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IMPLEMENTATION OF NEW PEDAGOGIES IN ONTARIO CANADA MATHEMATICS TEACHING, 1950 TO 2024

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Abstract:
This paper examines the implementation of a new curriculum and new pedagogies in mathematics education in Ontario between 1950 and 2024. This was a particularly important period for mathematics education, as reform mathematics based on a constructivist paradigm replaced the traditional transmission pedagogy of the behaviorist era. This radical departure from tradition clashed with the personal perceptual fields of many mathematics teachers, which impeded the successful implementation of newer, research-affirmed strategies. The paper examines historical pedagogical changes from 1950 to date and makes recommendations to improve the implementation of new pedagogies in mathematics education going forward.

Keywords: curriculum, pedagogy, Ontario, reform mathematics

1. Introduction

Fidelity of implementation in curriculum is a thorny issue, particularly when implementing new pedagogies. This paper examines issues around the implementation of new pedagogies in mathematics teaching in Ontario between 1950 and 2024. This issue is particularly important to me, as during much of this period, I was either a student, teacher, or educational researcher. The first part of this paper looks at issues around fidelity of implementation of any new curriculum. The second part focuses specifically on implementing new pedagogies in mathematics in Ontario. The period 1950 to 2024 saw tremendous change in pedagogies, frequently based on research around how students learn, but also driven by political and social changes. Quite often, these changes came to Ontario from the United States and other jurisdictions. It is unclear to me why Ontario chose to implement curriculum changes, especially imported from the United States, which has an unimpressive record of mathematics education (OECD, 2024).

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2. Theoretical Framework

Perceptual theory (Combs, 1999) posits that people's perceptions are colored by their previous experiences, prior knowledge, emotions, values, and beliefs. This set of personal characteristics comprise one's perceptual field. The perceptual field influences one's perceptions of reality, of other people, and of oneself. Each person's perceptual field is, perforce, unique. Thus, when one encounters a new experience, it is compared to one's existing perceptual field; the new experience will then be added to the perceptual field, modify one's perceptual field, or be rejected as conflicting with one's existing perceptual field.

Therefore, teachers' perception of curriculum is viewed through the lens of their perceptual field. Their perceptual field, in turn, is based on their beliefs about teaching and learning; their past experiences with teaching and learning; their personal philosophy of teaching; their personal repertoire of instructional strategies and the personal valuation of each strategy; their understanding of students and students' individual needs; and the degree to which the curriculum impacts each teacher's personal perceptual field.

So, if the curriculum does not impact the teacher's personal perceptual field, or impacts it only to a minor degree, there is no impetus for the teacher to assimilate the curriculum and enact it in the classroom, except perhaps in a perfunctory manner to satisfy external oversight of what occurs in the teacher's classroom. For example, a teacher might consider a curriculum expectation as "taught" if the topic is merely mentioned during class. A different teacher's understanding of the same expectation being taught might include multiple and varied activities that not only address the expectation but also deepen students' understanding of the expectation, with opportunities for the students to transfer their understanding of the expectation to new or different situations and demonstrate its use in solving unusual problems. These two very different considerations of "taught" will be based on the two teachers' differing perceptual fields with respect to the curriculum expectation. If a teacher adheres to the "teach as they were taught" paradigm, which in mathematics typically involves Socratic lessons or lectures, asking the teacher to radically change their pedagogy clashes with their personal perceptual field, and frequently results in resistance toward or rejection of the change.

In the same way, each student's own perceptual field influences their perceptions of every lesson they encounter. They may accept, partially accept, or reject the new information, depending on how it compares to their personal perceptual field. Students' perceptual fields will be modified when they perceive value in the activities and have an expectation that they can complete the activities successfully (Eccles & Wigfield, 2024). Therefore, the teacher's knowledge of their students is critical, in that they need to connect subject content to the students' various perceptual fields to both engage the students in the learning and also to ensure that students actually absorb, retain, and understand the subject matter being taught.

Perceptual theory also explains the findings of Rogers's (2003) research on the diffusion of innovations, which show that the number of adopters of a new innovation follows an "S"-shaped curve: Initially, there are a small number of innovators, who created the innovation; then, a larger (though still relatively small) number of early adopters take up the innovation, followed by a still greater number of adopters once the innovation is seen to be working. This produces a dramatic increase in the total number of adopters. Later, a relatively smaller number of late adopters accept the innovation. Penetration will never reach 100%, since there will always be resistors to the innovation.

Perceptual theory explains this pattern. Early adopters are generally risk-takers, for whom the innovation is seen as valuable, and congruent with their perceptual fields. As the early adopters demonstrate the usefulness of the innovation, more adopters accept the innovation and modify their perceptual fields to agree with it. Once there are many adopters, the late adopters, having seen evidence of the innovation's feasibility, modify their perceptual fields, accept the innovation, and begin to use it. This pattern can be seen in the acceptance of new pedagogies in education. Early adopters of a new pedagogy are the innovators and risk-takers. After other teachers see evidence of the efficacy of the new pedagogy, they are willing to modify their perceptual fields and try the new methods. Once there is evidence of the new pedagogy's efficacy, the late adopters come on board, to varying degrees. Of course, especially in mathematics teachers, some teachers refuse to embrace the new pedagogy and continue to teach the way they were taught. It is therefore clear that for a new pedagogy to be successfully implemented, teachers need to see evidence of the efficacy of the pedagogy, sufficient for them to modify their personal perceptual fields such that the new pedagogy becomes congruent with their perceptual fields and thus a reasonable strategy to attempt.

3. Research Questions

This study sought to answer the following research questions:

1) What has been the level of fidelity of implementation of curricula in Ontario and what have been impediments to its implementation?
2) How have new pedagogies in mathematics teaching been implemented over the period 1950 to 2024?
3) What professional learning for new pedagogies in mathematics teaching has been provided during that period?
4) What can be done to improve the fidelity of implementation and implementation of new pedagogies in mathematics education in Ontario?

4. Review of the Literature

This literature review focuses on mathematics curriculum and associated pedagogy. It also includes non-mathematical curriculum where appropriate; implementation of new pedagogy, especially in mathematics; and professional development (PD) initiatives associated with new curriculum.

4.1 New Curriculum Implementation

Reys and Reys (2011) point out that curriculum is more easily changed than teacher behaviors and is thus more politically expedient; as they put it, "*While most experts agree that teaching is more directly related to student learning, the curriculum is tangible and changeable—more easily legislated and governed than teaching*" (p. 101). Curriculum changes may be influenced by historical, social, political, and international factors, such as large-scale assessments like PISA (Bobis *et al.*, 2021; OECD, 2023; Popkewitz, 1988). By far the greatest influence on curriculum change should be research evidence (Osta *et al.*, 2023; Perell *et al.*, 2017). For example, Grouws *et al.* (2013) used hierarchical linear modeling to show that an integrated curriculum was superior to a subject-specific curriculum with respect to student achievement. Further, research demonstrated that dimensions such as student motivation and growth mindset should be considered in formulating curriculum (Perell *et al.*, 2017).

Linares *et al.* (2018) identify three levels of curriculum implementation: intended curriculum (system level), enacted curriculum (teacher level), and learned curriculum (student level). Similarly, Burton (2003) identifies three phases of curriculum implementation: formal, planned, and learned. In reality, there are many more levels involved, as discussed in Section 6: The Curriculum Telephone Game. At every interface between levels there is the opportunity for reduced fidelity of implementation (noise). For example, simply translating the intended curriculum into words (the written curriculum) introduces complications around language and nuances (Popkewitz, 1988).

4.2 Implementation of New Curriculum in Mathematics

Advances in learning theory resulted in a shift from a behaviorist to a constructivist paradigm, resulting in a rethinking of teachers' roles (Meader, 1995) and a focus on student-centered learning, in which students construct their own knowledge. In mathematics, this shift fostered dramatic changes in pedagogy and a revised curriculum, known collectively as *reform mathematics*. Reform mathematics involves active rather than passive student participation; the appropriate use of manipulatives and technology; making real-world connections for students; and supporting student autonomy through groups and choice. Such strategies were shown to increase and sustain student engagement (Moyer *et al.*, 2018; Smith & Star, 2007). The reform mathematics curriculum focused on four tenets (Meader, 1995):

- A focus on students' complex thinking,
- A change of the focus of authority in classrooms,
- A change in student roles,
- An emphasis on ongoing professional learning by teachers.

All of these tenets were dramatically different from traditional transmission teaching; they typically were a mismatch for many teachers' perceptual fields and their views of what constituted effective teaching in mathematics. As noted earlier, the new curriculum was influenced by historical, social, cultural, and international factors (Alper *et al.*, 1996; Popkewitz, 1988). In many cases, traditional views of mathematics curriculum and mathematics pedagogy were a major factor. This frequently made implementation of the new curriculum difficult.

The reform mathematics curriculum took different emphases depending on the overall educational goals of the country. For example, For example, China and Lebanon featured highly centralized education systems; Chile focused on quality education for all; South Africa's system emphasized anti-racist education; Denmark placed a heavy emphasis on mathematical competence; France's major goal was statistical competence; the Philippines focused on a curriculum and the necessary PD to implement it (Linares *et al.*, 2018; Osta *et al.*, 2023; Ruiz *et al.*, 2023; Spyker & Malone, 1993; Wilson & Goldenberg, 1998). Australia implemented a relatively pure version of reform mathematics (Grouws *et al.*, 2013). In the United States, the curriculum was heavily tied to textbooks, to the extent that many educators saw the mathematics courses and the textbooks as synonymous (Remillard, 1999; Tarr *et al.*, 2006).

