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THE EFFECT OF METACOGNITIVE STRATEGIES ON SPATIAL VISUALIZATION PERFORMANCE IN GEOMETRY

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Abstract:
Visualization is pivotal in geometric cognition, comprising three levels ranging from plane-level visualization to spatial visualization. It is at the spatial visualization level that learners demonstrate the capability to accurately translate two-dimensional images into three-dimensional objects and understand the relationships between the two dimensions. This study investigates the effect of metacognitive instruction on spatial visualization performance (SVP) among senior high school students. Through a post-test only control and experimental group design, three spatial visualization strategies (cognitive modeling, self-questioning, and reciprocal teaching) were assigned to three groups of students, while no instructions were given to a control group; all groups comprised 88 students. The findings challenge traditional stereotypes about gender differences in spatial abilities by revealing no significant performance difference between males and females in terms of enhancing spatial visualization using metacognitive approaches. Furthermore, the study found that the effect of metacognitive strategies in enhancing spatial visualization performance is positive, demonstrating that teaching students to be aware of and regulate their cognitive processes leads to improved performance across all experimental groups. The research also finds no significant differences among the metacognitive instructional strategies, suggesting that the core strategies of metacognition are universally effective. The findings emphasize the need for educational reforms that promote gender equity and the integration of metacognitive instruction into curricula.

Keywords: geometry; spatial visualization; metacognitive strategies; senior high school; Ghana

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1. Introduction

Geometry is an important and yet one of the oldest branches of mathematics (Gunhan, 2014). It focuses on the study of shape, size, relative position of figures, and properties of shapes (Tay, 2023). It is introduced to children even at the preschool level, making it an integral part of the mathematics curriculum across various levels. The practical applicability of geometric principles is well emphasized (English & King, 2015) in high school mathematics curricula worldwide (Akayuure, 2019). Despite improving the geometry aptitude of students, it presents challenges for teachers and schools (Baah-Duodu *et al.*, 2019; Yalley *et al.*, 2021; Acquah & Alhassan, 2018).

Learning geometry not only equips students with basic reasoning abilities but also encourages students' spatial abilities to visualize shapes and objects, and to solve real-life problems (Ozerem, 2012). Also, the study allows learners to understand the world around them by comparing shapes, objects, and their connections. Cognitively, the intertwinement of geometry and spatial reasoning is significant to interpret and reflect on the physical environment. Therefore, mathematics teachers are concerned with helping learners develop the spatial skills needed to interpret shapes and space (Clements, 2004). The research in geometry learning and spatial thinking has evolved from studies in psychology (Hegarty & Tarampi, 2015), when in the 1970s some researchers were interested in the relationship of spatial abilities to mathematical learning and problem solving (Owens & Outhred, 2006). Research on spatial ability as a single component has indicated that it has a strong connection with achievement in geometry (Hannafin *et al.*, 2008; Markey, 2009; Panaoura *et al.*, 2007; July, 2001; Clements, 2004; Cevirgen, 2012). However, it is not clear how certain factors connected to cognition affect spatial ability in students' geometry learning.

Spatial ability, as defined by Williams *et al.* (2010), refers to the capacity to process and conceptualize ideas based on spatial relationships among objects. It involves the cognitive processes of recognizing and manipulating shapes to solve spatial problems effectively. In simpler terms, spatial reasoning entails the mental capability to reach conclusions through cognitive processes such as imagining, representing, and transforming visual information within a spatial context (Komala *et al.*, 2021). According to Goos *et al.* (2007), and supported by Hershkowitz (1998), geometric thinking encompasses processes of visualization and reasoning. These scholars emphasize that visualization and reasoning are integral to mastering geometry. Van Hiele also contributed to this understanding by positing that both visualization and reasoning represent essential levels of geometric proficiency (Armah & Kissi, 2019). In cognitive processes, Giardino (2010) further asserts that visualization and reasoning are interconnected, and this interrelationship is critical in geometric problem-solving. Reasoning, in this context, involves mentally envisioning entities within spatial configurations. For instance, deducing that if a table is situated in the kitchen, then logically, the tabletop must also reside in the kitchen, since it is an integral part of the table itself. Even when contemplating empty spaces, our reasoning typically revolves around imagining potential occupants or structures within that space—a process that inherently involves visualizing spatial relationships and structures. Visualization, on the other hand, is a cognitive skill that facilitates the recognition of shapes, creation of new objects, and identification of relationships among them (Arcavi, 2003; Titus & Horsman, 2009). It enables individuals to transform or manipulate spatial patterns into visual representations of different shapes or configurations (Lowrie *et al.*, 2020). This ability is fundamental in geometric learning as it aids in comprehending geometric concepts, visualizing geometric transformations, and solving spatial problems effectively.

