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LIMITATIONS OF SECONDARY SCHOOL STUDENTS IN SOLVING A TYPE OF TASK RELATING TO THE EQUATION OF A CIRCLE: AN INVESTIGATION IN VIETNAM

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Abstract:
In Vietnam, secondary school students learn the equation of a circle in Grade 10. Based on how to present this equation in the textbook "Geometry 10" and types of task for students, we believe that some limitations happen to students when they solve problems related to the equation of a circle. This paper reports the investigation of 845 students from the Mekong Delta-Vietnam. The results show that our prediction is correct.

Keywords: the equation of circle, type of task, errors in solving problem, mathematics education

1. Theoretical background

1.1 A mathematics task

The definition of mathematical task is a classroom activity, project, question, problem, construction, application or exercise in which the purpose is to focus the students' learning on a specific mathematical concept (Cai et al., 2010; Stein et al., 1996). According to Gahamanyi, a mathematical task can be viewed, in general terms, as any piece of mathematical work to be done by an individual or a group. In mathematics education, especially in teaching-learning context, a mathematical task normally refers to mathematical work or problems that are assigned to students, teachers or other concerned people (such as parents and mathematics curriculum makers) to be performed for the purpose of societal knowledge development in the subject of mathematics. Chevallard (1999) considered a mathematical activity as a human activity situated in an institutional setting and any mathematics activity can be subsumed as a system of tasks.

1.2 The role of mathematics task

Mathematical tasks shape what students learn, how students think about mathematics, how students develop their understanding of mathematical concepts, and how students make sense of mathematics. A task is represented in multiple ways, including as it appears in the curriculum, how a teacher adapts the task and how the task is carried out during instruction (Stein et al., 1996). Mathematical tasks provide the contexts in which they learn to think about mathematics, and different tasks may place differing cognitive demands on students (Henningsen & M. K. Stein, 1997).

2. The equation of a circle in "Geometry 10" of Vietnam

2.1 How to introduce "The equation of a circle" in "Geometry 10" of Vietnam

The equation of a circle

In the textbook "Geometry 10" (Hao et al 2007), the equation of a circle is presented in two forms:

The first form: In the Oxy, given the circle (C) with center $I(a;b)$ and radius R. The equation of (C) is: $(x-a)^2+(y-b)^2=R^2$ (1).

The second form: $x^2+y^2-2ax-2by+c=0$ is the equation of a circle with center $I(a;\text{b})$ and radius $R=\sqrt{a^2+b^2-c} \Leftrightarrow a^2+b^2-c>0$.

2.2 Types of related task

Basing on exercises in the Textbook "Geometry 10", we generalized classified them into types of tasks as follows:

A. Type 1: Verify whether a quadratic equation $x^2+y^2-2ax-2by+c=0$ (1) is the circle equation or not?

Technique for solving:

- Calculating: $a^2 + b^2 - c$.
- Basing the sign of $a^2 + b^2 - c$ to reach conclusion.

If $a^2+b^2-c>0$, (1) is the equation of a circle with center I(a; b) and radius.

If $a^2+b^2-c\le 0$, (1) is not the equation of any circle.

B. Type 2: Verify whether quadratic equation $kx^2 + ky^2 - 2ax - 2by + c = 0$ (2) is the circle equation or not?

Technique for solving:

- Converting Type 2 to Type 1 by diving both sides of (2) by *k*: $x^2 + y^2 - 2\frac{a}{k}x - 2\frac{b}{k}y + \frac{c}{k} = 0$.

C. Type 3: Find values of a real parameter m in order that $x^2 + y^2 + px + qy + r = 0$ (3) becomes an equation of a circle

Technique for solving:

- Building the inequation: $\left(\frac{p}{-2}\right)^2 + \left(\frac{q}{-2}\right)^2 - r > 0$.

- Finding m by solving the inequation $\left(\frac{p}{-2}\right)^2 + \left(\frac{q}{-2}\right)^2 - r > 0$.

D. Type 4: Find coordinates of center and radius of a circle (C): $x^2 + y^2 + px + qy + r = 0$

Technique for solving:

- Determining the coordinates of the center Iof (C): $I\left(\frac{p}{-2}, \frac{q}{-2}\right)$.

- Computing the radius of (C): $R = \sqrt{\left(\frac{p}{-2}\right)^2 + \left(\frac{q}{-2}\right)^2 - r}$.

