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THE EDUCATIONAL ROLE OF HARMONICS IN PLATO'S *REPUBLIC VII*

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Abstract:
The purpose of this study is to discuss Plato's views concerning the mathematical aspects of music and its educational role, through the science of harmonics in *Republic* VII. Plato in his *Republic* includes harmonics as the fifth of the courses in a program of mathematical sciences (arithmetic, geometry, stereometry, astronomy, and harmonics) to be studied by the future philosopher-rulers, who are to become the best guardians and govern the ideal State. These mathematical sciences constitute the preparatory courses of the highest of all, which is dialectics. The prominent place that mathematical sciences occupy in Plato's educational system, as that stage of education which helps the soul to turn towards true philosophy, is based on the fact that they contribute to the cultivation of cognitive skills, teach people how to think and help in the exercise of abstract thought. Harmonics have an equal, if not superior position, among the other mathematical sciences. They are all connected with each other in terms of the result achieved by their study, that is, the conversion of the soul from the visible world to the intelligible world. This happens due to their mathematical way of thinking and their intelligible character, excluding their practical use and their involvement with empirical objects.

Keywords: Plato, harmonics, mathematical sciences, society, music education

1. Introduction

Plato's work is full of references to music (Pelosi, 2010; Moutsopoulos, 1959). His views concern ''*the role of music in moral education, the ''abstract analysis of musical structures'' and the mathematical relations of ''harmonic structures to the constitutions of human soul, and to that of the universe at large*'' (Barker, 1984, p. 124). The role of music in the moral education of citizens is presented in Plato's works *Republic* and *Laws* ''*works which describe in detail the political set-up of two ideal constitutions as well as the lifestyle and ethical values embraced by their citizen*'' (Lynch, 2020, p. 111). In the context of this paper, we will deal with *Republic*.

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In *Republic*, we distinguish two stages of education with music. The first concerns music education of all citizens at a young age (*Republic*, II, III) and the second stage the education of future philosopher-rulers (*Republic* VII, 522c-531d), the fine spirits that are to govern the ideal State.

Music (*mousiké)* at the first stage, has a broader concept than in our days, meaning the unity of melody, poetry, and dance (Petraki, 2008; Brann, 2004; Stamou, 2002). The aim of this stage is the cultivation of both the human soul and body through an educational program with music and gymnastics (*Republic,* 376 e). The goals of this stage through music education are focused on imparting harmony to the children's souls for the formation of a moral personality (Moreau, 2017; Jenkins, 2015; Bourgault, 2012; Woerther, 2008). Through music education, Plato looks forward to cultivating virtues such as soberness, courage, liberality, and high-mindedness (*Republic*, 401d–e 402c) which are acquired through habits and exercise (*Republic*, 518 e, 522 a).

However, Plato rejects gymnastics and music as exclusive subjects that can lead the soul to the truth and acquaintance with the Form of the Good. The conquest of the Form of the Good is what is required for the acquisition of knowledge and constitutes Plato's basic epistemology (Gwynneth, 1972). The reasons are related to the fact that gymnastics aims ''*to that which grows and perishes*'', as its work is to attend ''*the growth and decay of the body*'' (*Republic,* 521e). Moreover, the study of music aims to educate:

> ''*the guardians through habits, imparting by the melody a certain harmony of spirit that is not science, and by the rhythm measure and grace, and also qualities akin to these in the words of tales that are fables and those that are more nearly true*'' (*Republic,* 522 a).

By this way, it is clearly shown that a different approach to music as a scientific field with mathematical criteria is required, separating ''*art and science, of lower and higher education*'' (Lippman, 1964, p. 75).

