the mathematics course*." It is thought that pre-service teachers' knowledge of these Mathematics Curriculum skills and transferring them to interdisciplinary practices by effectively combining them is essential in developing these skills.

Based on this context, creative drama was chosen as an effective method for classroom practices, and M.C. Escher's works were examined using this method. Adıgüzel (2019) defined creative drama as the animation of a purpose or idea based on the group members' life experiences using certain techniques such as improvisation, role-playing, etc. The related literature contains various studies on drama (Duatepe, 2004; Duatepe &Paksu, 2009; Kariuki & Humphrey, 2006; Gadanidis, Hughes &Cordy, 2011). M.C. Escher's drawings which contain mathematics and art together, were examined from the art perspective (Yeral, 2006); they were not analyzed mathematically. However, there is no study interpreting mathematical concepts especially transformation geometry on the relationship between mathematics and art.

1.1 Purpose of the study

Unfortunately, mathematics and geometry have not gone beyond being boring subjects for most people. Considering the drama process's strong effect on expressing every person's creative potential (Üstündağ, 2006), it may be important for individuals to explore the art in mathematics and the mathematics in art. Based on the view that creativity can be developed through education, drama can be used to reveal the creativity in individuals (Üstündağ, 2006). Departing from this point, this study aims to examine

M.C. Escher's drawings with creative drama in the light of transformation geometry. For this purpose, the following questions were addressed.

1) Does the evaluation of M.C. Escher's drawings with creative drama in the light of transformation geometry affect pre-service elementary mathematics teachers' attitudes towards geometry? If yes, does this effect differentiate according to gender? What are the pre-service teachers' opinions about the reasons for significant differences?
2) What are the pre-post concepts of pre-service elementary mathematics teachers regarding the evaluation of M.C. Escher's drawings with creative drama in the light of transformation geometry? What are the factors affecting the pre-post concepts?

2. Methodology

2.1 Research model

This study aimed to examine M.C. Escher's drawings with creative drama in the light of transformation geometry. For this purpose, the mixed-method was used. The mixed-method combines qualitative and quantitative methods, approaches, and concepts within a study or subsequent studies for providing a comprehensive analysis of the research problem (Creswell, 2014).

The explanatory sequential was chosen from the mixed-method designs. The explanatory sequential is performed in two separate interactive stages; Quantitative data collection and analysis is the first stage, and the qualitative part is the second stage. Qualitative data are used to explain quantitative findings (Cresswell and Plano Clark, 2011).

In this study, quantitative data collection tools were administered to the group before starting the implementation. Qualitative data were then collected in each workshop, quantitative data collection tools were administered again after the application, and finally, interviews were conducted with the participants. This cycle can be expressed as supporting quantitative data with qualitative data.use this style when you need to begin a new paragraph.

2.2 Data collection tools

Quantitative data of the study were collected with the Geometry Attitude Scale (GAS) developed by Bindak (2004). A draft scale of 46 items, half of which was negative, was created by combining the literature on attitude scale development, students' writings about geometry, and expert opinions. This draft scale was applied to 113 people for item analysis; after three different items analyses, six items were determined to be insufficient to measure the attitude intended to be measured, and they were deleted.

The 40-item draft scale was applied to 131 people for factor analysis, which showed that the items were distributed and divided into 9 factors. 15 items with a factor load greater than 0.35 or with a low factor load difference (less than 0.1) were excluded

from the scale. Finally, the final form of the Geometry Attitude Scale consisted of 25 items (16 negatives & 9 positives) and 4 sub-factors. The Cronbach Alpha reliability coefficient of the data collection tool was .92.

Qualitative data were collected with the Workshop Evaluation Form consisting of three open-ended questions prepared by the researchers, the Academic Achievement Measurement Form (AAMF) consisting of 10 questions prepared by the researchers, participant diaries, and semi-structured interviews.

The workshop evaluation forms were prepared by the researchers, and necessary corrections were made according to the feedback received from two creative drama leaders who are experts in their fields. The participants were given a diary before the application. They were asked to keep learning diaries by writing down the knowledge they learned, their feelings, and the first thing they noticed after each workshop.

AAMF was also prepared by the researchers and was structured by taking the opinions of three mathematics educators who are experts in their fields. Before administering to the study group, a pilot was conducted with 10 pre-service teachers. Incomprehensible parts were identified, and necessary corrections were made.

The semi-structured interview form consisted of three open-ended questions prepared by the researchers. It was finalized after taking the opinions of 2 expert mathematics educators and two drama leaders.

For the first sub-problem of the study, the quantitative data of GAS and the qualitative data collected with the Workshop Evaluation Form and participant diaries were used. For the second sub-problem, quantitative data were collected with AAMF, and qualitative data were collected with semi-structured interviews. In order to avoid data loss during the study, the process was video-recorded.

2.3 Study group and implementation process

The study group consisted of 21 volunteer participants from 2nd -year students of the Elementary Mathematics Education Department of one of the leading universities in the Mediterranean Region. 18 (85.7%) of the participants were female, and 3 (14.3%) were male. Regarding the age of the participants, 66.7% were in the 18-19 age group, and 19.0% (4) were in the 20-21 age group. The study was conducted through 8 workshops (weeks) for 23 hours. Table 1 includes the titles (goals) and periods of the workshops.