Teachers were central to the successful implementation of the reform curriculum (Linares *et al.*, 2018; Spyker & Malone, 1993). There is a long tradition of teachers developing resources for their classes (Linares *et al.*, 2018) and adapting curriculum to suit their specific students (Drake & Sherin, 2006; Sherin & Drake, 2009). These decisions were (as expected) based on the teachers' explicit and implicit beliefs about teaching and learning (Remillard, 1999)—that is, the teachers' perceptual fields.

Abundant research has shown that the implementation of new pedagogies in mathematics is dramatically influenced by teachers' perceptual fields (Bobis *et al.*, 2021; Borko *et al.*, 2000; Eccles & Wigfield, 2024; Linares *et al.*, 2018; Remillard, 1999; Remillard & Bryans, 2004), although often not described using the language of perceptual theory (Remillard, 1999). Tyminski *et al.* (2010) provide an excellent example of an experienced mathematics department struggling with the new pedagogies of reform mathematics: some teachers attempt to embrace the changes, others attempt to implement portions of the new pedagogies, and still others totally reject the new pedagogies and maintain their traditional transmission-style teaching. Yet, all the teachers make their decisions based on their personal perceptions of the role of the teacher and their personal perceptual fields. Wilson and Goldenberg (1998) chronicled the dramatic clash between a veteran middle school mathematics teacher's personal perceptual field and the new pedagogies required in reform mathematics principles; the educator's views on the role of the teacher prevented him from enacting some of the most important pedagogies of reform mathematics despite his good-faith efforts over a period of 2 years.

This phenomenon is not limited to Ontario, Canada. Spyker and Malone (1993) describe the same barriers experienced in attempting to introduce new pedagogies in mathematics in Australia. Mathematics teaching in the United States is rife with similar attempts to increase the fidelity of the implementation of new pedagogies through a heavy reliance on textbooks (Grouws *et al.*, 2013; Rampal *et al.*, 2017; Tarr *et al.*, 2006). However, Tarr *et al.* (2006) point out that some teachers, based on their personal perceptual fields, will follow the textbook closely, some will adapt pedagogy to their own classes, and still others will throw out the textbook completely and revert to their traditional methods. Baron (2015) in turn emphasizes the need for teachers' reflective practice (Schön, 1983) because they need to see examples of successful implementation of the new pedagogies before modifying their personal perceptual fields and adopting new practices professionally.

Ontario supported the implementation of the new pedagogies in two ways. First, the pedagogy was explicitly contained in the specific content expectations (OME, 1999, 2000). Second, an extensive array of professional learning supports was produced, consisting of lesson plans, exemplars, suggested teaching strategies, ongoing workshops, and a website devoted to professional learning. However, as with other jurisdictions, teachers made decisions individually on what to implement based on their own perceptual fields. Table 1 outlines various teaching styles that have been identified in the literature.

Many mathematics teachers favor traditional teaching styles, which are congruent with a transmission pedagogy based on a behavioral paradigm. However, reform mathematics is based on a constructivist paradigm (Smith & Star, 2007) in which traditional teaching styles are mismatched. Reform mathematics requires teachers to utilize non-traditional teaching styles as teachers' roles move to a more student-centered environment.

Enacting an inquiry pedagogy is difficult and possibly requires significant time to implement. There is a long tradition of teachers creating resources when implementing new curriculum (Z1) and frequently there may be some loss of fidelity with these newly created resources since their creation will be heavily influenced by teachers' personal perceptual fields.

Table 1: Teaching styles

Category	Examples
Traditional	Command Practice Task assignment
Participatory	Small groups Reciprocal teaching Microteaching
Individualization	Individualization by groups Modular teaching Programmed teaching Individualized programs Self-evaluation Self-teaching Learner-initiated
Student cognition	Guided discovery Problem solving Divergent discovery
Socialization	Social involvement Cooperative learning
Creativity	Free exploration Problem posing

Note: Based on Fernandez-Rivas and Espada-Mateos (2019).

4.3 Professional Development

As reform mathematics curricula were implemented worldwide, the need for effective PD was clear (H, YEAR; T, YEAR). In the United States, PD revolved around textbooks that illustrated reform mathematics principles and pedagogy (Grouws *et al.*, 2013) This strategy was relatively ineffective because the new pedagogies clashed with teachers' perceptual fields and did not reflect teachers' tacit understanding of what constituted effective teaching (Linares *et al.*, 2018).

Baron (2015) emphasized the need to change teacher beliefs before attempting to change teacher behaviors. Beliefs are a part of teachers' perceptual fields, along with emotions and previous experiences, so a dramatic modification of teachers' perceptual fields was needed. Baron proposed PD that focused on modifying teacher beliefs, with a heuristic of enlightenment, empowerment, and emancipation: Enlightenment involves making beliefs visible since many parts of teachers' perceptual fields are tacit and comprise invisible barriers to change; empowerment honors teachers' beliefs and clarifies that the new pedagogies are (at least partially) congruent with those beliefs that are congruent with student success; and emancipation—the realization that the new pedagogies are good for students and worth trying.

Remillard (1999) proposed PD corresponding to collaborative inquiry, recognizing the centrality of teachers in implementing any new curriculum or pedagogy. Since reform mathematics was a dramatic new paradigm in pedagogy, collaboration with other teachers was seen as critical in successfully implementing the new pedagogy.

Several researchers recognized the need for job-embedded professional learning (Brown *et al.*, 1996; Gravemaijer & Rampal, 2015), given teachers' "patchy" understanding of the new pedagogies, which required a shift to student inquiry as opposed to traditional transmission methods (Gravemaijer & Rampal, 2015). PD initiatives frequently had limited success; for instance, Boeson *et al.* (2014) found that 5 years after initiating a new mathematics curriculum in Sweden, less than one-fifth of teachers were consistently using new pedagogies specified in the new curriculum.

Ontario undertook an extensive job-embedded PD program (Irvine & Telford, 2015) combined with online PD workshops and a website (EduGAINS) with detailed lesson plans for some mathematics courses and additional resources. Because teachers function in a professional bureaucracy, with top-down guidance but extensive bottom-up independence in implementation (Mintzberg, 1989), providing examples of successful implementation of new curricula was both appropriate and needed to effect changes in teachers' perceptual fields (Remillard & Bryans, 2004).

5. Methodology and Method

This analysis employed literature reviews and document analysis to examine how the implementation of new pedagogies in mathematics education occurred. The analysis was also informed by the author's 50 years of experience in education as a teacher, department head, administrator, provincial education officer, professor, and educational researcher.

The analysis was impeded by several factors. First, there is a paucity of Ontario-based or even Canadian-based research in this area. Secondly, many historical curriculum documents are difficult to access; the only curriculum documents accessible online are the current versions of curricula. When I contacted the Ontario Ministry of Education (OME), I was surprised to be told that it retains only current versions of curriculum documents, even in the OME library. The Ontario Institute for Studies in Education (OISE) library has some limited curriculum documents. The library at Western University (UWO) has a more extensive collection in its reference section, but it too is incomplete.

6. Fidelity of Curriculum Implementation: The Curriculum Telephone Game

The curriculum of a jurisdiction constitutes the skills and knowledge that students are expected to learn. It may also include requirements for pedagogy and assessment of these skills and knowledge, as well as overarching principles reflecting "enduring understandings" for students. There are several incarnations of curriculum. The first is the *intended curriculum*, usually designed by a school district, state, or province, and sometimes nationally. The intended curriculum, framed by policymakers and curriculum experts, typically reflects the jurisdiction's educational philosophy as well as current research, political goals related to education, and societal goals such as good citizenship, respect for others, and fostering personal goals of lifelong learning. In Ontario, the intended curriculum is designed at the provincial level.

The next level of curriculum is the *written curriculum*—the setting down in words of the intended curriculum. The written curriculum, a carefully worded document, is the basis on which educators determine what is expected to take place in classrooms. When the written curriculum is disseminated to districts, school boards, and schools, educators translate it into actionable expectations for teachers. This *interpreted curriculum* may be produced at the individual classroom teacher level, but often the interpretation occurs at the district or school board level, with intermediary curriculum consultants, and it may also occur at the school level by school administrators, based on their knowledge of what works best in their institutions. The interpreted curriculum typically consists of a significant subset of the written curriculum that has been rewritten for classrooms. Any interpretation of curriculum is influenced by the philosophy and experience of the interpreter, which can result in widely divergent versions of the interpreted curriculum being forwarded for implementation.

The interpreted curriculum is then enacted by classroom teachers. The *enacted curriculum* is what is actually taught in classrooms and may be dependent on individual student or classroom characteristics based on teachers' intimate knowledge of what works best for them as teachers as well as the characteristics of their students. Some subset of the enacted curriculum constitutes student assessment. The *assessed curriculum* is typically the "core learnings" or "key concepts" identified by the teacher. Since the entire curriculum is too large to be assessed in full, the teacher assesses a subset of the curriculum; student performance on this subset is then used by the teacher to infer a student's mastery of the entire enacted curriculum. Finally, the *learned curriculum* is the subset of the assessed curriculum, which the student has mastered to an acceptable degree. There is some debate over what constitutes mastery, whether it is reflected by an acceptable score or grade on teacher-designed or standardized tests (Milic *et al.*, 2016); by student understanding reflected on other measures such as interviews, portfolios, or productions (OME, 2010); or by other dimensions beyond content, such as motivation, engagement, and attitude (Elliott *et al.*, 2017).

All of these various curricula result in what I call the *curriculum telephone game*. Many of us played a variation of the telephone game as children: One child whispers a message to another child, who then passes the message along to a third child, and so on. When played in a classroom of children, the message received by the final child in the chain inevitably is very different from the original message conveyed by the first child. With curriculum, due to the multiple layers between the intended curriculum and the learned curriculum, the outcome is similar to the children's game, with the message frequently distorted along the way. Figure 1 illustrates the curriculum telephone game.