E. Type 5: Find coordinates of center and radius of a circle (C): $\alpha x^2 + \alpha y^2 + px + qy + r = 0$**.**

Technique for solving:

- Dividing both sides of (4) by α : $x^2 + y^2 + \frac{p}{\alpha}x + \frac{q}{\alpha}y + \frac{r}{\alpha} = 0$.

- (C) has the center $I\left(\frac{p}{-2\alpha}, \frac{q}{-2\alpha}\right)$, and the radius is $R = \sqrt{\left(\frac{p}{-2\alpha}\right)^2 + \left(\frac{q}{-2\alpha}\right)^2 - \frac{r}{\alpha}}$.

F. Type 6: Write the equation of a circle (C) with center I (a; b) and through a point A (x_A; y_A)

Technique for solving:

- Computing the radius of the circle $R = IA = \sqrt{(x_A - a)^2 + (y_A - b)^2}$.

- The equation of C) is: $(x-a)^2 + (y-b)^2 = R^2$.

G. Type 7 : Given A (x_A; y_A) and B (x_B; y_B). Write the equation of a circle (C) with diameter AB.

Technique for solving:

- Determining the coordinates of center of (C): $I(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2})$.
- Computing the radius of (C): $R = \frac{AB}{2} = \frac{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}{2}$.
- The equation of (C) is: $(x - \frac{x_A + x_B}{2})^2 + (y - \frac{y_A + y_B}{2})^2 = R^2$.

H. Type 8 : Write the equation of a circle (C) tangent to the straight line $\Delta: Ax + By + C = 0 \ (A^2 + B^2 \neq 0)$.

and whose center is I (a; b).

Technique for solving:

- Computing $R = d(I, \Delta) = \left| \frac{A.a + B.b + C}{\sqrt{A^2 + B^2}} \right|$.
- The equation of (C) is the one after plugging the values of a, b, R into the equation: $(x - a)^2 + (y - b)^2 = R^2$.

I. Type 9 : Write the equation of a circle (C) whose center I is on the straight line Δ: $Ax + By + C = 0 \ (A^2 + B^2 \neq 0)$.

and through two points A and B.

Technique for solving:

- General form of (C) is: $x^2 + y^2 - 2ax - 2by + c = 0$.
- Plugging coordinates of A, B into: $x^2 + y^2 - 2ax - 2by + c = 0$ and coordinates of I into: $Ax + By + C = 0 \ (A^2 + B^2 \neq 0)$, we obtain system of 3 equations in three unknowns: a, b, c. Solving this system to find out a, b, c.

J. Type 10: Write the equation of a circle (C) passing through three points A, B and C.

Technique for solving:

- General form of (C): $x^2 + y^2 - 2ax - 2by + c = 0$.
- Plugging coordinates of A, B, C into: $x^2 + y^2 - 2ax - 2by + c = 0$, we obtain system of 3 equations in three unknowns: a, b, c. Solving this system to find a, b, c.

K. Type 11: In coordinate plane Oxy, find a set of points which satisfies property p, where the solution is a circle.

Technique for solving:

- Let M(x; y) be in the set of points to find out.
- Make use of the property p, we can deduce an equation F(x; y) = 0.
- Prove F(x; y) = 0 to be the equation of a circle.

3. Research hypothesis

Basing on how to present the equation of a circle (but not to show in which case an equation $(x-a)^2+(y+b)^2=k$ becomes an equation of a circle) and types of related task as above, we formulated the research hypothesis as follows:

Hypothesis: For the problem "Find the values of real parameter k such that the equation $(x-a)^2+(y+b)^2=k$ are the circle equations", most students will convert it to the form $x^2+y^2-2ax-2by+c=0$ to solve; if not, they will commit errors or not to know how to solve.

4. Methodology

The problem to investigate: In order verify the above hypothesis, we used the following problem: "Find the values of real parameter such that the equation $(x-1)^2+(y-2)^2=m+2$ are the circle equations?"

The investigated students: 845 students from 5 secondary schools in the Mekong Delta – Vietnam.

Time: The investigation was carried out in 2015.

5. The results and discussion

After analyzing the students' answers, we found out three following strategies used:

A. Strategy 1 (correct): $(x-1)^2+(y-2)^2=m+2$ *are equations of circle with center* $I(1,2)$ *and radius* $R=\sqrt{m+2}$ *if and only if* $m+2>0 \Leftrightarrow m>-2$.

