INTRODUCING THE COORDINATE SYSTEM TO KINDERGARTEN CHILDREN: A DEVELOPMENTAL APPROACH

Kevser Koç¹, Ayşe Arzu Arı², Yusuf Koç²
¹Istanbul Medeniyet University, Türkiye
²Kocaeli University, Türkiye

Abstract:
The coordinate system is a fundamental tool for many daily activities, yet it is usually taught to children in middle school. Research suggests that children should be exposed to the coordinate system at a younger age, using methods appropriate to their developmental level. This paper presents a set of instructional activities designed to teach kindergarten children the basic concepts and principles of the coordinate system. The activities were implemented in two early childhood institutions in Istanbul, Türkiye. Arranged in a developmental progression from simple to complex and from concrete to abstract, they align with the children's developmental levels. We discussed what we learned from our implementations of the activities. Detailed suggestions are presented for teachers introducing young children to the coordinate system.

Keywords: coordinate system, early mathematics, kindergarten, mathematics teaching

1. Introduction

The importance of developing mathematical skills in early childhood is widely emphasized in the literature. However, due to low awareness of the subject’s importance, teachers’ lack of resources, and underestimation of children’s potential, mathematics must be sufficiently included in the kindergarten curriculum. Math topics are often limited to commonly-known topics such as counting, addition, subtraction, patterns, and basic geometric shapes (Ministry of National Education (MoNE), 2013). Among other topics, the coordinate system needs to be addressed in early mathematics programs. Given that children are inherently curious about their surroundings and eager to understand, it should be a fundamental goal of kindergarten programs to satisfy this

¹Correspondence: email kocyusuf@gmail.com
curiosity and demonstrate the relevance of mathematics in life through engaging in activities from an early age.

Children encounter structures like coordinate systems in various settings. From seating arrangements in cinemas, theaters, gymnasiums, or classrooms, to passenger placements in vehicles, from locating addresses on streets or avenues to using maps, the concept of the coordinate system is ubiquitous. Moreover, chess, similar strategy game boards, and activities such as coding involve layouts where vertical lines intersect.

The development of map reading skills depends on children’s understanding of the coordinate system (Blades & Spencer, 1994; Sarama et al., 2003; Somerville & Bryant, 1985) Understanding the coordinate system also contributes to developing spatial reasoning skills (National Research Council, 2006; Sarama & Clements, 2009). Research has demonstrated that children introduced to geographical and spatial concepts early on are consistently more successful than their peers in later years (Atkins, 1981; Muir, 1985). Therefore, education on spatial coordinates should prepare children for daily and academic life.

Young children can learn and use coordinates when developmentally appropriate games and activities are utilized. These games and activities stimulate as many senses as possible, appeal to diverse types of learners, and engage children physically, cognitively, and emotionally (Epstein, 2014). It is also essential for children to familiarize themselves with the materials used, spend time with them independently, and repeat the activities to comprehend the subject thoroughly. This allows them to use the materials as they wish, for as long and according to their plans. It provides opportunities for more profound thought on the topic—conditions necessary for an optimal learning environment (Epstein, 2014; Hyson, 2008).

At the heart of analytical geometry, the coordinate system is primarily used to determine the position of any point in the plane (or in space). In the plane, a horizontal axis (x-axis), a vertical axis (y-axis), and a point where they intersect (origin) form the coordinate system. In this system, the position of a particular point is determined by an ordered pair expressed as (x, y). The x-value represents the horizontal distance between this point and the y-axis, while the y-value indicates the vertical distance between this point and the x-axis. Therefore, all points in the plane are expressed as coordinates represented by ordered pairs (x, y). To locate a point, students need to envision lines extending vertically from the horizontal axis and horizontally from the vertical axis and find where they intersect.