Table 1: Workshop titles and periods

Workshop	Title (Goals)	Period
1st Workshop	The Relationship between Communication-Math and Art	120 min.
2nd Workshop	The Relationship between Harmony-Trust-Mathematics and Art	180 min.
3rd Workshop	The Life of M.C. Escher	180 min.
4th Workshop	Transformation Geometry and M.C. Escher	180 min.
5th Workshop	Symmetry in M.C. Escher's drawings	180 min.
6th Workshop	Reflection in M.C. Escher's drawings	180 min.
7th Workshop	Rotation in M.C. Escher's drawings	180 min.
8th Workshop	Translation in M.C. Escher's drawings	180 min.

Ethical permissions were obtained from the necessary organizations and the study group for the implementation process. First, GAS and AAMF were administered to the voluntarily formed group. The participants were then informed about how, where, and when each workshop would be held. A notebook was given to each participant to keep a diary, and they were informed about the points to pay attention to while keeping a diary. After each workshop, participants filled out the workshop evaluation form, and researchers collected it. VAS and AAMF were administered again after the completion of the 8th workshop. Afterwards, interviews were conducted with 6 pre-service teachers, 3 female and 3 male. Each interview lasted approximately 40-50 minutes. Interviewed participants were coded as P1, P2, P3, P4, P5 and P6.

2.4. Data analysis

The data obtained from GAS were analyzed in the computer environment with SPSS 21.0, and the findings were tabulated and interpreted. Percentage, frequency, arithmetic mean, and standard deviation were used to analyze and interpret the data. In addition, the independent samples t-test was used to test the differentiation according to gender, and the significance of the differences according to age, high school type, origin, mother's education level, and father's education level was tested with ANOVA (one-way analysis of variance). "Levene Test" results showed that the variances of the group scores were homogeneous (>.05), and t-test was used for comparing 2 groups and ANOVA in comparing 3 or more groups. Tukey test was used to reveal the groups with significant differences found in ANOVA. Regarding the significance, the p-value was taken as 0.05. Although there are different concepts and approaches related to qualitative data analysis in the literature, Yıldırım and Şimşek (2013) define data analysis in two categories, namely descriptive analysis and content analysis, as suggested by Strauss and Corbin (1990), according to the depth of the analysis. In this study, qualitative data were analyzed using descriptive and content analysis.

In the first stage of data analysis, the process suggested by Huberman and Miles (2002) was followed. This process includes the following phases; summarizing data for the first stage of data analysis, interpreting coding approaches that are more descriptive and interpretive, and creating short reminders. Each participant's workshop evaluation forms and diary were subjected to preliminary analysis in these stages. Preliminary codes were evaluated, focusing on scale items, the method used in the implementation, and the case. Based on this context and the "You know what you can show" approach of Huberman and Miles (2002), the products created by the participants were also included in the study.

3. Findings

The data obtained within the scope of the study are presented separately for each sub-problem.

3.1 Does the evaluation of M.C. Escher's drawings with creative drama in the light of transformation geometry affects pre-service elementary mathematics teachers' attitudes towards geometry? If yes, does this effect differentiate according to gender? What are the pre-service teachers' opinions about the reasons for significant differences?

Table 2 shows the dependent t-test results of the participants regarding whether the changes observed after the implementation of creative drama for geometry show a significant difference.

Table 2: Dependent t-Test Results of Participants' GAS Pretest-Posttest Scores

Group	N	$\bar{x}$	S	sd	t	p
Pre-test	21	90.85	15.23	20	27.33	.00*
Post-test	21	98.85	13.06			

*p<0,05

According to Table 2, participant' GAS scores differed significantly before and after the implementation, and this difference was significant ($t_{(20)}$=27.33, p>.05).

This finding shows that the creative drama method positively affected participants' attitude scores. Table 3 shows the independent t-Test results for the difference in the pretest GAS scores of the participants according to gender.

Table 3: Independent t-Test Results of Participants' GAS Pretest Scores according to Gender

Gender	N	$\bar{x}$	S	sd	t	p
Female	18	89.77	14.96	19	.788	.434*
Male	3	97.33	18.50			

*p<0,05

As seen in Table 3, there is no significant difference between the attitudes of male and female participants towards geometry ($t_{(19)}$=.788, p>.05). The mean pre-test score of female participants was ($\bar{x}$=89.88), and male participants ($\bar{x}$=97.33). This result can be interpreted as the attitudes of male and female participants toward geometry were similar.

Table 4 shows the independent t-Test results for the difference in the post-test GAS scores of the participants according to gender.

Table 4: Independent t-test results of participants' gas posttest scores according to gender

Gender	N	$\bar{x}$	S	sd	t	p
Female	18	97.16	13.28	19	1.49	.099
Male	3	109.00	5.29			

Regarding Table 4, the GAS scores of the participants did not differ significantly according to gender after the implementation, and the effect of gender on attitude was not significant ($t_{(19)}$=1.49, p>.05). This finding shows that the application has the same effects on the attitude of male and female participants. However, despite the lack of

significant difference according to gender, the mean GAS score of male participants was ($\bar{x}$ = 109.00), while female participants' mean GAS score was ($\bar{x}$ = 97.16).

The effects of examining M.C. Escher's drawings with creative drama in the light of transformation geometry on pre-service elementary mathematics teachers' attitudes towards geometry – that is supporting quantitative data with qualitative data - were categorized under three themes and sub-themes by analyzing daily evaluation forms and diaries, namely mathematics and geometry, creative drama and M.C. Escher.

The frequencies of each theme were determined separately for each workshop and presented in Table 5, Table 6, and Table 7. Table 5 shows the sub-themes of mathematics/geometry, Table 6 sub-themes of creative drama, and Table 7 sub-themes of M.C. Esher.