The approach to music as a scientific field with mathematical criteria is satisfied by the second stage of training through harmonics as a part of the mathematical sciences. This stage, which concerns higher education, is not intended for all citizens, but for the most capable young and future philosopher-rulers (*Republic*, 522c-531d), the fine spirits that are to govern the ideal State. The purpose of that stage of education through the mathematical sciences is the intellectual education of guardians since they have already taken moral training through music education at the first stage. At the second stage, which is the subject of this paper, five branches of mathematical studies are distinguished: arithmetic (*Republic,* 525 a -526 c), geometry (526 c -527 c), stereometry (solid geometry) (528 a-d), astronomy (528 d-530c) and harmonics (530d -531c). The study of the mathematical sciences is of particular importance because they constitute the preparatory courses for the highest course of all, which is dialectics (*Republic*, 531d-532e). Through dialectics, the guardians will ascend to an understanding of the immaterial entities which Plato calls forms. The category of music education of the second stage, "*is closely connected with the work of contemporary Pythagoreans, deals with the abstract analysis of*

musical structures, and sets out a programme for harmonics as a mathematical science" (Barker, 1984, p. 124).

The content of the ancient Greek science of harmonics examines issues concerning the mathematical relationships of the numbers (Rusk, 1967). In antiquity, two traditions of people existed who were occupied with the science of harmonics. Those who were occupied in an empirical way (empirical harmonicists) and those facing the mathematical point of view (mathematical harmonicists) (Burkert, 1972). The basic belief of the former is that the principles governing musical relationships are autonomous, while of the latter that, as they are not autonomous, they are subject to mathematical theories which are understood through numerical ratios (Crickmore, 2003). Plato's treatment of harmonics belongs to the second category, to which the Pythagorean tradition also belongs. These theories are mainly developed in the works of Philolaus (Diels Kranz i 44, B6=Stobaios) and Archytas (Diels Kranz i 47 B1). This tradition will later become the rival school of the empirical harmonicists of Aristoxenus (Burkert, 1972). Aristoxenus in his *Harmonic elements* "*claims that harmonic properties such as consonance are firstly subjects of experience by a musically trained ear and cannot be traced to numerical ratios*" (Isola, 2016, p. 222).

Plato's concern with harmonics is correlated with the belief that harmony and music are closely related to the order of the world. In *Cratylo* (404e-406a), Plato is presenting the idea that musicians and astronomers believe that Apollo, called God of harmony, makes all things move along by a harmonic force, whether it has to do with harmony or with music chords or with poles of heaven**.** Moreover, the significance given by Plato in harmonious relations takes place in the construction of the universe that is ordered musically, as presented by the myth of Hr, in the tenth book of his *Republic* (616d 6-617b 7) and in the construction of the world soul with musical proportions, as described in the *Timaeus* (34b-36d). However, the detailed presentation of these issues which belong to Plato's engagement with music concerning the harmonic structures in relation to human soul and the universe, goes beyond the purpose of this paper which is focusing on the educational role of harmonics as a mathematical science in Plato's *Republic*.

Before thoroughly examining the position of mathematical sciences in Platonic philosophy and the program of the five mathematical courses, of which harmonics was an integral part, a brief reference to the theory of forms and the determination of the Form of Good is considered necessary since the understanding of the later was the objective of the whole Plato's educational program.

2. The Theory of Forms in Plato's philosophy. The determination of the Form of Good through the Divided Line

Plato presents education which aims at the moral formation of future philosophers-rulers and the cultivation of their intellectual powers. Plato believed that human beings have the power to learn within their soul, in contrast to some others, meaning rather the sophists, who believed that they put science in an empty soul, as they put vision in the blind persons (*Republic,* 518 b-c). Thus, according to Plato, the purpose of education is not to put the power to see in the soul (*Republic,* 518 d), because it has it, but the conversion

of the soul towards the view of its brightest being, the embracement of the Good (*Republic* 518 c-d), or in terms of Platonic philosophy, the sight of the Form of Good (*Republic*, 505 a). In order to approach Platonic philosophy in depth and understand the theory that Plato develops about the citizen in the *Republic* and the *Laws*, it is necessary to understand the theory of Forms, since this is the focus of Platonic thought.