For a meaningful learning experience, children should be allowed to explore and understand concepts of the coordinate system rather than being taught the coordinate system as a pre-established construct. Children must learn about spatial configuration to know that the plane is organized into grids or coordinate systems. Spatial structuring is the mental process of constructing an organization or form for an object or a set of things in space. Children may initially perceive a grid as a collection of squares rather than a set of horizontal and vertical lines. However, as their learning progresses and their developmental maturity increases, they gradually start to see it as organized in rows and
columns, and they then learn the relationships of order and distance within the grid (Sarama & Clements, 2009).

Research provides many insights regarding when, under what conditions, and at what level children begin to understand the coordinate system. According to the findings of (Piaget & Inhelder, 1967), eight-year-old children do not spontaneously construct and use coordinates when faced with a coordinate system problem. Recent research, however, has shown that even four-year-olds can understand the coordinate system to a certain extent and can complete simple tasks related to coordinate system-like shapes. Children perform better when lessons are presented in contexts that are more familiar and meaningful to them, such as when two models are to meet at a point (Somerville & Bryant, 1985). In another study, four-year-old children had difficulty finding the corresponding area when a point was described with numbers and letters (e.g., (1, a)). Still, their success increased when colors were used (e.g., (red, yellow)) (Blades & Spencer, 1994). According to the findings of Lidster and Bremner (1999), when children were asked to point to a specific location with arrows placed perpendicular to each other, the success rate was high. However, when the same area was indicated by hand, and the child was asked to find it, the success rate decreased, especially if the arrows were positioned at a distance from the location. Children were more successful when structuring the task because they considered both coordinates sequentially. When the errors made by the children in these studies were analyzed, it was observed that they primarily resulted from the consideration of only one coordinate. Furthermore, children are more successful when the coordinates are closer, and their success levels decrease as they move away from the origin (Lidster & Bremner, 1999).

This study introduces activities designed according to child development stages to build the concept of coordinates in early childhood years, guided by scientific research findings.

2. Materials and Methods

2.1. Participants
The study involved five kindergarten teachers and their children from a private kindergarten serving middle-income families in Istanbul, Türkiye. The children were 35 girls and 27 boys, with an average age of 5.2 years.

2.2. Materials
The study primarily utilized everyday classroom materials such as Legos, toy cars, and animal figures. Additionally, the teachers and researchers prepared educational tools using resources available within the classroom.

2.3. Procedure
The activities presented in this study were developed over approximately two years of observation and participation in the school where they were implemented. During this
period, one or two researchers made weekly visits to the school, providing in-service training in teaching preschool mathematics. This study encompasses a portion of the activities carried out in these schools.

During the preparation phase, a team composed of one early childhood researcher educator and two mathematics education researchers first conducted a literature review. They identified a hypothetical learning trajectory through which children acquire skills necessary to understand the basics of the coordinate system. Subsequently, potential activities were developed for each level. When preparing these activities, the team adhered to the following principles, informed by scientific data:

- Emphasize processes where children are actively engaged.
- Ensure activities align with play-based perspectives.
- Gradually introduce games to children, beginning with teacher-guided tasks and progressing towards independent problem-solving.
- Order activities from simple to complex and easy to difficult.
- Prioritize materials that children are familiar with and use regularly.
- Adjust activities according to children’s skill levels.
- Initially, prioritize concrete materials, transitioning to more abstract materials at advanced levels.
- Encourage beginner-level children to engage in activities using as many senses as possible, particularly involving large motor skills.
- Select beginner-level tasks from units close to the origin on the coordinate system, gradually introducing tasks using units farther away from the source as children gain proficiency.
- After each session, leave the materials used in the activities on open shelves in the mathematics learning center, allowing children to interact with the materials independently and at their discretion.
- Encourage children to articulate their thoughts while working under the teacher’s guidance, asking probing questions like, "Why did you decide this way? Can you explain?"

One of the researchers initially experimented with the activities with a few children, with sessions videotaped. Those videos were reviewed and discussed in weekly in-service meetings with the teachers. Based on the children’s skill levels, decisions were made regarding which activities would be initiated at what level. In the days following, the teachers, occasionally accompanied by a researcher, implemented, and recorded the activities and shared the videos in a WhatsApp group. The researchers provided feedback on those videos and guided the teachers to ensure that children understood the subject. The teachers’ classroom implementation was reviewed, evaluated, and revised in subsequent meetings as a whole group.