Table 5 shows participants' statements about what they noticed about mathematics/geometry, what they learned from the workshop, and what this workshop added to them. It includes positive factors that may increase their attitudes.

Table 5: Sub-themes and frequencies for mathematics/geometry

Workshops	**Mathematics/Geometry**	**f**
1st Workshop	Mathematics-Geometry and Art are intertwined	5
	Mathematics is more meaningful when combined with other subjects	2
	Mathematics is not monotonous	1
	Mathematics is more fun when we associate it with art	3
	Mathematics is not just about numbers	1
	Mathematics lessons can be fun too	2
	I never paid any attention to the mathematics around me	1
	I started to think about some mathematical concepts	1
	I started to establish the relationship between mathematics and art.	2
2nd Workshop	Mathematics-Geometry and Art are intertwined	12
	Mathematics includes everything	7
	It is the first time that I cannot wait for the mathematics lesson to come	5
3rd Workshop	There are people and works that combine mathematics and geometry with art.	3
	Transformations can have both mathematical, geometric, and artistic meanings.	2
	Geometry and art combine so beautifully.	7
	Mobius strip	2
	Understanding that it is fun to mix mathematics and art	2
	The ability to combine mathematics, art, and geometry	3
	I was unaware of Escher's mathematical works	14
4th Workshop	The concepts of reflection, symmetry, rotation, and translation and learning them through play.	7
	How you can encounter Transformation Geometry in daily life	4
	The concepts belonging to Transformation Geometry can be concretized.	5
	The relationship between Play and Mathematics	2
5th Workshop	The difference between the axis of symmetry and the line of symmetry	15
	Symmetry in Escher's works	14
	His shortcomings in symmetry	2
	The image of an image is the object	1
6th Workshop	The difference between reflection and symmetry	15

	The concept of reflection and its types	14
	Their shortcomings in reflection	5
	Interpreting the reflection in Escher's drawings mathematically	6
7th Workshop	The parameters of the rotation.	14
	The meaning of Mathematical Parameter	3
	Learning that Esher has works that contain rotation.	9
8th Workshop	The translation in mathematics is different from the translation in daily life.	2
	The translation is easy to remember when learned through the play.	12
	The translation is vector-dependent.	5
	The properties of translation	11

As seen in Table 5, the participants understood the relationship between mathematics and art and made sense of mathematical expressions after this process. Pre-service teachers expressed reflection and symmetry as the same concepts, understood the mathematical meaning of transformation, and learned them in a fun and enjoyable setting.

Recognizing or making sense of the mathematical concepts of transformation geometry in M.C. Escher's drawings is also a tool that allows pre-service teachers to see the mathematics in daily life practices. In addition, the products (Figure 2) that the participants created after the workshop show that they can predict the transformation geometry mathematically.

Figure 2: Examples of products structured by participants

In addition, the following statements were drawn from participants' diaries;

- "*Mathematics-Geometry and Art are intertwined.*" (15),
- "*Mathematics is more fun when it is associated with art.*" (13),
- "*It is the first time that I cannot wait for the mathematics lesson to come.*" (5),
- "*I understood both the geometric and artistic meanings of the transformations.*" (12),
- "*I became aware of Escher's mathematical works.*" (14),
- "*I can learn/teach transformation geometry through play.*" (17),
- "*I understood the Mathematical Parameter.*" (13).

Table 6 shows participants' statements about what they noticed about drama, what they learned from the workshop, and what this workshop added to them. It includes positive factors that may increase their attitudes.

Table 6: Sub-Themes and Frequencies for Creative Drama

Workshops	**Creative Drama**	**f**
1st Workshop	Expressing oneself more easily with drama	3
	To what drama can contribute	1
	There is a need for different methods, such as drama.	1
	Drama is a good tool for socializing	3
	Learning in drama occurs when participants are active.	1
	The lesson is understood much better by having fun	4
	Helping to learn by communicating not only with the leader but also with others	1
2nd Workshop	I felt comfortable doing drama	2
	I enjoyed drama	3
	Thanks to drama, one can have fun even when they are sick.	1
	The importance of task distribution in groups	2
	One can generate nice and creative ideas	2
	It entertains while making you think	2
3rd Workshop	I can improvise	1
	I overcome my excitement with drama	4
4th Workshop	The importance of space	15
	The lesson is more fun with drama	10
	The relationship between Play and Mathematics	2
	One can also make drama with friends they are not in contact with.	2
5th Workshop	Symmetry is more memorable with drama	3
	Have fun making drama	3
6th Workshop	One can use drama while teaching	7
7th Workshop	I make progress in drama	7
	I am much better at animation now	4
8th Workshop	I make progress in drama	10
	I am much better at animation now	5

Regarding Table 6, the frequency of pre-service teachers' statements about the sub-themes of drama is high in the first three workshops. Pre-service teachers, who encountered drama for the first time, stated that drama is a tool for socialization in the acquaintance, trust, and harmony workshops, i.e., the first three workshops. The idea evolved to using drama in mathematics lessons in the following workshops, and pre-service teachers expressed it as a teaching method while having fun. Figure 3 shows the products made by the participants in the 7th workshop, inspired by the works of M.C. Esher, and emerged during the evaluation phase of the drama.

Figure 3: The Products made by the participant in the 7th workshop

In addition, the following statements were drawn from participants' diaries;

- "*I can express myself more easily with drama*." (13),
- "*I needed different methods such as drama*." (11),
- "*Drama is a nice tool for socializing*." (8),
- "*I feel comfortable doing drama*." (12),
- "*I can produce nice and creative ideas*." (8),
- "*I was able to overcome my thrill with drama*." (16),
- "*Space is very important when doing drama*." (15),
- "*The lesson is more fun with drama*." (10),
- "*I was able to establish the relationship between play and mathematics with drama*" (6),
- "*I was also able to make drama with the friends with whom I was not in contact"* (6).