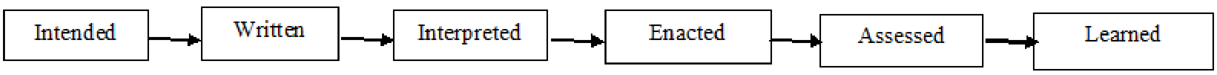

Figure 1: The curriculum telephone game, indicating loss of fidelity between stages

Every layer is subject to noise in transmission. In moving from intended to written, written words are always subject to personal interpretation—no matter how carefully the wording is chosen. The move from written to interpreted is expected to result in deviations from the original intended curriculum, since "interpreting" means to change, typically with the intent of clarifying. As described above, the enacted curriculum is a modified version of the interpreted curriculum, based on teacher beliefs, personal philosophy, and student and classroom characteristics. The assessed curriculum is perforce, a subset of the enacted curriculum, for two reasons: the entire enacted curriculum is too large to be fully assessed, and the assessed curriculum is a subset based on teachers' perceptions of core concepts or key ideas. In addition, because students' mastery of the enacted curriculum must be imputed from the assessed curriculum, this step is also subject to interpretation and noise. Finally, the learned curriculum is influenced to a large extent by student characteristics.

At every level, the curriculum telephone game is influenced by perceptual theory (Combs, 1999). As outlined in the Theoretical Framework section, individuals' perceptions impact how the curriculum is perceived, received, and interpreted. The result is inevitably a distortion, to some degree, of the information received. This has very significant implications: a mere 10% distortion at each level would result in a learned curriculum that is less than 60% congruent with the intended curriculum. In reality, the distortion at some levels is much greater, such as the link from the interpreted curriculum to the enacted curriculum, since enactment depends on so many influences beyond the original curriculum creators' control (Figure 2). Boeson *et al.* (2014) reported relatively low levels of fidelity in the implementation of a new mathematics curriculum, with only 15% to 20% of teachers demonstrating adequate congruence with the intended curriculum, and significant teacher pushback on changes to pedagogy. A Canadian study on curriculum alignment found a very high level of alignment for mathematics content (97%) but a very low level of alignment for cognitive processes (7.3%) (Seitz, 2017). Kurz *et al.* (2010), in turn, found non-significant correlations between the intended and enacted curriculum and student achievement, except in the case of special education students.

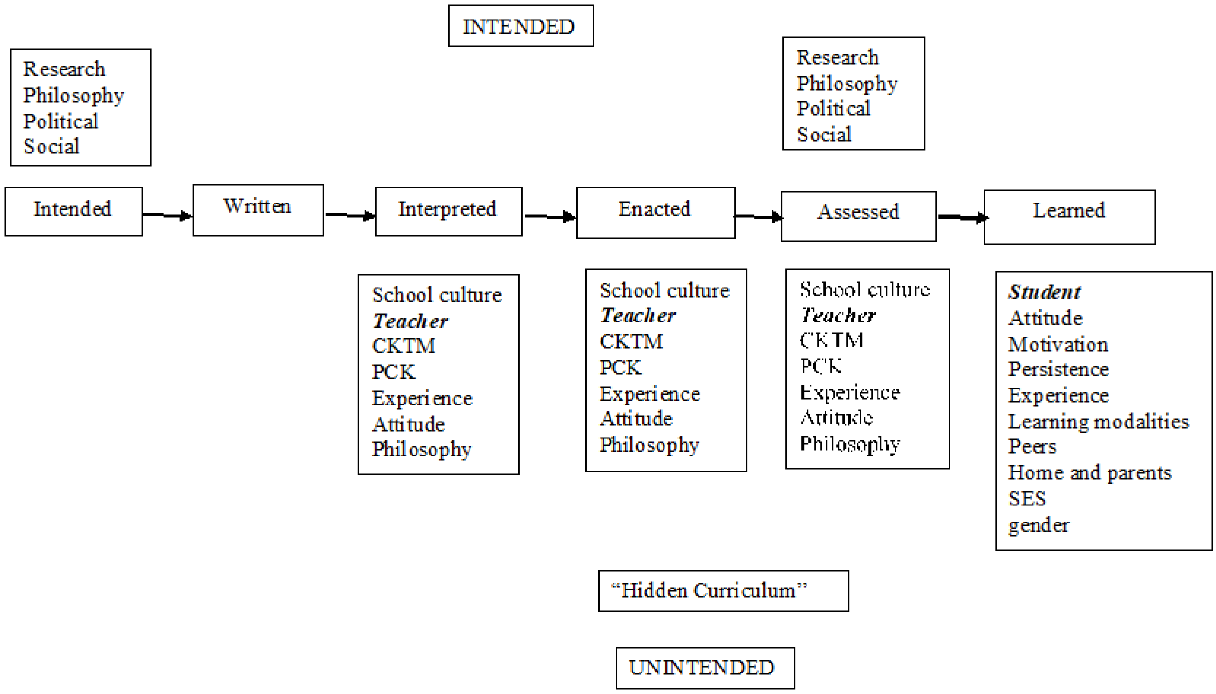

Figure 2: The curriculum telephone game showing influencers

In Figure 2, PCK refers to pedagogical content knowledge (Shulman, 1986), and CKTM to content knowledge for teaching mathematics. PCK is the specialized knowledge necessary for a teacher to effectively teach their subject matter to students. PCK is sometimes conceptualized as having various subdomains, such as knowledge of content and students, knowledge of content and teaching, or knowledge of content and curriculum, while subject matter knowledge may have subdivisions such as common content knowledge, specialized content knowledge, and horizon content knowledge (Ball *et al.*, 2008). However, what is critical is recognizing that pedagogy and content are interrelated and mutually supportive (Seagall, 2004). Research into PCK is ongoing and extensive (e.g., Kind, 2009; Zeng *et al.*, 2023). CKTM (Ball *et al.*, 2005) is the specialized content knowledge necessary for teaching mathematics, which includes all the aforementioned dimensions for both subject matter knowledge and PCK, with a special focus on how the uniqueness of mathematics impacts the teaching of the subject (Hill *et al.*, 2008).

The influence of the hidden curriculum (Figure 2) is pervasive. Perceptual theory predicts that this hidden curriculum influence is unavoidable and extensive, particularly at the level of enacted curriculum and beyond, given the large number of influencers at those levels. We see this, for example, in the trope "teach as they were taught," describing teachers' instructional philosophies heavily influenced by their experience as students—

also known as the apprenticeship of observation (Lortie, 2002). Thus, the teacher's instructional philosophy may or may not be congruent with the intended curriculum; however, this philosophy will importantly impact the enacted curriculum and what follows (Burton, 2003).

This paper explores potential ways of reducing the noise generated between the intended and enacted curriculum, increasing the clarity and thus increasing the congruence between the intended and learned curriculum. This is particularly important given the interrelationship between content and pedagogy. While this paper focuses on pedagogy in the curriculum, pedagogy cannot be totally divorced from content. Some content requires specific modifications to pedagogy in order to become clear and relevant to students.

7. Pedagogy in Ontario Mathematics

This section examines the place of pedagogy in mathematics curriculum documents, the implementation process in Ontario, and the influences on Ontario mathematics curriculum development.

7.1 Curriculum Policy Documents and Related Reports

From 1950 to 1972, the Ontario Department of Education (ODE) released curriculum guidelines for mathematics courses in the province. Because they were guidelines and not requirements, teachers did not have to teach or assess all the topics listed. The guidelines provided a list of topics for each course, with one or two pages of front matter. Textbooks of the era covered all the topics listed and were the primary teaching resource. For example, the 1950 guidelines had two pages of front matter (ODE, 1950). Because lists of topics were comfortable and familiar to teachers, the front matter was frequently ignored.

The ministry guidelines provided little or no guidance on teaching techniques. The entire mathematics section of the 1950 guidelines for mathematics Grades 7 to 10 comprised 10 pages. The 1950 guidelines had suggestions regarding pedagogy: relate the content to pupils' daily life, start by reviewing the previous year's content to identify where remediation was necessary, teach the pupil to solve problems, and use numerous examples to develop understanding. Interestingly, there is recognition of the racial dimension, although presumably a Eurocentric focus: "*While the ideal method is to develop concepts in mathematics from the personal experience of the pupil, it should be remembered that mathematics is the product of racial experience—the collective result of a large number of thinkers*" (ODE, 1950, p. 57).

There is no further guidance on pedagogy. The presumed pedagogy, euphemistically called Socratic teaching, had not changed since the early 20th century. The method involved teacher-led whole-class instruction, using question and answer strategies, although sometimes this devolved into teacher lectures. A typical mathematics class, which ran for an entire school year, was 37 minutes in length; the class began with a review of the previous day's homework, followed by the teacher working through a number of examples of new material, and the final portion of the class devoted to students working individually on large numbers of questions similar to the worked examples. Any questions that were not completed in class were expected to be finished as homework.

Evaluation during the period consisted of unit tests and three sets of examinations, except in Grade 13 in which departmental examinations set by ODE were administered across the province. Over the years, there were some changes to content topics but pedagogy remained the same throughout the period. In the 1960s, there were significant content changes with the introduction of the "new math," which included topics such as set theory, logic, and the study of algebraic systems (ODE, 1961). All these topics were treated abstractly and were intentionally divorced from students' real life. Although these were dramatic content changes, no new pedagogies were developed for teachers, and the Socratic method remained the standard.