The activities were designed in various formats: large groups, small groups, and individual sessions. Teachers first conducted individual sessions, allowing them to assess children individually and determine their level while facilitating their transition to group work. As children mastered the activities at one level, they were introduced to the next
level. This process fostered the children’s development and led to the finalization of the activities.

2.4. Differentiation
When introducing an activity in a group setting, the teacher demonstrated the expected performance in detail, and the children observed. A higher-level child was then asked to complete the activity, allowing the rest of the group to perform robustly. A child at another level was then asked to work through the activity. The rest of the group observed their peers. This group session concluded by selecting another higher-level child who would perform well for the rest of the group. These procedures aimed to reach all children with varying needs by providing them with many opportunities to observe the performance of others and to complete the task. The teacher worked with the children individually or in pairs later.

The children’s differences were noted and considered throughout the teaching and assessment processes. The activity plans were designed to meet the unique needs of young children. The teacher's objective was to match each child with a level that would provide a slight challenge but still be achievable, thus contributing to their development and progress. Activities were designed to have a moderate level of cognitive difficulty, which was then adjusted based on the children's progress. When a child completes an intermediate-level activity, the teacher assigns a more complex or higher-level task. Conversely, if a child struggled with a task during the game, the teacher reduced the difficulty by giving a lower-level task. A guiding principle of the study was to ensure that all participating children completed the activities with a sense of accomplishment.

3. Activities

3.1. Activity 1
In this introductory activity, an EVA puzzle mat was used to design a game to introduce the basics of the coordinate system to young children in an exciting and captivating way (Photo 1). All the photographs in the study were taken by the researchers.
3.1.1. Method of Implementation
Initially, the game was played one-on-one with each child to assess their progress and familiarize them with the game. Later, the game was played in small groups.

3.1.2. Materials
A 4x5 rectangular EVA puzzle mat was created by combining twenty squares with animal images. Various animal figures and cars of assorted colors were positioned on the two perpendicular sides of the 126 cm x 157 cm mat. An important consideration here is that the objects placed on one side should be the same type, animals on one side (or axis) and cars on the other.

3.1.3. Targeted Skills
The activities aim to teach children to identify the intersection points of two perpendicular lines and imagine another line extending perpendicularly from a point along a given line.

3.1.4. Individual Work
A child was invited to participate voluntarily in the game. Initially, the child was asked to name the animals and colored cars placed on the two sides of the mat. Additionally, the child was asked if she could recognize the animal images on the puzzle mat squares. In the next step, the child must identify the animal images on the squares to focus on the
game’s primary purpose adequately. At the beginning stage, the teacher started with a meaningful short story or drama, suggesting a move for an animal and a car to meet. For example, the teacher asked, "The elephant on the horizontal side and the red car on the vertical side are friends, and they want to meet, but they can only move forward on their paths.

Within the game, each animal and car can only move forward in a straight line or path in front of them. The car and the animal can only meet at the intersection point where their paths intersect. For instance, when the dog and the blue car move to meet, the dog moves on its path and stops when it reaches the point of intersection. Next, the blue car arrives, and they meet at the point.

3.1.5. Play-Based
The teacher suggests moves to ensure the child understands the rules. Once the child understands the rules, the teacher and the child take turns suggesting new actions. In this second phase, the teacher occasionally makes mistakes and provides opportunities for the child to correct them. During this process, the teacher reinforces the child’s self-confidence by encouraging active participation and attention with phrases like, "Oh! How did I make such a mistake? Can you correct me?"