Table 7 shows the frequency of participants' statements about what they noticed about M.C. Escher's drawings, what they learned from the workshop, and what this workshop added to them.

Table 7: Sub-themes and frequencies for M.C. Escher's drawings

Workshops	**M.C. Escher's drawings**	**f**
1st Workshop	-	
2nd Workshop	-	
3rd Workshop	M.C. Escher's drawings have mathematical meaning	2
	There is an artist named MC Escher	3
	The life and works of MC Escher	2
	M.C. Escher combined mathematics and geometry, and art in a very meaningful way in his works	4
	M.C. Escher's life was interesting	3
	M.C. Escher was a great artist	2
4th Workshop	M.C. Escher's works contain the concepts of transformation geometry	8
	I did not know about M.C. Escher's drawings	12
5th Workshop	Symmetry in the works of M.C. Escher	14

	Examining and commenting on the works of M.C. Escher	14
6th Workshop	Reflection in the works of M.C. Escher	12
	Interpreting the reflection in drawings	11
7th Workshop	Rotation in the works of M.C. Escher	17
8th Workshop	Translation in the works of M.C. Escher	18

Regarding Table 7, in the first two workshops, the participants associated the relationship between mathematics/geometry and art with different artworks. After the 3rd workshop, they predicted and expressed the concepts of transformation geometry in M.C. Escher's drawings. Recognizing the relationship between mathematics and art and structuring it with the help of the works of a famous painter made the participants feel important; they felt like Escher. In addition, the mathematical meaning in M.C. Escher's drawings and the ability to predict this meaning developed after the 4th workshop. In addition, Figure 4 shows the workshop product, in which the mathematical meaning of translation and rotation is predicted by using the works of M.C. Escher.

Figure 4: The workshop product, in which the mathematical meaning of translation and rotation is predicted

In addition, the following statements were drawn from participants' diaries;

- *"M.C. Escher was an amazing painter and mathematician."* (17),
- *"I am curious about mathematicians who have artworks other than M.C. Escher."* (4),
- *"I often felt as I am M.C. Escher."* (8),
- *"I noticed math is not monotonous with M.C. Escher's drawings."* (4).

3.2 What are the pre-post concepts of pre-service elementary mathematics teachers regarding the evaluation of M.C. Escher's drawings with creative drama in the light of transformation geometry? What are the factors affecting the pre-post concepts?

Table 8 includes the participants' preconcepts about the transformation geometry before the implementation. Frequencies belong to all participants, and the statements of the interviewed participants are also included in the table.

Table 8: Preconcepts about the transformation geometry

Concepts	**Preconcepts**	**f**	**Participants**
Symmetry	Reflection by a point	12	P1, P2, P5
	Overlap when folded	14	P1, P2, P6
	Displacement in the coordinate axis	8	P1, P2, P3, P4, P5
	The object being symmetrical	7	P1, P3, P4, P5,
Reflection	Mirror image of an object	23	P1, P2, P3, P4, P5
	Projecting the shape elsewhere	9	P6
Rotation	I did not hear it	17	P2, P3, P5
	Flip the shape, move to another position	11	P1, P4, P6
Translation	I did not hear it	12	P2, P3, P4
	Going beyond, translating	10	P1, P52, P6

According to Table 8, participants defined the concepts without using mathematical language. In addition to the pre-service teachers who stated that they did not hear the rotation and translation, some pre-service teachers associated them using daily language away from the mathematical language. P3 expressed the symmetry as being symmetrical with the example given in Figure 5. In this example, P3 seems to be confused about making the symmetric shape and the shape having an axis of symmetry.

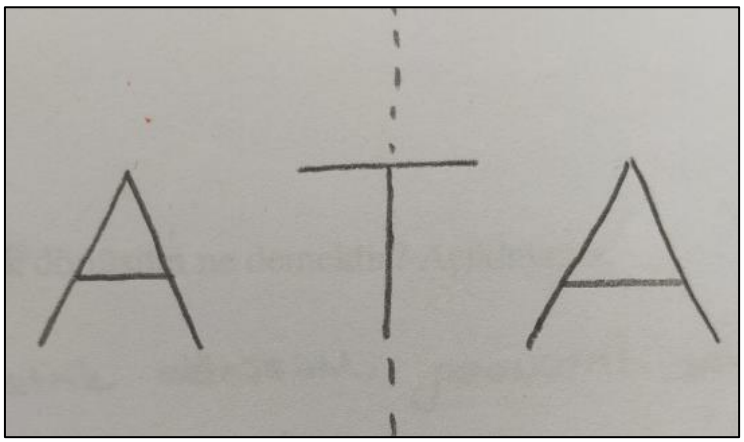

Figure 5: Example configured by P3

The situation that P2 expresses as overlapping when folded is shown in Figure 6. Although this statement is acceptable, it is far from a mathematical representation and illustration. In other words, no attention was paid to distance conservation and vertical projection.