According to Plato, form is called the essence of things. "*The forms are not first principles deducible-and thus definable-through logic. Rather, they are higher realities grasped by intuition*" (Geddert, 2021, p. 53). All perceptible objects are considered as imitation of forms since for each object that belongs to the visible world there is a corresponding idea in the intelligible world (Ross, 1971). Forms are ''*immaterial entities*'' and constitute ''*the objects of that true knowledge which the mind can grasp quite independently of perceptual representations, and of which the perceived properties of everyday things are imperfect reflections in a misleading medium*'' (Barker, 1989, p. 53). As Plato writes in *Phaedo* (78 d-e) beautiful things, beautiful people, beautiful horses, and beautiful clothes are far from the form of the Beautiful since they are nothing but imitations of it. According to the theory of forms, ''*individual men are instances of the Form of man. Political states can thereby also have Forms[..]. These were all imperfect, hence none could be called a just society*'' (Bowen, 1972, p. 103).

The knowledge of the Form of the Good, as well as the other values will lead man to the true being since the Form of the Good is ''*the pinnacle of philosophical inquiry*'' (Pappas, 1998, p. 117). Real knowledge can be attained only with forms that are immaterial entities. Therefore, the purpose of science must be the investigation and understanding of these forms (Sarton, 1980). What Plato is most interested in for the future philosopher-rulers of his ideal State is the understanding of the supreme Idea, the Form of the Good, which nature determines of all other forms. The future philosopher, after having started with education in music and gymnastics, is led to the form of the Good, with the contribution of the mathematical sciences (arithmetic, geometry, stereometry, astronomy, harmonics) and then with dialectics, the highest of all the courses.

The position of Forms in Plato's philosophy is better understood through the Divided Line, where the mental stages and the levels of knowledge are determined (Benson, 2010; Foley, 2008; Mueller, 2005). Through the Divided Line Plato ''*arranges the entire structure into a path toward the Form of the Good*'' (Pappas, 1998, p. 139). Plato defines a divided line as that which intersects in two unequal segments. Then, each segment is again divided into two unequal subsegments. The first segment of the line corresponds to the visible world, the world of opinion. The lower subsegment of the first segment corresponds to the lowest level of knowledge, and it includes the shadows and the reflections of objects. It is the mental stage of imagination. The upper subsegment of the first segment represents the physical objects of the reflections and shadows (plants, animals, artifacts). It is the mental state of trust *(Republic,* 509 d-510 a).

The second segment of the line corresponds to the intelligible world, to the world of Knowledge. The lower subsegment represents the mathematical objects. It is the mental stage of thought. The upper subsegment of the second segment, but also the upper

subsegment of all the subsegments, corresponds to the Forms. It is the mental stage of the intellection (*Republic*, 510b-511d).

In Figure 1, the two worlds are described, the intelligible and the visible, the mental stages, and the objects of each of them according to the divided line.

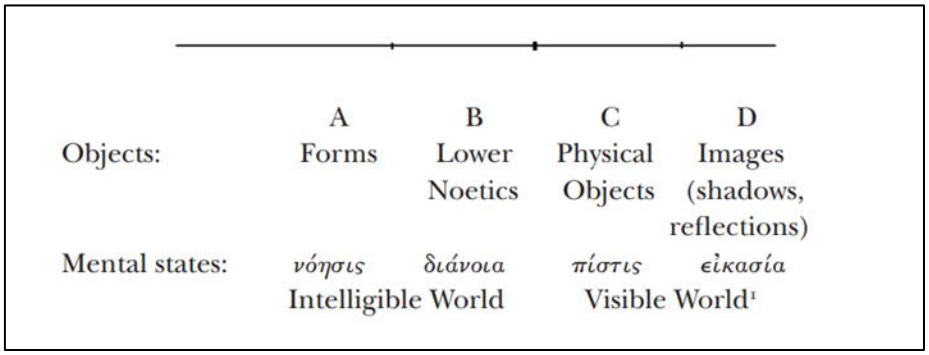

Figure 1: The divided line in Plato's Philosophy (Foley, 2008, p. 1)

With the Divided Line, ''*the relations among the Form of the Good, all other Forms, and the objects of the visible world*'', become apparent (Pappas, 1998, p. 117). Plato's conception of the Divided Line demonstrates, on the one hand, the continuity between the two worlds, the visible and the intelligible since it is a line, but at the same time, the fact that the two worlds are separate since the line is divided (Pappas, 1998).