3.1.6. Details
Based on scientific findings, the teacher initially moved the animals and cars closer to the origin. The vertical and horizontal sides of the puzzle mat meet at the origin. If the child accomplishes such tasks, they are given tasks that include points of intersection further away from the origin of the puzzle mat, the coordinate plane. Based on the literature and our observations, we realized that children struggle to think in two dimensions. Therefore, in the beginning, the children were asked to focus on the cars (considering the horizontal side as the x-axis), and the teacher guided the movements of the animals on the other side. For each move suggested by the teacher, the child’s progress was monitored and encouraged. Instead of using guiding statements like "You should move first," the teacher used expressions that enabled the child to engage in the game or implied that they gave priority to the child, such as "I need to think a little. Could you go first?" or "Where will they meet? Let the elephant start." The teacher first names the points on the horizontal side, the x-axis, then the points on the vertical side, the y-axis. Although we do not expect children to learn or understand such formal concepts at the preschool level, we want to prepare them for learning situations they will encounter later. Thus, the concepts of the x-axis and y-axis have yet to be introduced to the children.

In a game with a child, the teacher first suggests a move, waits for the child’s progress, and checks if the child brings the car to a stop at the meeting point. Then, the teacher moves her animal to the point. After a few steps, when the child grasps this phase, the teacher asks, "Now it is your turn. Which animal and car should we move?" The child is allowed to suggest moves. Empirical evidence shows that children better grasp
cognitive tasks they initiate and manage themselves (Lidster & Bremner, 1999), so efforts have been made to make the children actively participate.

In the next phase, the teacher gives the following instruction:

“You did well. Now, let us try something different. Can you predict where they will meet without moving the animals or the cars? Take a step back and tell me which square they will meet on.”

Then, as in the previous phases, the teacher asks the child to spot the intersection points where animals and cars will meet, starting from points closer to the origin and moving to points further away. Then, the child is asked to suggest a move, and the teacher finds the area where the suggested animal and car will meet.

Both to make the game more motivational and to understand the child’s thinking, the teacher made deliberate mistakes, and then the following interaction took place:

Teacher: “Do you think it was correct?”
Child: (laughing) “No.”
Teacher: “Oh dear! I thought it was correct. Where do you think I made a mistake?”
Child: “Here. Move the car two steps further.”

3.1.7. Group Work

After working individually with each child and once they learned the rules of the game, the activities were also conducted in small groups. The groups consisted of three or four children. First, the teacher asked the children to take turns making moves:

Teacher: “Can you tell us where the horse and the blue car will meet?”
Child: (after thinking for a few seconds) “On the rabbit.”
Teacher: “Children, is there a rabbit in the intersection area of the horse and the blue car?”
Several children speak simultaneously: “Yes, there is a rabbit.”
Teacher: “How many squares must the blue car move to meet the rabbit?”
Child: “Four squares.”
Teacher: “Are you sure? Let us count, children? (The teacher points to each square with her hand while the children count.) One, two, three, and four. Yes, precisely four squares! How many squares must the rabbit move to meet the blue car?”
Two children simultaneously: “Two squares!”
Teacher: “Let us count and see.”
Together: “One, two.”

In all activities, opportunities were provided for the child to think and monitor their moves as well as the moves of their peers. The children were expected to reflect on their own and each other's moves and justify each step after each move.
3.2. Activity 2
Following Activity 1, children worked on a material resembling a coordinate plane consisting of unit squares. The main difference between the first and the second activities is the material. Initially, the teacher asked them to arrange small animal toys horizontally and the corresponding food items (or other toys) vertically on the game mat. They were asked to determine how the animals should move to align with the food items. The children matched the animals with the food items by counting the squares (Photo 2).