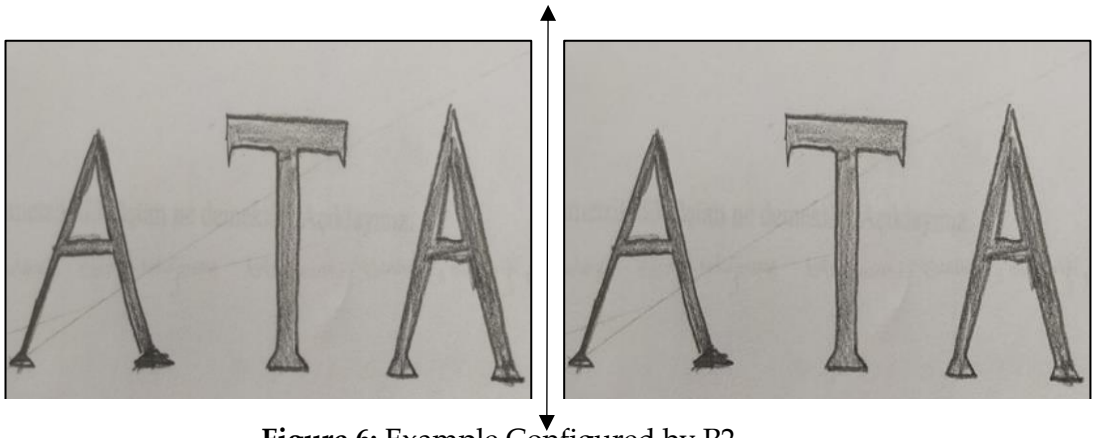

Figure 6: Example Configured by P2

Table 9 includes the participants' post-concepts about the transformation geometry after the implementation, the frequencies, and the statements of the interviewed participants.

Table 9: Post-concepts about the transformation geometry

Concepts	**Post-concepts**	**f**
Symmetry	Horizontal and vertical axes of symmetry	6
	Formation of the image of the shape	7
Reflection	There is a line of symmetry (line of reflection), and it is a parameter.	11
	As a result of distance conservation, the dimensions and shapes of objects are preserved.	7
	Image and shape are equal.	12
	The points are equidistant from the line of symmetry. Distances to the line of symmetry are preserved	11
	The shape's side lengths, angles, perimeter, and area are preserved.	9
	The direction of the angles, in other words, the direction of the shape, is not preserved; it may change.	8
Rotation	The parameters are the center point and the angle.	16
	There is no movement in the literal sense, but there is a mathematical match.	17
	It is important to know the properties of the circle and use them.	11
	The center point is the intersection of the midpoints of all beams.	4
	The rotation has a direction	5
Translation	The distance is preserved.	14
	The parameter is the vector.	11
	The shape's side lengths, angles, direction, perimeter, and area are preserved.	17
	There is no physical movement; it is a function that preserves distance and property using vectors.	13

Regarding Table 9, participants used the mathematical language for the transformation geometry after the implementation. In addition, the term "parameter" is used, and the parameters belonging to each concept are known. The parameters expressed by the participants are the line of symmetry for reflection, center point and angle for rotation, and vector for translation. Moreover, participants also stated that rotation and translation

are not physical movements; they have a mathematical meaning. In addition, they emphasized what is preserved and what is not preserved after each transformation and integrated it into the practices while examining M.C. Escher's drawings. Some of these products are shown in Figure 7.

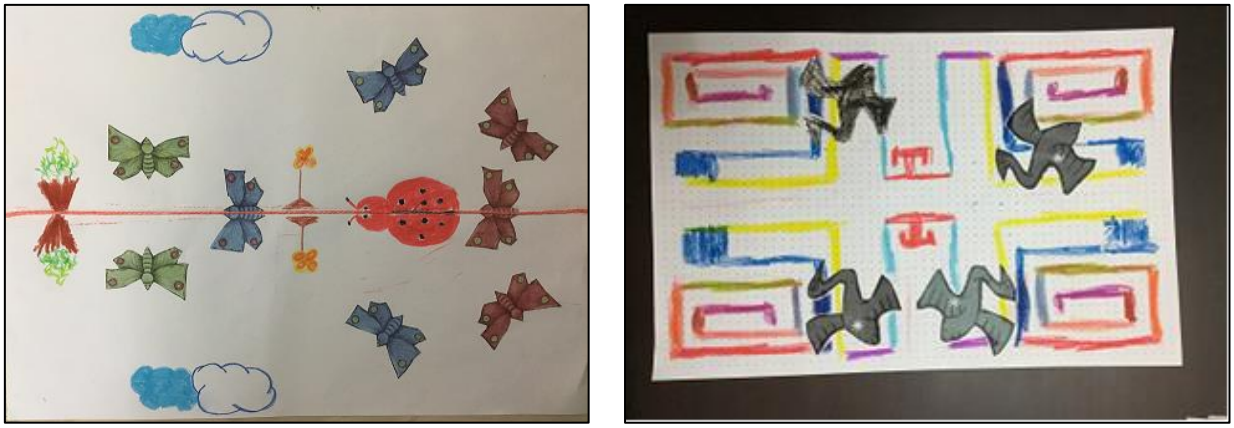

Figure 7: Product samples obtained after implementation

The examples in Figure 7 have mathematical meanings. Particular attention was paid to the conservation of shape, distance, and parameters. The images of the object are constructed with the figures cut from M.C. Escher's drawings; the points are equidistant from the line of symmetry. Participants used rulers and/or dotted paper for distance conservation while taking the horizontal and vertical symmetries of the shapes. The factors affecting pre and post-concepts are listed in Table 10.