B. Strategy 2 (correct): $(x-1)^2+(y-2)^2=m+2 \Leftrightarrow x^2+y^2-2x-4y-m+3=0$ (*)

(*) are the equations of circle $a^2+b^2-c>0 \Leftrightarrow m+2>0 \Leftrightarrow m>-2$ ($a=1,\ b=2,\ c=-m+3$).

C. Strategy 3 (incorrect): $(x-1)^2+(y-2)^2=m+2$ *are equations of circle with center* $I(1,2)$ *and radius* $R=\sqrt{m+2}$ *if and only if* $m+2\geq 0 \Leftrightarrow m\geq -2$. This strategy is not correct because in the case of $m=-2$, $(x-1)^2+(y-2)^2=0$ (3) $\Leftrightarrow \begin{cases} x=1 \\ y=2 \end{cases}$. Therefore, (3) is not an equation of a circle.

Table 1: The results of students' answers

Answers of students	Strategy 1	Strategy 2	Strategy 3	No solution/don't find out any strategy to solve	Sum
The number of students	180 (21.3%)	295 (34. 91%)	89 (10. 53%)	222 (26. 27%)	845 (100%)

Statistical table 1 shows that strategy 2 was dominant strategy where 34.91% of students, namely 295 (out of 845) students, used to solve the problem, which means that they converted the given equation into the form $x^2+y^2-2ax-2by+c=0$, then computed m. Below is the answers presented by students (see Figure 1-2).

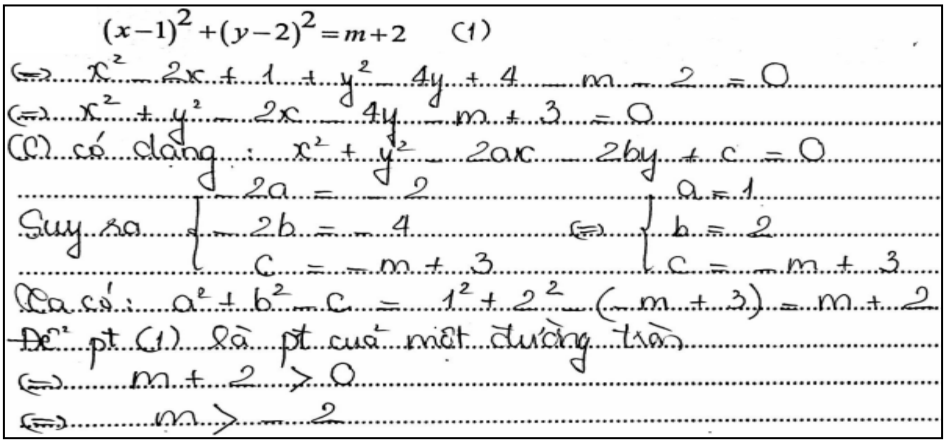
$(x-1)^2+(y-2)^2=m+2$ (1)
$\Leftrightarrow x^2-2x+1+y^2-4y+4-m-2=0$
$\Leftrightarrow x^2+y^2-2x-4y-m+3=0$
(1) có dạng: $x^2+y^2-2ax-2by+c=0$
Suy ra $\begin{cases} -2a=-2 \\ -2b=-4 \\ c=-m+3 \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=2 \\ c=-m+3 \end{cases}$
Ta có: $a^2+b^2-c=1^2+2^2-(-m+3)=m+2$
Để pt (1) là pt của một đường tròn
$\Leftrightarrow m+2>0$
$\Leftrightarrow m>-2$

Figure 1: Written answer of T.M.T

(Class 10, Secondary School "Bình Thủy", Can Tho)