Photo 2: Implementation of Activity 2

3.3. Activity 3
Having completed Activity 2, the children engaged in slightly more abstract representations at this level. They placed animal figures on a plane with columns and rows labeled with numbers and shapes (Photo 3). The teacher was free to create a meaningful context for this game. She constructed a story where characters had to navigate the coordinate plane to reach their goals. In the story, when the characters reach specific coordinates on the plane, they encounter a surprise situation or another character. Sometimes the teacher provided the coordinates, while other times, the children were asked to provide them.
3.4. Activity 4
We implemented this activity with the children who had completed earlier activities. The coordinate system is introduced here in concrete (theater hall) and more abstract (sketch of the hall) contexts. As in the coordinate system, in theaters, seats are labeled with numbers and letters, and each seat is an intersection point of a row and column. Thus, the seating configuration for the audience was found to be a meaningful and motivational context to enhance children’s understanding of coordinates. We conducted the activity during World Theatre Week to make it more meaningful. It took three days to complete this five-step activity.

The first step of the activity was conducted during circle time, where the teacher engaged in a conversation with the children to focus their attention on the activity. We observed that children better understood the coordinate system when interacting with their peers in problem-solving situations. Questions such as "Have you ever been to a theater?" "How did you choose your seat?" and "Did you see numbers and letters on the seats?" were asked to facilitate children’s interest. Then, the children began to share their experiences:
Teacher: “So, folks! When you went to a cinema or theater, did you have a specific seat assigned to you, or did you sit wherever you wanted?”
Child #1: “We had seats already assigned to us.”
Child #2: “My sister had a piece of paper, and she looked at it.”
Teacher: “So, why do you think such a seating configuration exists in theaters? What would happen if there was not?”
Child #3: “The audience would need clarification about where to sit, wondering where their seat is.”

Next, the teacher suggested,

Teacher: “How about creating a theater hall in our classroom? What do you think?”

After the children agreed, she continued,

Teacher: “How should we arrange the chairs? First, we need to make a plan. Let us form three groups, and each group will come up with an idea. Each group will receive a bowl of blocks representing chairs. The table tops will represent the classroom floor. Arrange the blocks to represent the chairs. Show how you want the seating arrangement to be when watching a theater performance in the classroom.”

The children collaboratively brainstormed ideas on how fifteen chairs could be arranged as a team. Afterward, the teacher gave each child a sheet of paper and asked them to draw a floor plan of the seating arrangement based on the blocks on the table (Photo 4). The children then selected a spokesperson for each group, and each group presented their proposal, explaining why they made such choices.
In the second stage (the next day), the children were reminded of the theater hall models they had previously created, and then they examined the photographs and drafts.

Teacher: “Let us make a theater hall in our classroom and number the seats. Does anyone know how many chairs we have in our class?”
Child: “15.”
Teacher: “Very well. Please discuss this again in your groups. How should we arrange the chairs?”

The children initially discussed within their groups and then, as a whole group, attempted to determine how to arrange the 15 chairs. They decided to have five chairs in three rows. Next, they organized their classroom accordingly (Photos 5 & 6).
Following the arrangement of the chairs, the teacher suggested, “Children, let us give these chairs a number and a letter, just like in the theater hall. What do you think?” The teacher then formed the coordinate system on the classroom floor with horizontal and vertical lines using colored tape. She gave the children letter and number cards and asked children to place the letters on the horizontal line and the numbers on the vertical line at the beginning of each row (Photo 7). Next, the children labeled the chairs vertically with numbers and horizontally with letters (Photo 8).
The children were asked to pick a chair, sit down, and write its location on paper. Some children verbalized the coordinates of chairs, such as “My chair is A4”, “My chair is B1”, and “My chair is C2” (Photo 9).
Then, they attached the papers with the coordinates on the back of the chairs. Afterward, the children compared chair coordinates (Photo 10).

This activity progressed by asking the children individually about the labels assigned to the chairs. When the teacher asked a child the chair’s name, she also asked to explain how it was named. The child explained this by indicating the letter and number on the sides. Next, the teacher distributed the sketch of the classroom and asked the children to write the names of the chairs on the sketch (Photo 11).
On the final day (Day 3), children performed a drama that simulated attending a theater play. The teacher had already prepared a sketch of the seating arrangement and tickets. The children came to the ticket office and chose their seats by looking at the sketch, just like buying tickets in a theater or cinema. They used play money to buy tickets and found their seat (Photo 12).