Table 10: Factors affecting pre and post-concepts

Theme	Sub-theme	Participant
Transformation Geometry	Lack of mathematical meaning in previous experience with transformation geometry	P1, P2, P3, P4, P5, P6
	Transformation geometry is not meaningful because it is left to the end of the term.	P3, P4
	Failing to realize that the knowledge they tend to memorize because the lessons are boring is the knowledge that could be learned.	P1, P2, P4, P5
	Transformation geometry may be needed to predict works of art.	P5
Drama	Learning by having fun with drama is more permanent.	P1, P2, P3, P4, P5, P6
	Drama helps to come up with creative ideas	P1, P4, P5
	Drama offers appropriate learning opportunities.	P2, P3, P4
Art	Recognizing the relationship between mathematics and art	P1, P2, P3, P4, P5
	The applicability of mathematics motivates them	P1, P2, P5

As seen in Table 10, the factors that may cause the participants' preconcepts to change are grouped under three themes: transformation geometry, drama, and art. Regarding the

sub-themes, the following statements were expressed by all the participants: "Lack of mathematical meaning in previous experience with transformation geometry " and "Learning by having fun with drama is more permanent." Among the participants, only P5 expressed the reason for the change of preconcepts as follows;

> *"I realized that I have an interest in works of art. For example, I want to go and look at the Mona Lisa; I want to understand it, so I think I learned because I thought transformation geometry might be needed to predict works of art."*

4. Discussion and Conclusion

In this study, it can be said that the desired application gains have been achieved from the analysis of both qualitative and quantitative data about the evaluation of M.C. Escher's drawings with creative drama in the light of transformation geometry.

Quantitative data were collected with the scale developed by Bindak (2004), and qualitative data were collected with the "Workshop evaluation form," "Diaries," and "Academic achievement evaluation form" developed by the researchers. Regarding the quantitative data obtained from this study, which consisted of 8 workshops in total, an increase was observed in participants' attitudes towards geometry after participating in the workshop. It was also found that this situation did not differ according to gender, which is the desired result from the perspective of the creative drama method. Because based on the concept of equality, creative drama is a method that values everyone just because they are "individuals," the members of the study group are expected to reach the desired result as they are in the same environment. It is possible to say that the attitude towards geometry has changed with this study that included both creative drama and the interpretation of works of art.

In the context of the art-mathematics relationship, we encounter STEAM (Science, Technology, Engineering, Art, and Mathematics) studies (Dejarnette, 2018; Glass & Wilson, 2016; Liao, 2016) that blend disciplines. In these studies, works of art are predicted mathematically and associated with other disciplines. This study did not include science, technology, and engineering fields, and only mathematics was used to predict art. Researchers who want to improve this study are recommended to combine it with the other three disciplines.

There are various studies in the literature in which works of art are predicted by using mathematics (Jeong, 2010; Johnson, & Steinerberger, 2019; Kovacs, 2020). One of these studies is Bennett's doctoral dissertation in 2019 titled "More than a visual: mathematical concepts in the artworks of Lewitt and Escher." Bennett's study is similar to the mentioned studies and partially overlaps with this study. Bennet aimed to reveal the relationship between mathematics and art through Sol LeWitt and M.C. Escher's works and stated that both artists used mathematical concepts to create rule-based works of art. In the study, mathematical ideas were exemplified by the works of artists and included in the literature, which strengthened the link between art and mathematics.

Being aware of the strength of the connection between art and mathematics, this study aimed to improve participants' attitudes and concepts using this power.

In addition to studies that develop attitudes towards mathematics by using artworks (Ffolkes-Bryant, 2008; Forseth, 1980; Healy, 2004), the results of the studies that support creativity by using art in mathematics teaching (Kattou et al., 2013; Leikin & Pitta-Pantazi, 2003; Pitta- Pantazi et al., 2018) are also in line with the findings of the study. The statements of the participant group, who are newly acquainted with such an application, about the development of their creativity, and the resulting products can be given as examples of these findings.

This study is also similar to the studies in which drama is used as a method of teaching mathematics (Al-deeb & Aladini 2021; Fleming, Merrel and Tymms, 2010; Karapınarlı 2007) and geometry (Duatepe-Paksu & Ubuz, 2009; Ünlü, Avcu & Avcu, 2010; Uygun & Güner, 2021). In addition to these studies, the findings overlap with studies in which drama and art are used together, positively affecting mathematical achievement (Kotarinou & Ch, 2008; Kotarinou & Stathopoulou, 2012; Omniewski 1999). In other words, in this study, the works of M.C. Escher were predicted by using creative drama, and as a result of this prediction, the attitude and academic achievement increased.

The gains of the transformation geometry learning domain should support participants' creativity and thus help them comprehend how geometric shapes can be transformed in a two-dimensional plane; help discover the rules and shapes' properties (İnce, 2012); support their thinking on translation, reflection, and rotation transformations (National Council of Teachers of Mathematics [NCTM], 2000); and mediate them to gain a spatial perspective (Hollebrans, 2003). In this context, the drawings of M.C.Escher serve to structure the concepts of transformation geometry and overlap with the mentioned studies.

After the 8-week implementation, the participants stated that they achieved gains of transformation geometry, developed a positive attitude towards geometry, gained practical experience in using the mathematical language, and could structure the lesson using both artistic works and drama when they became teachers. Future researchers are recommended to develop, implement and predict applications involving different grade levels and different works of art.

We, educators, emphasize the need to associate mathematics teaching with daily life. Although this study is a study without generalization anxiety, it is worth emphasizing that any study that can be created and applied for each sub-field of mathematics can become functional by integrating it into the curriculum. Although it is an assertive sentence, it is obvious that every sub-learning field can be associated with any field that supports art, music and creativity.

Conflict of interest statement

The authors declare no conflicts of interest.