During this period, a notable exception to the outside influences on education was the Hall-Dennis Report (ODE, 1968) commissioned by the Ontario Conservative government, which recommended sweeping changes to education in Ontario (not specifically for mathematics education). In many ways, the report foreshadowed the advent of student-centered education, analogous to the philosophy of Dewey (1916); this was a very different vision than the then-current view of teaching. The Hall-Dennis Report's recommendations included: education that addresses the whole child, including their social, emotional, and academic needs; a more relaxed teacher-pupil relationship that encouraged student discussion, inquiry, and experimentation; education that enhances the dignity of the child and encourages their curiosity; replacing lock-step learning, in which every student is expected to learn the same content at the same time, with individualized instruction with relaxed timelines; and schools that were involved with and informed by the communities they serve, rather than being isolated silos of learning. This report led to experimental educational settings in the 1970s, such as open classrooms and team teaching. However, the overall impact on mathematics pedagogy was very limited, as mathematics teachers generally continued to teach as they were taught (i.e., Socratic questioning and lectures).

Pedagogically, the 1968 mathematics curriculum (ODE, 1968) was noteworthy for espousing a broad range of pedagogies, including team teaching, group work, audio-visual resources, student assignments, student presentations, and other strategies, including Socratic lessons and lectures. As usual, because the guidelines were not compulsory and the recommendations were in curriculum front matter frequently ignored by teachers, implementation of these new pedagogies was sporadic.

In 1972, the ODE, now renamed the Ontario Ministry of Education (OME), issued new guidelines for mathematics. As before, these were guidelines and not compulsory expectations. This document introduced themes of problem-solving and applications but provided no indications of pedagogies appropriate for teaching these concepts.

The 1985 guidelines (OME, 1985) are mainly remembered for the elimination of Grade 13 and the introduction of the Ontario Academic Credits (OAC), which, in mathematics, resulted in the former Grade 13 courses being repackaged as OAC credits.

The themes of problem-solving and applications were continued, and experiential learning was introduced, with the advent of investigations being highlighted. As usual, this information was in the curriculum front matter and was not compulsory. No information on appropriate pedagogies was included. The process of issuing non-compulsory guidelines would continue until the publication of the Common Curriculum in 1995, which specified expected outcomes to be achieved by students by the end of Grades 3, 6, and 9 (OME, 1995b).

In 1987, the then-Liberal government commissioned newspaper columnist George Radwanski to produce a report on dropouts, which found that many students considered the Ontario educational system irrelevant to their lives, and that the education system had not reacted to the shift from manufacturing to a service economy. Among Radwanski's recommendations were destreaming secondary school, early childhood education, standardized testing, an outcome-based curriculum, and replacing the credit system with a common curriculum. As usual, the report said very little about the pedagogy required to implement these recommendations.

In 1995, Ontario's NDP government produced the aforementioned Common Curriculum, which contained some relatively forward-thinking ideas about assessment, such as open-ended questions, investigations, journals, observations, conferences, interviews, portfolios, and student self-assessments. However, the pedagogy required to teach a destreamed Grade 9 class, with its wide array of student abilities and needs, was not directly addressed. This led to a high degree of teacher frustration, exhaustion, and burnout (Irvine, 2021). In addition, the 3-year outcomes requirement, while attempting to recognize that students do not all learn at the same rate, resulted in the teachers of grades other than Grades 3, 6, and 9 feeling less accountable for student learning. Some school boards and individual teachers developed implementation supports (e.g., Kristensen-Irvine, 1996), but overall, implementation of this curriculum was somewhat erratic.

When the Conservative government came to power in the next election, the mathematics curriculum was again changed. This new curriculum was notable for several reasons. First, it was the first mathematics curriculum that was written by teachers rather than by OME staff. It was also based on extensive research in mathematics education and was a radical departure from the traditional curriculum and pedagogy common in mathematics classrooms. This curriculum (OME, 1999, 2000) required extensive use of technology, especially graphing calculators. Its specific expectations and requirements also included the use of investigations, inquiry, and multiple methods of achieving content mastery, in a very active and student-centered way. For example:

> *"Determine, through investigation, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use graphing software to obtain the graphs of a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; focus on the characteristics of the equations of linear relations and how they differ from the characteristics of the equations of non-linear relations)."* (OME, 1999, p. 14)

"Collect data using appropriate equipment and/or technology (e.g., measuring tools, graphing calculators, scientific probes, the internet (Sample problem: Drop a ball from varying heights, measuring the rebound height each time)." (OME, 1999, p. 21)

"Construct a variety of cylinders for a given volume and determine the minimum surface area for a cylinder with a given volume." (OME, 1999, p. 24)

The expectations in this curriculum also required students to apply the scientific method as well as higher-order thinking skills, which was significantly different than previous curricula:

"Pose and solve a problem involving the relationship between the perimeter and the area of a figure when one of the measures is fixed." (OME, 1999, p.16)

"Pose problems, identify variables, and formulate hypotheses associated with relationships (Sample problem: Does the rebound height of a ball depend on the height from which it is dropped? Make a hypothesis then design an experiment to test it)." (OME, 1999, p. 21)

In addition, student assessment and evaluation were significantly changed, requiring a shift away from assessing only knowledge of mathematical facts and procedures to the evaluation of students across four categories: thinking, application, communication, and knowledge (TACK). This was also the first curriculum to be supplemented with extensive resources for teachers. Among these resources were course profiles that provided student activities for many expectations, with some activities fleshed out completely and others outlined; exemplars, which were samples of graded student work to assist teachers in the new assessment strategies; targeted implementation and planning supports (TIPS), with complete lesson plans for Grades 7, 8, 9 Applied and 10 Applied, together with suggestions for other grades; and the EduGAINS website, with a wealth of teaching resources for teacher PD. In addition, the OME sponsored numerous PD workshops across the province, which, due to limited staff, used the train-the-trainer model—a less than optimal mode of professional learning (Pearce *et al.*, 2012; Yarber *et al.*, 2015).

Due to abysmal EQAO results in Grade 9 Applied Mathematics, the curriculum was revised again in 2005 and 2007 (OME, 2005, 2007). The majority of these revisions were content-related, although one significant change was to move the mathematical processes into the front matter. The stated reason for this move was to give the mathematical processes a higher profile and greater emphasis (Mueller, 2019). However, participants who created the revised curriculum also admitted that most teachers ignored the front matter and went directly to the lists of expectations. Consequently, the mathematical processes tended to receive less emphasis (Irvine, 2022). This was an unfortunate development, since the mathematical process is arguably the big process ideas of mathematics and should stand out as the enduring understanding of mathematics education.

This curriculum maintained the practice of embedding pedagogy in specific expectations. For example: "Multiply a polynomial by a monomial involving the same variable [e.g., $2x(x+4)$, $2x^2(3x^2-2x+1)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil)" (OME, 2005, p. 30). To support teachers in implementing this revised curriculum, the OME modified the TIPS materials to reflect the curriculum changes, producing the TIPS For Revised Mathematics (TIPS4RM), and offering interactive online professional development sessions.

In 2008, the OME embarked on an ambitious professional learning initiative called MathGAINS (GAINS being an acronym for Growing Accessible Interactive Networked Supports). With program funding of $7 million, each school board was required to develop a job-embedded PD program and submit their respective plans to the OME. For school boards without sufficient internal capacity to develop such a program, the OME provided a cadre of Provincial Math Coaches—teachers who had been trained in the coaching cycle of co-plan, co-teach, and co-debrief (Irvine & Telford, 2015). The OME also ran summer institutes while continuing the online workshops for teachers. These research-affirmed strategies resulted in improved implementation of the new pedagogies.

During the COVID-19 pandemic, many educational jurisdictions reduced curricula and focused on the big ideas of their curricula (Irvine *et al.*, 2023). However, in both 2020 and 2021, the OME issued revised or new curricula for Ontario mathematics; in June 2020, the ministry issued a revised mathematics curriculum for Grades 1 to 8 that was required to be implemented in September of that same year. The revised curriculum featured several new strands, including social-emotional learning, mathematical modeling, financial mathematics, and coding. All these strands consisted of new content for elementary and middle school teachers; no appropriate pedagogies were identified. This almost 400-page document had 110 pages of front matter, of which five pages (4.5% of the front matter) discussed pedagogies (OME, 2020b). No teacher resources were available until later in 2021.

In June 2021, OME issued a new curriculum for destreamed Grade 9 mathematics. The required implementation date was September 2021. The curriculum was very content-heavy, including almost every expectation from the previous Grade 9 Applied and Grade 9 Academic courses, as well as new units on social-emotional learning, mathematical modeling, financial mathematics, and coding. As with the revised Grades 1 to 8, no teacher resources were available when the curriculum was implemented. Mathematics teachers, exhausted from teaching during the pandemic, were expected to implement the new curriculum with only 2 months to plan.

Later that year, the OME contracted the Ontario Association for Mathematics Education (OAME) to develop teaching resources for both the revised Grades 1 to 8 curriculum and the new destreamed Grade 9 curriculum. There was a huge demand for these resources, with over 1,000,000 visits to the resource site by May 2022 (OAME, personal communication, May 27, 2022), although the number of unique visitors to the site was unknown. Overall, the resources that were developed were disappointing. For the Grade 9 destreamed course, many lesson plans were long (often over 20 pages), with a large amount of front matter. Some lessons for the Grade 1 to 8 courses were folders containing up to eight files. The exhausted mathematics teachers frequently found these resources of limited use.

The curriculum document of over 300 pages had 62 pages of front matter. Five pages of front matter were devoted to pedagogy. Within the pedagogy section, there are descriptions of nine high-impact instructional strategies (Irvine, 2024; OME, 2020b, 2021). These are notable because they are research-affirmed pedagogical strategies with significant research evidence on their effectiveness. The high-impact pedagogies are:

- Learning goals, success criteria, and descriptive feedback,
- Direct instruction,
- Problem-solving tasks and experiences,
- Teaching about problem solving,
- Tools and representations,
- Math conversations,
- Small-group instruction,
- Deliberate practice,
- Flexible groupings.