Most children quickly found the intersection points of the horizontal and vertical rows and took their seats. Their friends assisted those who had difficulty finding seats. At the end of the activity, a few children went on stage and sang a song.
4. Assessment

Teachers conducted classroom observations throughout these games and documented children’s work through recording and videotaping for evaluation purposes. These assessment records were later analyzed, and video clips were reviewed to assess children’s learning levels and inform instructional decisions. Next, the tasks each child could accomplish and those they struggled with were identified. Efforts were made to create conducive environments to facilitate children learning. Additionally, the initial activities enhanced the children’s awareness and comprehension of the subject matter. Most children succeeded in the final Theater Hall Activity except for a few.

5. Conclusion and Recommendations

This study was developed and presented as a project component, including various mathematical activities for kindergarten children. The activities aimed at teaching the basics of the coordinate system. Teachers should introduce the activities to children only after learning basic mathematical counting concepts.

We observed that children willingly engage with these activities, exhibit interest in participation, and quickly grasp the concepts. Mathematical activities that evolve incrementally through engaging games attract children’s interest, and acquiring new mathematical concepts improves their self-confidence. However, teachers should structure the activities in a progressive order of complexity, as presented here. It is essential for children to encounter challenges commensurate with their abilities and to experience a sense of achievement. The teacher should facilitate the child’s engagement through questions to elicit thinking.

Throughout the implementation of these play-based activities, teachers made ongoing adaptations and implemented differentiated instruction to address individual needs. Children rotated to receive personalized instruction and engage with challenges of varying complexity. The learning environment encouraged peer learning and cognitive development. Teachers must comprehensively understand the developmental stages of coordinate concepts to provide practical guidance and differentiation.

Furthermore, the teacher’s attitude and rapport with the children during the activities were pivotal. Teachers should ensure that all children are cognitively and physically engaged in the activities. We observed that children were more open and made faster progress when teachers showed enthusiasm and trust in their instructional methods.

In conclusion, when presented through developmentally appropriate activities, even mathematics topics not typically included in the kindergarten programs, such as coordinates, are willingly embraced by children. We encourage teachers to have more mathematical concepts in their programs and expose children to diverse topics.
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About the Authors
Kevser Koç is an early childhood education associate professor at Istanbul Medeniyet University. She received her doctorate from Indiana University Bloomington and completed her Master’s in Early Childhood Education. She completed her undergraduate studies in Psychology at Ankara University. Her doctoral dissertation was titled “Moral Dilemmas in Early Childhood Settings: Turkish Teachers’ Accounts.” Her research interests include early childhood education, teacher education, children’s literature, and early mathematics. She has published articles in prestigious journals, including Young Children, Journal of Early Childhood Research, International Journal of Special Education, and SAGE Open. e-mail: koc.kevser@gmail.com

Ayşe Arzu Arı is an assistant professor of mathematics education at Kocaeli University with a doctorate from Kocaeli University in mathematics. Her dissertation was on topological spaces. She completed her undergraduate studies in Mathematics at Ankara University. She has taught middle school mathematics in public schools for several years. Her research interests include teacher education, mathematics education, mathematical modeling, and ethnomathematics. She has published several research articles, including “Mathematics teacher candidates' perceptions of mathematical modeling and modeling: A metaphor study” and “Content analysis of meta-analysis studies made in mathematics education in Turkey.” e-mail: abural@kocaeli.edu.tr

Yusuf Koç is an associate professor of mathematics education at Kocaeli University with a doctorate from Indiana University Bloomington. He completed his undergraduate studies in Teaching Secondary School Mathematics at Middle East Technical University. His doctoral dissertation was on using case studies in mathematics teacher education. His research interests include early mathematics, mathematics education, teacher education, and teaching and learning geometry. He has published several articles in prestigious journals, including Teaching and Teacher Education, Journal of Mathematics Education, The Educational Researcher, and SAGE Open. e-mail: kocyusuf@gmail.com
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