About the Authors

Gözdegül Arik Karamik is an Assistant Professor at the Department of Mathematics and Science Education at Akdeniz University, Turkey. She became a Creative Drama Leader in 2017 after completing the 320-hour program and a drama project. She researches and authored research articles published in international journals and book chapters about different approaches and applications in mathematics teaching, number sense, transformation geometry, algebra and creative drama.

Murtaza Aykaç is an Associate Professor at the Early Childhood Education Department at Niğde Ömer Halis Demir University, Turkey. He became Creative Drama Leader in 2007 after completing the 320-hour program and a drama project too. He researches and authored research articles published in international journals and book chapters about lifelong learning, creative drama and social sciences.

References

Adıgüzel, Ö. (2019). *Eğitimde Yaratıcı Drama* (4th Edition). Yapı Kredi Yayınları. Ankara.

Al-deeb, M., & Aladini, A. (2021). CLIL and drama on conceptual knowledge and logical thinking in mathematics. T*urkish Journal of Computer and Mathematics Education (TURCOMAT)*, 12(13), 6897-6910. https://www.turcomat.org/index.php/turkbilmat/article/view/10078

Bennett, K. (2019). Mathematical concepts within the artwork of LeWitt and Escher. [Unpublished Doctoral dissertation, The Faculty of Alfred University]. New York. https://aura.alfred.edu/handle/10829/23396

Bindak, R. (2004). Geometri tutum ölçeği güvenirlik geçerlik çalışması ve bir uygulama. [Unpublished doctoral dissertation, University of Dicle] https://tez.yok.gov.tr/UlusalTezMerkezi/

Bool, F. (1982). M*C Escher, His Life and Complete Graphic Work: With A Fully Illustrated Catalogue*. Abradale Press/Harry N. Abrams.

Cresswell, J. W., & Plano Clark, V. L. (2011). *Designing and Conducting Mixed Method Research*; 2nd Sage: Thousand Oaks.

Creswell, J. W. (2014). *A concise introduction to mixed methods research*. SAGE publications.

Dejarnette, N. K. (2018). Implementing STEAM in the early childhood classroom. *European Journal of STEM Education, 3(3), 18.* https://doi.org/10.20897/ejsteme/3878

Duatepe, A. (2004). The effects of drama-based instruction on seventh grade students' geometry achievement, van Hiele geometric thinking levels, attitude toward mathematics and geometry. [Unpublished doctoral dissertation, METU] Ankara, Turkey. https://tez.yok.gov.tr/UlusalTezMerkezi/

Duatepe-Paksu, A., & Ubuz, B. (2009). Effects of drama-based geometry instruction on student achievement, attitudes, and thinking levels. *The Journal of Educational Research, 102(4),* 272-286. https://doi.org/10.3200/JOER.102.4.272-286

Ffolkes-Bryant, B. (2008). Investigating the effect on student attitudes, motivation and self-esteem in performing arts elementary schools incorporating arts integration. [Unpublished doctoral dissertation, St. John's University School of Education and Human Services]. New York. https://www.proquest.com/docview/304817308?pq-origsite=gscholar&fromopenview=true

Fleming, M.; Merrel, C. and Tymms, P. (2010). The impact of drama on pupils' language, mathematics, and attitude in two primary schools. *Research in Drama Education: The Journal of Applied Theatre and Performance.9(2)* https://doi.org/10.1080/1356978042000255067

Forseth, S. D. (1980). Art activities, attitudes, and achievement in elementary mathematics. *Studies in Art Education, 21(2), 22-27*. https://www.jstor.org/stable/pdf/1320145.pdf

Gadanidis, G., Hughes, J., & Cordy, M. (2011). Mathematics for gifted students in an arts- and technology-rich setting. J*ournal for the Education of the Gifted, 34(3), 397-433*. https://doi.org/10.1177/016235321103400303

Glass, D., & Wilson, C. (2016). The art and science of looking: Collaboratively learning our way to improved STEAM integration. *Art Education, 69(6), 8-14*. https://doi.org/10.1080/00043125.2016.1224822

Hardy, G. H. (2005). *Bir matematikçinin savunması* (22. Basım). (N. Arık, Trans.). Tübitak Yayınları, Ankara. (1940).

Healy, K. G. (2004). The effects of integrating visual art on middle school students' attitude toward mathematics [Unpublished master dissertation, University of Alaska Anchorage]. USA. https://www.proquest.com/docview/305051220?pq-origsite=gscholar&fromopenview=true

Hollebrands, K. F. (2003). High school students' understandings of geometric transformations in the context of a technological environment. *The Journal of Mathematical Behavior, 22(1), 55-72.* https://doi.org/10.1016/S0732-3123(03)00004-X

Huberman, M., & Miles, M. B. (2002). *The qualitative researcher's companion.* Sage.

İnce, H. (2012). Kırsal bölgelerde ve şehir merkezindeki öğrencilerin dönüşüm geometrisi anlama düzeylerinin ve uzamsal görselleştirme yeteneklerinin incelenmesi [Unpublished master dissertation, Eskişehir Osmangazi Üniversitesi]. Eskişehir.

Jeong, K. C. (2010). Art education through artwork: focusing on the mathematical principle. T*he Journal of the Korea Contents Association, 10(4),* 447-457. https://doi.org/10.5392/JKCA.2010.10.4.447

Johnson, S. G., & Steinerberger, S. (2019). Intuitions about mathematical beauty: A case study in the aesthetic experience of ideas. *Cognition, 189, 242-259*. https://doi.org/10.1016/j.cognition.2019.04.008

Kappraff, J. (1991). Conecttons: The geometric bridge between art and sciences, Mc GrawHill Pub.