This marks a return to advocating pedagogy based on research. Unfortunately, the restrictively short 2-month implementation lead time precluded many teachers, exhausted from 2 years of pandemic teaching, from implementing these strategies; most teachers ignored the entire front matter and moved immediately to the list of expectations. A significant change with respect to pedagogy is the inclusion of examples, teacher prompts, instructional suggestions, and sample tasks in the curriculum's specific expectations, which acknowledges that many teachers ignore the front matter and move directly to the lists of expectations. This moves pedagogy to a more prominent position in the curriculum.

Anecdotal evidence from practicing teachers and from a symposium on the destreamed Grade 9 course held by the Fields Mathematics Education Forum in November 2023 indicated that the destreamed Grade 9 course had limited success. Comments included the lack of pedagogical support for teaching a course with such a wide range of abilities and readiness, far too much content to be adequately addressed in one semester, massive workloads, a high level of frustration with both the course and the resources on the OAME site, and increased teacher burnout. To date, the OME has not addressed teacher concerns about this course.

8. Pedagogies to Connect Students with Mathematical Content

Over the period 1950 to 2024, mathematics curriculum documents identified several pedagogies that could allow teachers to connect mathematical content to their students. Among the suggestions are using real-world situations and problems (ODE, 1950; OME, 1972, 1985); problem-solving (OME, 1985, 1999, 2000, 2005, 2007); mathematical modeling (OME, 2020a, 2020b, 2021); investigation (OME, 1999, 2000, 2005, 2007); and social-emotional learning (OME. 2020a, 2021). Yet, the curriculum documents (with the exceptions of 1999/2000 and 2005/2007) offer no indications of how teachers are to implement any of these strategies.

8.1 Real-world Applications

Clearly, it is not possible for most mathematics teachers to be experts in a wide range of applications of mathematics to the students' worlds. However, real-world applications need to be more than problems dressed up with some out-of-classroom verbiage. For example, a question about parabolas does not become "real-world" for students simply because it involves the trajectory of a satellite. A satellite is unlikely in the realm of a student's real world. In his discussion of real-world problems, Irvine (2015b) identifies the pitfalls of a simplistic view of real-world:

> *"Lee [2012] identifies a number of ways that mathematics and problem-solving are connected to the "real world." He enumerates simple analogies (e.g., relating temperature to negative numbers); classic "word problems"; analysis of real data; discussions of mathematics in society, such as misuse of statistics in the media; hands-on representations, such as physical models; and, mathematical modeling of real phenomena. I take issue with the categorization of classic word problems as "real world." Even Lee's example (Two trains leave the station...) is an example of, at most, an exercise of an algorithm, dressed up in some verbiage. Word problems, as taught in most classrooms, have no connection to the real world. At best, they provide some practice at literary decoding. At worst, they confuse, demotivate, and perpetuate the myth that mathematics is difficult, that problem solving involves the application of a known algorithm once the "clues" in the wording are deciphered; that mathematics problems should be solvable in a few minutes, otherwise give up as unsolvable; that students of mathematics should maintain an ability attribution model, rather than an effort attribution model."* (pp. 108–109)

Irvine (2015b) identifies that for a student, real-world problems must have at least one of these attributes, in descending order of real-world power: the mathematics flows from an investigation, experiment, or model in which the students were involved; students can use the mathematics immediately (e.g., in their part-time jobs, budgeting, or sports); students can use the mathematics in another subject, in the near term, such as in science, geography, technical shops, family studies; someone close to the student can or does use the math content, such as a family member, relative, adult acquaintance; there are examples in the real world of people using the mathematics. Utilizing real-world situations, and not just assuming that problems using non-school words such as Lee's (2012) "two trains…" can stimulate students' interests and promote engagement in their own learning (Arthur *et al.*, 2018).

8.2 Problem-solving

Problem-solving appears as a recommended strategy in numerous curriculum documents (OME, 1980, 1985, 1995b, 2021). As discussed above, problem-solving is not about words attached to the application of algorithms. OME (1980) provides a flowchart for problem-solving with the following steps:

- A problematic situation exists when a person attempts to make sense of something but is unable to do so
- A model of the problematic situation is constructed
- A known algorithm is applied to the model
- The elements of the model are restructured to make the unknown parts determinate
- The determined parts are then projected back into the real world.

The flowchart is not only confusing but assumes also that problem-solving is about using an already-known algorithm. It also minimizes the mathematical model-making involved.

A survey of over 600 mathematics teachers in the United States showed that over half of the respondents taught problem-solving; however, in investigating further, the majority of these teachers reported problem-solving involved word problems from the textbook, and relatively few taught actual problem-solving beyond that (Council of Chief State School Officers, 2000).

True problem-solving involves the cognitive processes directed at achieving a goal when no solution method is obvious to the problem solver (Mayer & Witrock, 1996). A mathematical problem involves an initial undesired situation, a desired end situation, and an obstacle preventing the movement from the initial situation to the end situation (Cotic & Zuljan, 2009). These descriptions identify actual mathematical problems, but are likely unknown to most mathematics teachers, and differ radically from traditional word problems. In suggesting problem-solving as a pedagogy, the OME provided no further information on what a true problem is or how to use real problems in teaching.

Problem-solving also involves the issue of transfer, which requires students to apply their knowledge to a new or different situation (Barnett & Ceci, 2002; Butterfield & Nelson, 1991). Often, problems posed in mathematics classrooms involve near transfer, with situations very similar to ones already encountered in class. True problem-solving should involve a mix of near and far transfer, with far transfer coming into play as students become proficient at addressing problem-solving situations.

Teaching problem-solving strategies has been shown to be very effective. In his meta-analysis of over 800 studies involving more than 2 million students, Hattie (2009) found an effect size of 0.61 for teaching problem-solving strategies; however, Hattie also found a disappointing effect size of 0.15 for teaching through problem-solving. Thus, teaching problem-solving heuristics such as Polya's (1957) four-step sequence of understand the problem, make a plan, do the plan, and look back, or the Harvard Balanced Assessment Project's heuristic of explore, model/formulate, transform/manipulate, infer/conclude, communicate/look back are effective ways of engaging students' metacognitive functions and of improving performance (Harvard University, 2003).

The OME could easily have included in the curriculum documents some of these heuristics, as well as a problem-solving flowchart (Figure 3). Note that a problem-solving flowchart is most effective when co-developed with students.

Real-world problems, either through teacher-provided situations or through students' problem-posing (Irvine, 2016; Stoyanova, 2003, 2005) have been shown to increase student motivation, engagement, self-efficacy, attitudes towards mathematics, attitudes towards learning, interest in mathematics, and achievement.

An exception to the OME's lack of pedagogies in curriculum documents was the 1999/2000 and 2005/2007 documents. Both of these are included in the front matter of Polya's problem-solving heuristic.

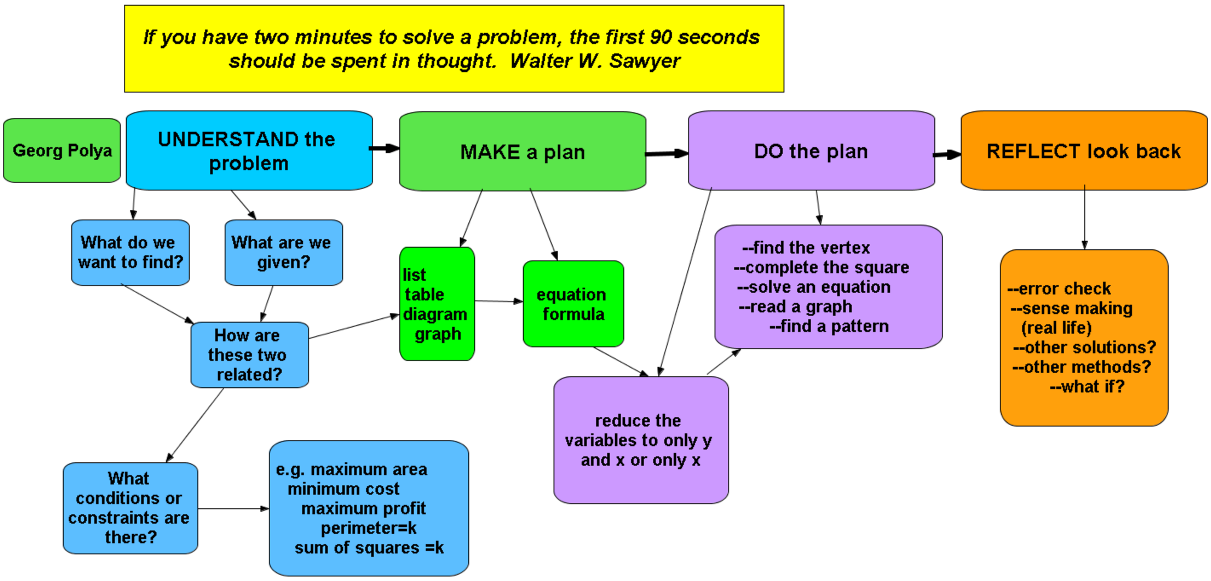

Figure 3: Problem-solving flowchart

8.3 Mathematical Modeling

Mathematical modeling is another area that is problematic if no additional information or examples are given. Many mathematics teachers assume that mathematical modeling immediately leads to an algebraic equation. However, mathematical modeling is far richer than that.