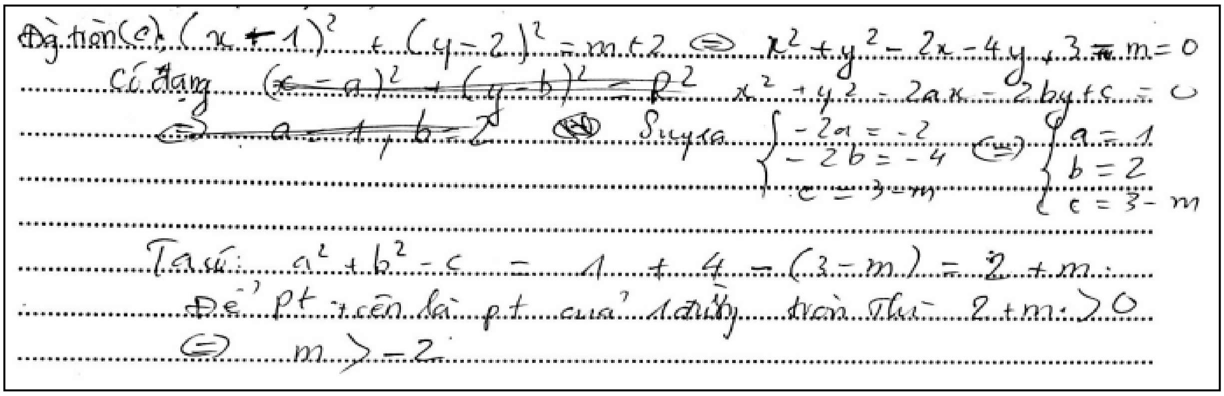
Đg tròn (c): $(x+1)^2+(y-2)^2=m+2 \Leftrightarrow x^2+y^2-2x-4y+3-m=0$
Có dạng $x^2+y^2-2ax-2by+c=0$
Suy ra $\begin{cases} -2a=-2 \\ -2b=-4 \\ c=3-m \end{cases} \Leftrightarrow \begin{cases} a=1 \\ b=2 \\ c=3-m \end{cases}$
Ta có: $a^2+b^2-c=1+4-(3-m)=2+m$
Để pt trên là pt của 1 đường tròn thì $2+m>0$
$\Leftrightarrow m>-2$

Figure 2: Written answers of N.P.N.T
(Class 10, Secondary School "Trương Định", Tien Giang)

Table 1 gave fact that many students (89/845 (10.53%)) used strategy 3 to solve (see Figure3). This strategy led to $m \geq -2$ (not true if $m = -2$).

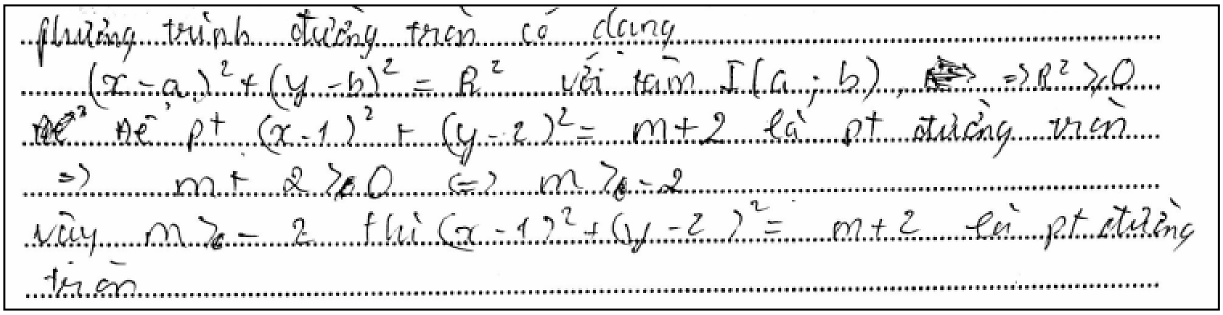

Figure 3: Written answers of T.H.M.
(Class 10, Secondary School "Trương Định", Tien Giang)

Table 1 also showed that 222 (26, 27%) students did not know how to solve the given problem. From the above analysis, the research hypothesis is true. It means that to solve that problem *"Find the values of real parameter such that the equation $(x-1)^2+(y-2)^2=m+2$ are the circle equations?"*, most students will convert it to the form $x^2+y^2-2ax-2by+c=0$ to solve; if not, they will commit errors or not to know how to solve.

6. Conclusion

The results of the above investigation, in our opinion, students made some errors and chose the strategy 2 to solve the given problem (not optimal) because of the following reasons:

1. The teachers depended much on contents of knowledge presented in the textbook. They do not actively complement the knowledge and skills necessary for teaching students in a mathematical knowledge; in this case, the teacher should guides their students how to find values of k such that $(x-a)^2+(y+b)^2=k$ becomes an the equation of a circle.
2. Students often have a habit of doing what the teacher teach and inability to apply flexibly in specific situations.

Therefore, in the process of teaching mathematics, teachers should have pedagogical measures to overcome the above limitations in order to increase effectiveness of their own teaching.

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