Karapınarlı, R. (2007). İlköğretim 7. sınıf matematik dersinde yaratıcı drama yönteminin öğrencilerin başarı ve kalıcılık düzeyine etkisi [Master's thesis, Muğla University]. https://tez.yok.gov.tr/UlusalTezMerkezi/

Kariuki, P. N., & Humphrey, S. G. (2006, November, 8-10). The Effects of Drama on the Performance of At-Risk Elementary Math Students [Oral presentation] Annual Meeting of the Mid-South Educational Research Association. Birmingham, AL.

Kattou, M., Kontoyianni, K., Pitta-Pantazi, D., & Christou, C. (2013). Connecting mathematical creativity to mathematical ability. *ZDM, 45(2), 167-181*. https://doi.org/10.1007/s11858-012-0467-1

Knuchel, C. (2004). Teaching symmetry in the elementary curriculum. *The Mathematics Enthusiast, 1(1), 3-8.* https://files.eric.ed.gov/fulltext/ED493996.pdf

Kotarinou, P., & Ch, S. (2008, November). Role–playing in Mathematics instruction [Oral presentation] International congress of Drama/Theatre in Education, Ankara. Turkey

Kotarinou, P., & Stathopoulou, C. (2012, February, 6-10). The History of 5th postulate: linking mathematics with other disciplines through Drama techniques [Oral presentation] CERME 8. Antalya, Turkey

Kovács, Z., Lichtenegger, B., Recio, T., Richard, P. R., & Vélez, M. P. (2020, September). Exploring artwork through Delaunay triangulations. In Proceedings of the 2019 MACAS (Mathematics and its Connections to the Arts and Sciences) Symposium (pp.63- 69), Montreal, Quebec.

Leikin, R., & Pitta-Pantazi, D. (2013). Creativity and mathematics education: The state of the art. *Zdm, 45(2), 159-166*. https://doi.org/10.1007/s11858-012-0459-1

Lesh, R. (1976). Transformation geometry in elementary school: some research issues. Martin (1976a), 185-243. https://files.eric.ed.gov/fulltext/ED132033.pdf#page=191

Liao, C. (2016). From interdisciplinary to transdisciplinary: An arts-integrated approach to STEAM education. *Art Education*, 69(6), 44-49. https://doi.org/10.1080/00043125.2016.1224873

Ministry of National Education (MoNE, 2018a). 1-8. Sınıf Güzel Sanatlar Dersi Öğretim Programı, Ankara. http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=358

Ministry of National Education (MoNE, 2018b). 1-8. Sınıf Ortaokul Matematik Öğretimi Programı, Ankara. http://mufredat.meb.gov.tr/ProgramDetay.aspx?PID=329

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Omniewski, R. A. (1999). The effects of an arts infusion approach on the mathematics achievement of second-grade students. [Unpublished Doctoral dissertation, College of Fine and Professional Arts]. Kent State University. https://www.proquest.com/docview/304530991?pq-origsite=gscholar&fromopenview=true

Pitta-Pantazi, D., Kattou, M., & Christou, C. (2018). Mathematical creativity: Product, person, process and press. *In Mathematical Creativity and Mathematical Giftedness* (pp. 27-53). Springer, Cham.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research*. Sage publications.

Uygun, T., & Güner, P. (2021). Van hiele levels of geometric thinking and constructivist-based teaching practices. *Mersin Üniversitesi Eğitim Fakültesi Dergisi,* 17(1), 22-40. https://doi.org/10.17860/mersinefd.684571

Ünlü, M., Avcu, S., & Avcu, R. (2010). The relationship between geometry attitudes and self-efficacy beliefs towards geometry. *Procedia-Social and Behavioral Sciences, 9, 1325-1329.* https://doi.org/10.1016/j.sbspro.2010.12.328

Üstündağ, T. (2006). Yaratıcı drama eğitim programının öğeleri. Adıgüzel, Ö (Ed.), Yaratıcı Drama 1985–1998 Yazılar. Ankara: Naturel Kitap Yayıncılık.

Yeral, Ö. (2006). Hasan Ali Toptaş'ın gölgesizler romanı ve olanaksızlık. [Unpublished Master's thesis, Social Sciences Institute]. Yeditepe University. İstanbul. https://tez.yok.gov.tr/UlusalTezMerkezi/

Yıldırım, A. & Şimşek, H. (2013). *Sosyal Bilimlerde Nitel Araştırma Yöntemleri*. (9. Baskı). SeçkinYayıncılık. Ankara.

Zembat, İ. Ö. (2007). Yansıma dönüşümü, doğrudan öğretim ve yapılandırmacılığın temel bileşenleri. *Gazi University Journal of Gazi Educational Faculty (GUJGEF), (1)*. https://dergipark.org.tr/en/download/article-file/77182

Zembat, İ. Ö. (2013a). Geometrik dönüşümlerden öteleme ve farklı anlamları. In Bingölbali E., Zembat İ. Ö., Özmantar M.F., Şandır H., & Delice A., (Eds), *Tanımları ve Matematiksel Gelişimleriyle Matematiksel Kavramlar* (pp.629-644). Pegem, Ankara.

Zembat, İ.Ö. (2013b). Geometrik dönüşümlerden dönme özellikleri. In Zembat İ. Ö., Özmantar M. F., Bingölbali E., Şandır H., & Delice A., (Eds), *Tanımları ve Matematiksel Gelişimleriyle Matematiksel Kavramlar* (pp.645-658). Pegem, Ankara.Pegem, Ankara.