Modeling can be considered either a noun or a verb. As a noun, modeling is described as "*purposeful mathematical descriptions of situations, embedded in particular systems of practice*" (Lesh & Lehrer, 2003, p. 109) or "*a variety of representational media, which may involve written symbols, spoken language, computer-based graphics, paper-based diagrams or graphs, or experience-based metaphors*" (Lesh & Harel, 2003, p. 159). As a verb, modeling is "*the process of translating between the real world and mathematics in both directions*" (Blum & Ferri, 2009, p. 45). Modeling can also be defined by the steps involved in the modeling process, "*a process whereby a situation has to be problematized and understood, translated into mathematics, worked out mathematically, translated back into the original (real-world) situation, evaluated, and communicated*" (Bonotto, 2010, p. 20). Modeling can be conceived as a key step in problem-solving (Lesh & Harel, 2003; Temur, 2012).

A model-eliciting activity (MEA) is needed to present a situation requiring students to construct a mathematical model. An MEA is a task that reveals student thinking processes through their descriptions, explanations, justifications, and representations throughout the modeling activity (Doerr & English, 2006). Lesh and Caylor (2007) note that an MEA requires the following criteria:

- Self-evaluation (students judge usefulness)
- Model generalizability (shareability and reusability)
- Model documentation (the need for students to reveal their thinking)
- Simplest prototype (the situation is as simple as possible while still requiring a model)

In addition, Doerr and English (2006) add two more criteria for an MEA:

- Explicit links to existing knowledge
- A requirement for high-level thinking and reasoning.

Therefore, to qualify as an MEA, the task must satisfy the six criteria listed above. Irvine *et al.* (2016) provide a sample of an MEA in mathematics.

So, a teacher wishing to utilize a mathematical modeling task in their classrooms must create or find satisfactory MEAs. This is a non-trivial task, and one for which the OME provides no additional guidance or support beyond the stated expectations in the curriculum policy document. Without guidance and exemplars, the mathematical modeling strand may degenerate into a simplistic provision of an equation and a few words of context, like the projectile problems in many textbooks. To address an MEA, students need clear, explicit instructions and clarification for the resultant product, such as exemplars or a rubric. MEAs have the potential to increase student interest and agency, but they require a great deal of preparation from teachers. The inclusion of a mathematical modeling strand is a significant addition to an already crowded curriculum. The 2021 destreamed Grade 9 curriculum (OME, 2021) provides some excellent development of mathematical modeling, with a number of sample tasks and suggestions. This section of the curriculum is also noteworthy for its student-centered approach, although the teacher prompts section still accommodates a teacher-centered classroom, leading to some ambiguity.

There are also clear links between mathematical modeling and the seven mathematical processes—problem solving, communication, representing, connecting, reasoning and proving, reflecting, and selecting tools and strategies (OME, 2005)—as well as to deep learning (Campbell & Cabrera, 2014) through the need for higher-order thinking skills. It is therefore puzzling that the OME links the mathematical processes to the new social-emotional learning strand rather than the mathematical modeling strand.

8.4 Investigations

The 1999/2000 curriculum was a major paradigm shift in pedagogy. Firmly rooted in social constructivism (Vygotsky, 1978), this curriculum required a shift from teacher-centered to student-centered pedagogy by including in the expectations student investigations as a principle of pedagogy. This shift was supported by the rise of technologies such as graphing calculators and motion detectors. Many mathematics classrooms resembled science classes, with students collecting data, making and verifying hypotheses, and communicating their findings. Through data collection of relevant, real-world data (e.g., students' height, arm span, hand span, etc.), students constructed scatterplots, found lines of best fit, and used regression analysis to justify conclusions. Applications of the scientific method (Nurlelah *et al.*, 2023) resulted in students applying higher-order thinking skills (HOTS) and metacognitive skills to design and carry out their investigations. In considering "What If?" questions, students posed and solved related problems. This shift to active student learning resulted in significantly increased student engagement and motivation, and in many cases also positively changed student attitudes toward mathematics as a subject.

8.5 Social-emotional Learning (SEL)

This new strand (OME, 2020b, 2021) is a significant departure from previous curricula since it is non-mathematical and speaks to educating the whole child by recognizing the affective dimensions of learning (Collie, 2020). Abundant research links student affective dimensions to achievement. In a study of over 90,000 students who wrote the standardized Education and Quality and Accountability Office (EQAO) assessments in 2012, Pang and Rogers (2014) found significant links to achievement for the affective dimensions of student attitudes towards mathematics, student self-confidence in mathematics, and student effort and engagement with homework. The same study reinforced the reciprocal relationships among achievement and affective variables of motivation, self-confidence, and attitude, with low or negative values adversely affecting student achievement. A study of PISA 2003 mathematics data found that affective variables such as self-efficacy, intrinsic motivation, goal orientation, and instrumental versus relational view of instruction all had positive effects on student achievement, with self-efficacy being the most influential (Ross, 2008). Overall, student motivation in school typically declines, beginning on the very first day of school, until it reaches a minimum around age 16, but then never increases (Middleton & Spanias, 1999; Steinmayr & Spinath, 2009). Clearly, this is a concern.

In mathematics, classes were found to have very high cognitive demands but very low levels of motivation and engagement (Shernoff *et al.*, 2003). In their study, Shernoff *et al.* (2003) found that students were more negative about mathematics and less engaged than in any other subject. Traditional methods of teaching in mathematics, such as Socratic lessons or lectures, that focus on content and ignore affective dimensions such as motivation, attitudes, and emotions have done very poorly in effecting changes in these areas (Clarkson, 2013; Smith & Starr, 2007).

The Social-Emotional Learning (SEL) strand has one overall expectation with six subdimensions: identify and manage emotions; recognize sources of stress and cope with challenges; maintain positive motivation and perseverance; build relationships and communicate effectively; develop self-awareness and sense of identity; and think critically and creatively. Although these concepts are important, mathematics teachers typically have no training or expertise in teaching them. The curriculum policy document (OME, 2021) provides some guidance, with six pages of discussion in the front matter, and teacher supports, consisting of various scenarios, in the strand descriptions. SEL is not a standalone strand, and the expectation is that the subdimensions will be interwoven with the content expectations of the course. In addition, the SEL strand is not to be evaluated; on a practical dimension, this is problematic, since many mathematics teachers have an evaluative focus and thus may downplay or ignore the SEL strand. Kilpatrick *et al.* (2011) relate these affective dimensions to productive *mathematical disposition*, "*the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner ad doer of mathematics*" (p. 116).

9. Curriculum Implementation in Ontario

Until the Common Curriculum in 1995, the ODE/OME were very clear that their role was to set curriculum policy, but implementation of the policies was the responsibility of the school boards. Thus, large school boards with sufficient internal capacity interpreted the curriculum documents, developed courses of study and supporting resources, and provided in-service professional learning for their teachers. In smaller school boards, which lacked the internal capacity and resources of the larger boards, the interpretation of the curriculum guidelines fell to individual teachers. This resulted in inconsistent fidelity of implementation, and frequently pedagogies remained unchanged, in what Lortie (2002) called the apprenticeship of observation. Teachers taught in the same way as they had been taught (i.e., Socratic lessons and lectures). At the same time, even though larger school boards provided in-service PD opportunities, there was little incentive for teachers to change their pedagogies and move away from their comfort zones.

The ODE/OME encouraged experimental courses, and most curriculum documents included suggested optional topics. For example, the 1960 ODE mathematics curriculum document was informed by experimental courses run in the 1951 and 1952 school years. Thus, the new curriculum had already been field-tested and refined prior to the issuance of the new document. Because the ODE/OME issued guidelines and not compulsory curricula, teachers and school boards could tailor their mathematics courses to better fit the communities in which they resided. This was encouraged.

> *"Although this guideline places a clear-cut emphasis on applications and problem-solving, it is the teachers' responsibility to ensure that this emphasis is realized in the classroom. … Virtually every part of Ontario has some local industry, commercial enterprise, or other activity that involves mathematics worthy of exploration."* (OME, 1980, p. 3)

For example, schools in a mining town could develop mathematics resources around mining; therefore, students were more likely to become engaged in the material since they could readily recognize its utility.

This flexibility changed dramatically with the advent of the Common Curriculum in 1995. This short-lived curriculum mandated specific content outcomes but said little about which pedagogies were best suited to achieve these outcomes. No professional learning resources were produced by the OME, and school boards had insufficient time to develop resources on their own. No longer were there optional topics in the documents, and no experimental courses were encouraged.

A sea change in mathematics pedagogy occurred with the 1999/2000 curriculum documents. As outlined above, there was a wealth of support documents produced. In addition, since the curriculum mandated extensive use of technology, the OME provided funds to school boards to purchase technologies such as graphing calculators and motion detectors; this encouraged using the inquiry process in teaching mathematics. This was the first time that recommended changes in pedagogy were supported with OME-developed resources and funding. Clearly, having a mathematics curriculum developed by practicing teachers recognized the need for supporting resources.

However, this curriculum was rushed into implementation for political reasons. In September 1999, teachers had only two units of the four-unit Grade 9 course, with the final two units arriving later in the semester. Given that this curriculum introduced massive changes in content, pedagogy, and assessment, the initial implementation of the curriculum was somewhat spotty (Suurtamm & Graves, 2007). The OME commissioned Suurtamm and Graves (2007) to determine the success of the implementation of the new curriculum, and the researchers recognized that this curriculum constituted a significant paradigm shift for teachers:

This curriculum challenges teachers with new ways of thinking about mathematics teaching and learning. While the implementation of a reform mathematics approach requires the alignment of several components including curriculum, resources, activities, classroom structures, teaching approaches, and the role of the teacher, the research suggests that the actual practice of teaching, such as the way a teacher poses questions or responds to students' understandings, is the most critical element In our view the kinds of changes teachers are asked to undertake in order to successfully engage all learners in mathematical inquiry, are not simple and require a substantive re-orientation of their basic beliefs about mathematical ideas as well as mathematics teaching and learning. Even when curricula are available and are supported by professional development and resources, reform-oriented teaching practices are not necessarily evident. The posing of problems, the facilitation of discussion, and the consolidation of mathematical concepts are teacher practices that require a great deal of knowledge and attention. In many cases, teachers themselves, have not learned mathematics in this way, nor have they had opportunities to learn and teach in inquiry-oriented settings. We know that even when teachers have had some inquiry-oriented learning experiences and acknowledge this approach as supporting their understanding of mathematics, there remains a visible tension between the reform-oriented and traditional approaches to teaching mathematics. (p. 5)

Suurtamm and Graves found fairly good penetration for technology and teaching through inquiry; less penetration of manipulatives, where teachers felt more PD supports were necessary; fairly limited penetration for new assessment practices, such as performance tasks; and generally, a greater desire for more PD opportunities, even beyond those already provided by the OME.