3. Mathematical sciences in Plato's thought

Plato founded the Academy in 385 BC. His interest in mathematical sciences became evident at the time he started traveling. He had visited Egypt, Cyrene, together with Theodore, Plato's teacher in mathematics, but also Sicily, at the invitation of the tyrant Dion around 388 BC (Gow, 1968), where he became friends with Archytas and Timaeus of Locri. At this time, he had begun to write dialogues in a different style, a style that betrayed the fact that his interests were turning in a new direction that of the organization of knowledge in a scientific corpus. He considered, therefore, that the mathematical sciences cover his quest, as they comprise pure science and can lead to true knowledge (Bowen, 1972), or in other words they can ''*draw the soul away from the world of becoming to the world of being*'' (*Republic,* 521d). Therefore, searching for the path that will lead to knowledge and with the help of mathematics, Plato began to turn away from the sensible objects towards the intelligible things that can be understood through the mind. What draws Plato on the mathematical sciences is not only their content itself, but the philosophy of mathematics and its contribution to ''*the right way of thinking and the conception of mental processes*'' (Gow, 1968, p. 174).

The study of the mathematical sciences in Plato's educational system occupies a prominent place as it is included in that stage of education which helps the soul to turn towards true philosophy (*Republic,* 521c). Plato believed that the path to the world of Forms goes through mathematics. The philosopher and Plato's student, Aristotle, in his work *Metaphysics* writes that for Plato:

> ''*besides sensible things and the Forms there exists an intermediate class, the objects of mathematics, which differ from sensible things in being eternal and immutable, and from the Forms in that there are many similar objects of mathematics, whereas each Form is itself unique.*'' (987 b 14-18)

The main object of mathematical sciences, according to Plato, is to teach people how to think and to help in the exercise of abstract thought. The significance of mathematics in Plato's educational system is highlighted by the fact that someone who did not know mathematics will not be accepted in the Academy (*Academic Index*, p.15). On a sign at the Academy entrance, it was written that ''*those who do not know geometry are forbidden to enter*''. Diogenes Laertius (iv 10) also mentions that Xenocrates, philosopher, and student of Plato, did not accept a young man to the Academy, telling him that he had no basis for philosophy, since he did not know music, geometry, and astronomy. Another fact that shows the important position of the mathematical sciences for Plato, is the succession of Plato in the Academy by Speusippus (Fowler, 1999), not only because he was his nephew, but because he excelled at mathematical knowledge. By Plato, ''*the first serious attempt at a philosophy of mathematics*'' was performed (Heath, 1981, p. 288).

4. Harmonics as the fifth course in the program of the mathematical sciences: arithmetic, geometry, stereometry, astronomy, and harmonics

The five mathematical courses (arithmetic, *Republic*, 525 a -526 c, geometry, 526 c -527 c, stereometry, 528 a-d, astronomy, 528 d-530c and harmonics, 530d -531c) targeted the most capable young people (*Republic*, 526 c, 535 b), who are to become the best guardians of the Ideal State (Mourelatos, 1980). These courses are preparatory of dialectics which is the supreme course, as it leads to the Knowledge of the Good (Burnyeat, 2000), which is essential for the future philosopher-rulers.

Plato, on his mathematical sciences program, seems to be influenced by the Pythagoreans. The Pythagorean quadrivium includes arithmetic, geometry, sphaeric, and harmony. Archytas was the first of the Pythagoreans who dealt with these subjects. In the passage Diels Kranz 47 B1, it is stated:

> ''*The mathematicians («τοὶ περὶ τὰ μαθήματα») […] have handed down to us clear knowledge about the speed of the stars, their risings and settings, and about geometry, arithmetic, and sphaeric, and last, not least, about music; for these branches of knowledge-courses (μαθήματα) seem to be sisters.*'' (Heath, 1981, p.11)

Archytas obviously means astronomy with sphaeric, as Heath (1981, pp. 11-12) also supports: ''*sphaeric being the geometry of the sphere considered solely with reference to the problems of accounting for the motions of the heavenly bodies*''. The course of music had the content of harmonics (Heath, 1981), a course for which Archytas showed his admiration (Lippman, 1964).