The rushed implementation was to become the norm for new mathematics curricula. In 2020, at the height of the COVID-19 pandemic, the OME issued a revised Grades 1 to 8 mathematics curriculum. Teachers received the new curriculum in June, and implementation was expected to occur in September, with no additional teacher support. This pattern of rushed implementation occurred again in June 2021, when a new destreamed Grade 9 Mathematics curriculum was received, again for September implementation. This new curriculum, over 300 pages, contained within each expectation teacher supports, consisting of examples, instructional tips, teacher prompts, and sample activities. Sampling these supports showed that many examples were rudimentary, possibly to support non-mathematics teachers who were assigned to teach this course; instructional tips often favored teacher-centered instruction, as did the teacher prompts; sample activities tended to be more student-centered, which clashed with the other supports. These supports, especially the teacher prompts, seemed to somewhat mirror mathematics instruction in the United States, which often features whole teacher scripts for teaching mathematical concepts, and are usually linked to approved textbooks.

10. Influences on Ontario Curriculum

During the period prior to the New Democratic Party (NDP) taking power in 1990, the mathematics curriculum in Ontario was mainly influenced by the mathematics policies of other countries, primarily the United States. During the 1960s, this resulted in the rise of the new math, which was also influenced by the Bourbaki movement arising in France (Corry, 1997). The failure of the new math gave rise to a back-to-basics backlash, especially in the United States. This influence was evident in the 1972 curriculum guidelines (OME, 1972). Once again, this policy was heavily influenced by curriculum and policy changes arising in the United States.

It is unclear why Ontario would choose to follow the lead of mathematics education in the United States, which is not recognized as a leading educational jurisdiction in the world and in fact, is frequently held up as an example of a weaker educational structure. The United States education system ranked 34th in 2022 while Canada ranked 6th (OECD, 2023).

In 1980, the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action*, which was to fundamentally change mathematics education in both the United States and Ontario, as well as in the rest of Canada. This document advocated problem-solving as a focus of mathematics education, and a paradigm shift from traditional, teacher-centered education to student-centered learning, congruent with a constructivist theory of learning. It also advocated a more whole-child vision of education. This marked a shift towards a research-affirmed curriculum, which was somewhat short-lived, as political considerations began to significantly influence curriculum policy.

The Radwanshi (1987) report was very politically motivated and contained virtually no educational research. It is unclear how many of his recommendations, such as standardized testing, might alleviate the dropout problem, but many of his recommendations were politically popular. This began an era in which political considerations would become the drivers of curriculum change, culminating in the Common Curriculum of 1995, the establishment of standardized testing administered by the EQAO, and the destreaming of Grade 9. It appears that almost all the changes were made without extensive consultation with educational research and the psychology of how students learn.

11. Discussion

As outlined earlier in the Curriculum Telephone Game section, the fidelity of curriculum implementation will always be less than 100%, particularly at the interface between the interpreted curriculum and the enacted curriculum. It is each teacher's duty as a professional to meet the needs of their students, resulting in an enacted curriculum that will differ from previous curriculum formats. Perceptual theory will, perforce, result in each teacher's perceptions of the appropriate curriculum being influenced by the teacher's own hidden curriculum (Figure 2).

It is therefore imperative that teachers be given sufficient support to increase, as much as possible, the fidelity of the implementation of any curricula. When curriculum is presented as a list of expectations, the implication is that all expectations are of equal importance and equal value. This is clearly not the case. Curriculum policy documents need to provide teachers with clear indications of which expectations are most important, as shown in Figure 3.

Thus, in a crowded curriculum, teachers would be able to identify the most important content and process expectations on which to focus their time. In addition, with several new strands such as mathematical modeling and social-emotional learning expected to be integrated into other content strands, teachers need examples and support of how this can be accomplished.

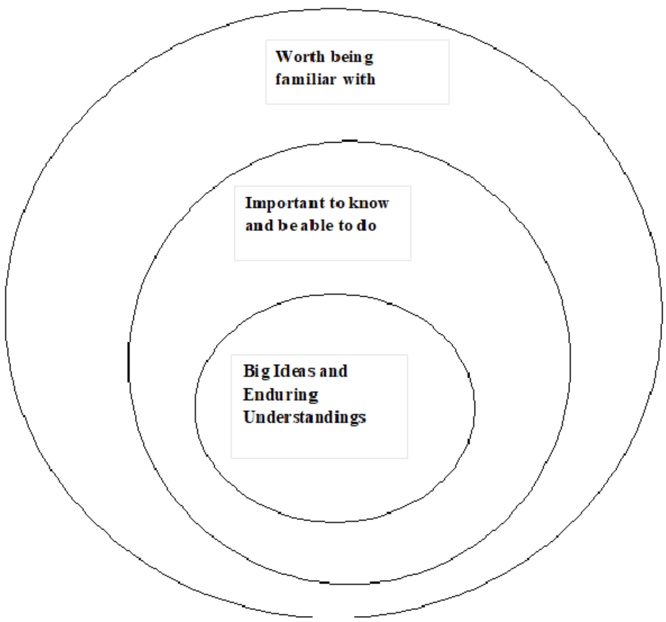

Figure 3: Prioritizing curriculum (reproduced with permission from Irvine, 2023)

In addition, providing teachers with context for each strand would increase the fidelity of implementation by helping teachers recognize where each strand fits into the overall curriculum continuum. Figure 4, called a From-To diagram, is one such way to provide this context.

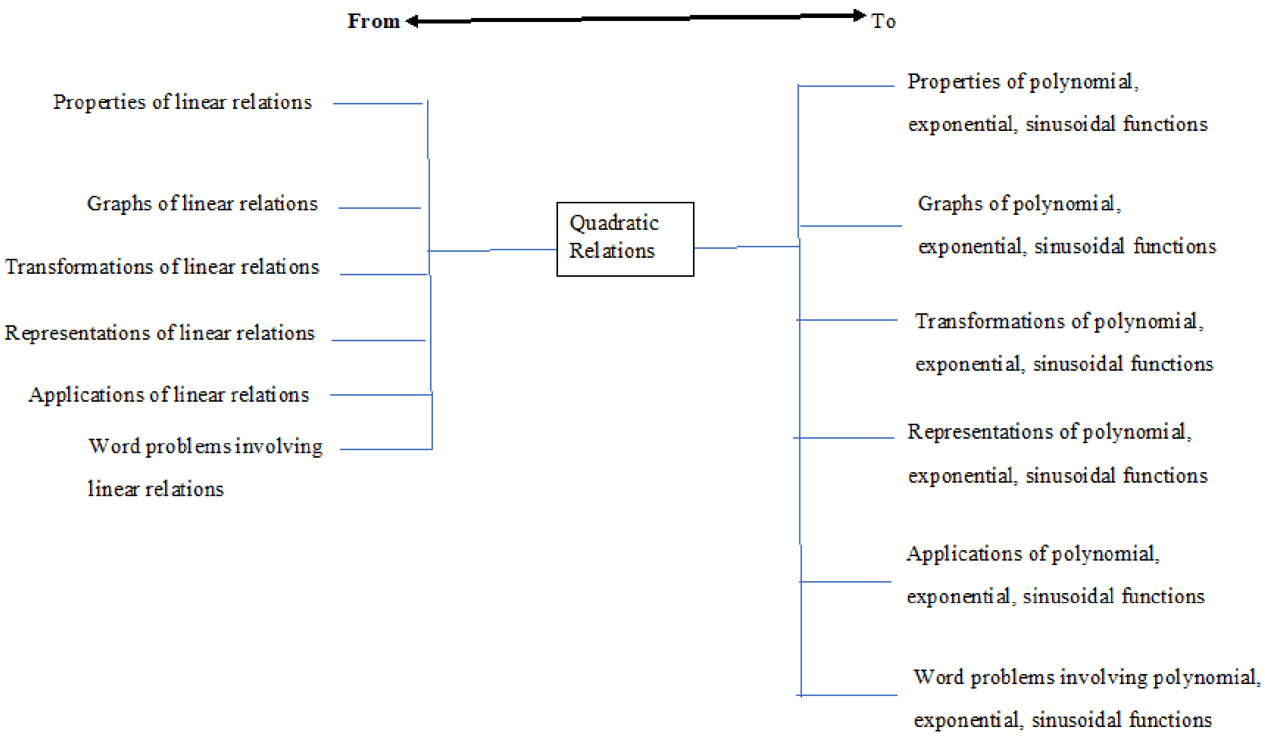

Figure 4: From-To diagram for the Grade 10 quadratic relations strand

Over the period 1950 to 2021, the number of pages of front matter in the curriculum documents has grown exponentially (r^2 =0.70) from two pages in 1950 to 62 pages in 2021, with a high of 110 pages in 2020. However, the proportion of pages devoted to pedagogy has not kept pace. Anecdotal evidence suggests that many (perhaps most) mathematics teachers ignore the front matter and move directly to the lists of content expectations. Therefore, changes in pedagogy need to be included in the curriculum expectations, as was done in 1999/2000, and now in the 2021 destreamed Grade 9 document.

Until the 1999/2000 curriculum, pedagogies were treated in the front matter of the curriculum documents, if at all. Preceding the more familiar and comfortable lists of topics, this front matter was frequently ignored by teachers. In any case, the pedagogical recommendations were merely suggestions and were not monitored in any way. Consequently, pedagogies in mathematics classrooms remained, for the most part, very traditional, utilizing Socratic lessons and lectures.

By moving the mathematical processes to the front matter, there is a risk that these important big process ideas will be further downplayed. Therefore, the OME needs to provide teachers with additional support for the mathematical processes, such as constellation diagrams (Figure 5) to indicate how specific expectations relate to each mathematical process.

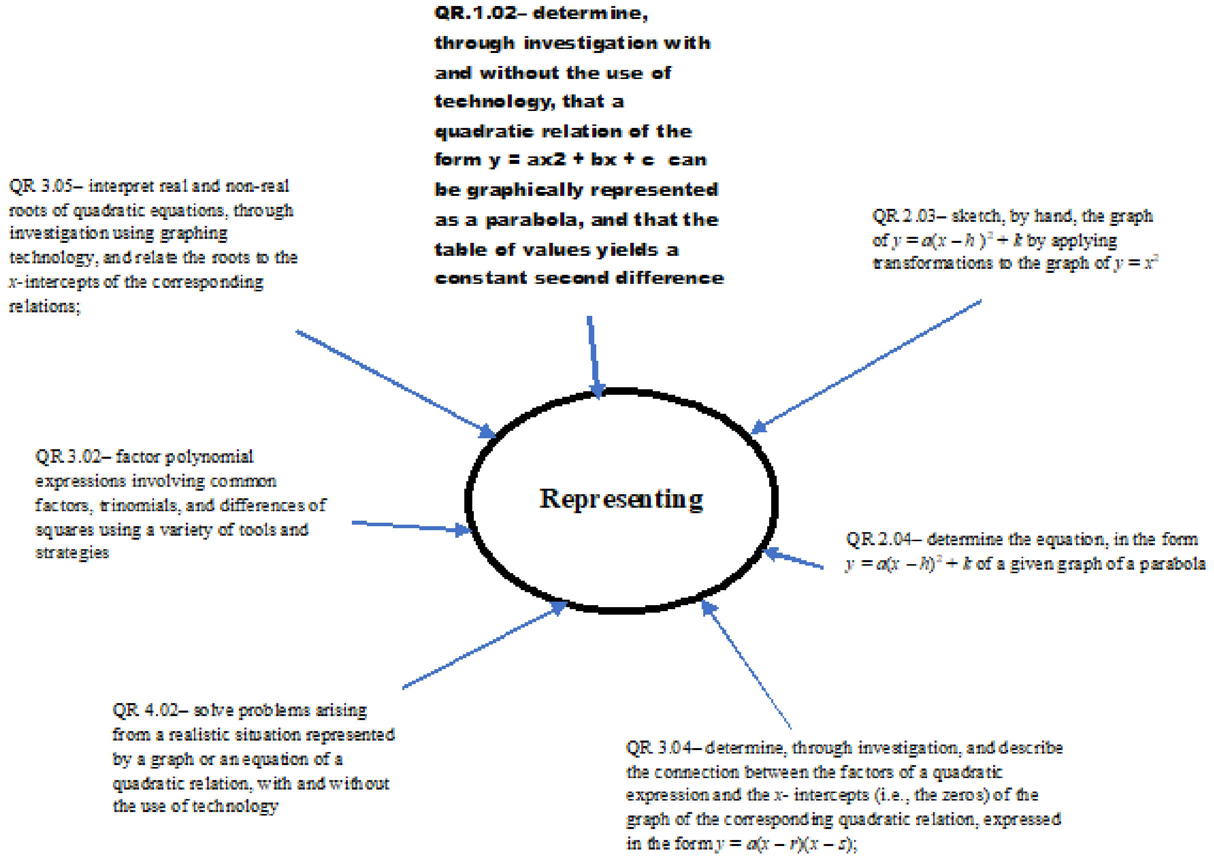

Figure 5: Constellation diagram for the mathematical process Representing, in the Grade 10 Quadratic Relations strand

Two initiatives with respect to pedagogy stand out. The first is the 1999/2000 curriculum. First, the curriculum policy document embedded pedagogies into the specific expectations of the curriculum, using verbs such as "by investigation" and "using a variety of methods." Secondly, multiple pedagogical supports were put in place to assist teachers in implementing the significantly different pedagogies that the new curriculum required.

The second curriculum policy document with the potential to significantly influence pedagogy is the 2021 Grade 9 destreamed mathematics document. For each expectation in this document, suggested pedagogies are embedded in each expectation, along with instructional tips, examples, teacher prompts, and sample activities. In addition, online supports developed by the OAME are available to teachers, although these supports tend to be extremely long and wordy. Issues with new curriculum implementation include the recent extremely rushed timelines, with new curriculum appearing in June for September implementation; the trend away from research-affirmed curriculum to heavily politicized documents; the massive increase in front matter in the curriculum documents; and the continued use of non-prioritized lists of expectations.

Teachers clearly need help if they are to move away from the "teach as they were taught" paradigm. In organizational theory, teaching is classed as a professional bureaucracy (Mintzberg, 1989). This means that teachers are given top-down direction but have a great deal of leeway in implementation in their classrooms. Therefore, teachers need to be convinced that whatever changes are being recommended will have a positive impact on their students. This can be accomplished through sharing research that confirms the effectiveness of the recommended pedagogies, and seeing examples of the pedagogies being successfully implemented in actual classroom situations.

It is insufficient to simply state new pedagogies, such as "use problem-solving" or "use cooperative learning." Teachers may not be aware of what these pedagogies entail, sometimes equating cooperative learning with simple group work, and not considering the five key dimensions of mutual positive interdependence, individual accountability, face-to-face interactions, social skills, and group processing (Johnson & Johnson, 1994). Supporting documents must be produced that outline the key elements of cooperative learning; provide details on cooperative learning strategies such as jigsaw, placemat, four corners, and others; and supply examples of these strategies applied to mathematics curriculum content.

In the move to standards-based education, opportunities to tailor curriculum to local needs appear to have been lost. Part of the rationale for a standard Ontario-wide curriculum was that students who change schools would not be disadvantaged. It would be hoped that the move to a standardized curriculum for all students was based more on advances in learning theory than on the relatively small number of students who change schools during the year. Research indicates that student motivation and engagement are important factors in student achievement. Being able to tailor curriculum to local needs plays a significant role in motivating students to engage in their own learning.

12. Recommendations

To increase the fidelity of the implementation of new pedagogies, several dimensions need to be addressed:

- While political overtones can never be totally removed, curriculum changes as much as possible should be based on research-affirmed advances in learning and teaching.
- As professionals, teachers require evidence-based examples of the new pedagogies' benefits to their students. To modify their perceptual fields, teachers need supports that enable them to recognize the congruence between their perceptual fields and the new pedagogies, as well as assistance in melding the mathematics content with the new ways of teaching.
- The pedagogies cannot be introduced only in the front matter. The 1999/2000 curriculum and the 2021 curriculum give us two examples of how the pedagogy and the mathematical content can be combined to clearly indicate the how and the why of the new pedagogies.
- The curriculum policy documents cannot be stand-alone artifacts. The 1999/2000 curriculum illustrated the need for all teacher support, and this example should be replicated with new curricula and new pedagogies. The discussion section of this paper illustrates three possible ways that teachers can be supported in implementing changes in pedagogies in an informed way. In this way, there

would be greater adherence to the intent of the new curriculum while still meeting the needs of the students in each class.

- It is insufficient to simply state in a curriculum document "use problem-solving" or "use cooperative learning"; supplementary materials need to describe in detail exactly what each strategy entails and provide classroom-based examples of the strategy in action. These supplementary materials may be in print, video, or on-demand examples for teachers to access.
- Reinstitute the seven-year curriculum review cycle that was used in the early 2000s. Regular review of curriculum is required since curriculum cannot remain static but rather must respond to ongoing research in both content and pedagogy. For example, the growth in artificial intelligence will have a major impact on education going forward and curriculum changes will need to respond to this reality.
- Provide sufficient lead time for the curriculum to be field tested and for teachers to become familiar with any changes. It is not realistic to introduce new content and pedagogies in June and expect successful implementation in September of the same year, with no other supports in place.
- The greatest loss of fidelity of implementation is at the interface between the interpreted to enacted curriculum. Therefore, providing additional support, such as those described in the discussion, would serve to increase the fidelity of the implementation of the intended curriculum.
- Employing local contexts for applications and investigations should be encouraged, as this will increase student motivation and engagement. Teachers should be encouraged to apply concepts in the specific expectations to situations familiar to students and situations that are clearly real-world applications that students will recognize.
- It has been shown that job-embedded professional development is the most effective way of changing teacher practice. Therefore, job-embedded professional learning should be attached to any new curriculum, especially if the new curriculum requires significant modifications to pedagogy.

13. Conclusion

Researchers and curriculum writers must continue to seek out strategies to increase fidelity of implementation of curriculum, and to increase penetration of new pedagogies. In doing so, teachers' beliefs and philosophies of teaching and learning must be considered strongly, as to truly implement change requires modifications to teachers' perceptual fields. Evidence-based change must be paramount. Teachers, as professionals, must see sufficient evidence that the changes are in the best interests of their students. Only in this way will teachers' perceptual fields be modified to not only accept but also embrace research-affirmed new pedagogies.

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