Harmonics was organized as a mathematical science in the 5th century BC along with the other Pythagorean courses. Quadrivium (arithmetic, geometry, music, and astronomy) has its roots in the Hellenic countries, but its success is shown most fully in the West, beginning with Boethius. There is no separate Greek word corresponding to quadrivium. The only related title can be found in the treatise of George Pachymeris (1242-1310) entitled *Compendium of Four Mathematics.* In the last work, music is directly connected to mathematics (Sarton, 1980). Quadrivium (arithmetic, geometry, music, and astronomy) and trivium (grammar, logic, and rhetoric) constituted the seven 'liberal arts' (Isola, 2016, p. 225).

Plato, as a general criterion for the suitability of any science that the future philosopher will be taught, sets nothing else but compelling the soul to contemplate essence, and not genesis (*Republic*, 526 e). Thus, within the aforementioned context, the essence of the first course, arithmetic, is not only its use for practical reasons, but mainly to ''*directs the soul upward and compels it to discourse about pure numbers*'', away from visible and tangible bodies (*Republic*, 525 d). Above all, their knowledge is necessary for personal cultivation and the promotion of human nature (*Republic,* 522 e), but also the training of even the slowest minds, giving their spirits greater sharpness than they have by nature (*Republic*, 526 b).

In the same context, the study of geometry is being understood, which is considered useful for its practical applications such as to the conduct of the war (*Republic,* 526 d). It is also observed that those who have studied geometry perform better in other subjects (*Republic,* 527 c), an ascertainment that also applies for Plato to all mathematical sciences in general. However, geometry's true meaning is that it would:

> ''*tend to draw the soul to truth and would be productive of a philosophic attitude of mind, directing upward the faculties that now wrongly are turned earthward.*'' (Republic, 527 b 7-9)

The third course is stereometry (*Republic*, 528 a-d), or solid geometry which is a new science and should be placed before astronomy. In this way, Plato deals first with the solids themselves and then with the solids in motion. Rusk states (1967, p. 24) that Plato does not deal with the science of stereometry in detail, because he might not have had enough knowledge of this new mathematical branch. However, the addition of stereometry offers a dynamic evolution in the standard of the Pythagorean quadrivium (Lippman, 1964).

The next course is the science of astronomy. As Plato states:

> ''*it is obvious to everybody, that this study certainly compels the soul to look upward and leads it away from things here to those higher things*'' (*Republic*, 529 a 1-3).

In astronomy, we should act as in the study of geometry:

> ''*we will let be the things in the heavens if we are to have a part in the true science of astronomy and so convert to right use from uselessness that natural indwelling intelligence of the soul*'' (*Republic*, 530 c 1-3).

Harmonics completes the mathematical program of five courses. Harmonics and astronomy, according to Plato ''*are in some sort of kindred sciences*'' (*Republic*, 530 d 7-8). The reason is that ''*as the eyes are framed for astronomy, so the ears are framed for the movements of harmony''* (*Republic*, 530 d 5-6). The prerequisites that were set for the other sciences, which are not to deal only with the practical side of science, work for harmonics as well. Plato considers that Pythagoreans are suitable to discuss about harmony as their theories are dealing with audible sounds and giving priority to the hearing rather than the intellect since:

> ''*their method exactly corresponds to that of the astronomer: for the numbers they seek are those found in these heard concords, but they do not ascend to generalized problems and the consideration which numbers are inherently concordant and which not and why in each case*'' (*Republic*, 531 c 1-6).

In the work of Archytas (Diels Kranz i 47 B1), to which Plato is probably referring, his criticism does not focus on his views on harmonics, but on the fact that he looks for numerical ratios in audible sounds (Burnyeat, 2000; Robins, 1995), while Plato seeks for them in a more abstract level that will demonstrate values such as the beautiful and the good. This is actually the challenge that Plato poses to harmonic scientists, to go beyond sensible experience. Plato, therefore, advocates that auditory ability cannot be enough to determine the correctness of musical relationships. The ascertainment of the relationships of musical intervals may begin with the sense of hearing, but their study should be the work of the intellect alone. However, Plato's criticism of Pythagoreans does not mean disrespect of their theories. He considers them as specialists, but he believes that he should be differentiated from them, even if he appreciates their work (Barker, 1989).

Dialectics follows the study of the guardians in the mathematical sciences. Plato defines ''*the man who is able to exact an account of the essence of each thing*'' (*Republic,* 534 b 3-4) "*as dialectician, and he sets dialectics above all other studies to be as it were the copingstone and that no other higher kind of study could rightly be placed above it''* (*Republic*, 534 e1-5). However, dialectics can offer only to the mind which has been perfectly prepared by the study of the previous five mathematical sciences. Moreover, it is also important the mathematical courses which are ''*the preliminary studies that are indispensable preparation for dialectics*'' should not be in the form of compulsory instruction (*Republic,* 536 d) *''as nothing that is learned under compulsion stays with the mind*'' (*Republic,* 536 e).

Mathematical studies began in the twenty-first year and lasted ten years. They were followed by a five-year study in dialectics from thirty to thirty-five and were fulfilled with practical training in military and civil posts (*Republic*, 539c-540a) that lasted fifteen years (Bowen, 1972; Rusk, 1967). ''*Only at the age of fifty are they brought to a vision*

of the Form of the Good, and once they have seen that they divide their time between philosophy of the highest order and government at the highest rank'' (Pappas, 1998, p. 120).

Taylor (1990) states that Plato refers to the mathematical sciences, as they were the only ones systematically developed branches of the fifth century. That's why dialectics was necessary as there was a need ''*to existing knowledge to stand in more stable ground*'' (Taylor, 1990, p. 339). However, although mathematics held such an important position in Classical Greece, in the Hellenistic period, its prominence is limited by literature studies, as great importance is given to rhetoric, the study of which did not require the teaching of mathematics. It is certain that an important reason for that was the need of the people of the Hellenistic period who spoke the Attic Greek, to receive some help in daily practice and, naturally, this help concerning the language, would come from literature studies and not from mathematics (Giannikopoulos, 1989).

5. Conclusion

The term "harmonics" in Plato's *Republic* needs to be read in the context of Plato's philosophy. This context is the theory of the Divided Line, where it is argued that the world can be divided into the visible and the intelligible, but there is a continuum in it, in which the varying representations of the absolute Form appear. The ontological coherence of all levels of being makes possible the participation of harmonics on absolute Forms.

Harmonics has an equal, if not superior position, among the other mathematical sciences. They are all connected with each other in terms of the result achieved by their study, that is, the conversion of the soul from the visible world to the intelligible world. This happens due to their mathematical way of thinking and their intelligible character, excluding their practical use and their involvement with the empirical objects. For Plato, the aim of harmonics, is not the analysis of musical systems used in practice but, as with other mathematical sciences, its aim should be to overcome the handling of its use in everyday practice and to come to the revelation of truth, beyond our perception. The placement of harmonics at the bottom line of the mathematical sciences is perhaps not accidental, as the subject of harmonics might be considered that it is more general than the others and can better explain the nature of mathematics. Even dialectics, the superior of all studies, could be described as the one seeking the harmony of man, and the harmony of the state and of the world, as well (Lippman, 1964).

Conflict of Interest Statement
The author declares no conflicts of interest.

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Dr. Eirini Nikolaou graduated as a musicologist from the Department of Music Studies (National and Kapodistrian University of Athens). She received the Msc degree and PhD degree from the Department of Philosophy-Pedagogy-Psychology of Ioannina University (Greece). Furthermore, she has a Diploma as a Piano soloist and Composer, where she

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