Visualization is pivotal in geometric cognition (Jones & Tzekaki, 2016), comprising three levels ranging from lower-level visualization, also referred to as plane visualization, to spatial visualization (Contreras, 2018). It is at the spatial visualization level where learners demonstrate the capability to translate two-dimensional images into three-dimensional objects accurately and figure out the relationships between the two dimensions. Spatial visualization is the skill needed to solve spatial problems effectively while providing appropriate explanations. This ability is crucial as learners attempt problems by making informed decisions and conceptualizing geometric scenarios. Spatial visualization, defined by Hegarty & Waller (2005), involves mentally manipulating and navigating spatial information, a skill indispensable in geometry. It entails modifying spatial patterns into various forms to comprehend and manipulate objects in all dimensions.

Studies indicate that students heavily rely on their spatial visualization abilities for geometric reasoning (Anci & Aslan-Tutak, 2015), with improved visualization leading to improvements in geometric reasoning capabilities. As students enhance their spatial visualization skills, they become adept at formulating and evaluating conjectures concerning geometric relationships. The significance of visualization in geometry has been emphasized by previous research: it facilitates students' comprehension of abstract geometric concepts (Sorby, 2009). Spatial visualization is positively related to achievement in mathematics (e.g., Guay & McDaniel, 1977; Haciomeroglu, 2016; Wang *et al.*, 2021; Hawes *et al.*, 2019). Typically, it has correlated with performance. More recently, and perhaps guided by the thought that spatial visualization is a factor that underlies geometric aptitude, researchers have attempted to discover fully the nuanced role that it plays in geometry learning (Uttal *et al.*, 2012), with studies suggesting that visual-spatial thinking regresses geometry performance (Hegarty, 2010). Kospentaris, Spyrou and Lappas (2011) stated in their study that visualization is an important factor that affects the choice of strategy when students undertake geometry tasks. Gestalt psychologists argue that all human thinking is visual in nature (Goldsmith *et al.*, 2016). Researchers have suggested that visualization skills can be enhanced through specific interventions such as training and the use of manipulatives (e.g., Eastman & Barnnet, 1979; Onyancha *et al.*, 2009; Ha & Fang, 2019; Lowrie *et al.*, 2019; Gilligan *et al.*, 2023). Additionally, there is both theoretical and empirical support for the effectiveness of Dynamic Geometry Software (DGS), such as GeoGebra (Ibili, 2019; Denbel, 2015; Bulbul & Guler, 2023), in fostering visualization abilities and enhancing geometric understanding. Furthermore, instructional methods have been identified as effective means to improve visualization skills (Jones & Tzekaki, 2016; Pedrosa *et al.*, 2014; Park *et al.*, 2021; Goos *et al.*, 2007). Studies exploring aptitude-treatment interactions have indicated that students' spatial visualization abilities are influenced by the efficacy of instructional strategies in mathematics (Wang *et al.*, 2007; Kaufman *et al.*, 2005; Hegarty & Waller, 2005). These studies have focused on enhancing students' geometry performance through targeted teaching on spatial tasks, especially through deliberate instructional practices and the use of supportive tools in developing students' visualization skills and geometric proficiency.

The development of spatial visualization skills requires substantial effort, as highlighted by recommendations from cognitive psychologists (Hegarty, 2011; Hoffler, 2010). In educational psychology and cognitive science research, the intersection of metacognition and visualization development has become a prominent area of study (Titus & Horsman, 2009; Mathewson, 1999; Shelton & Hedley, 2004). Metacognition, defined by Veenman (2015), involves the processes of monitoring, regulating, and evaluating one's cognitive processes and performance. It enables learners to reflect on their thinking, strategize, and adjust their approaches based on feedback and outcomes. Studies have indicated that metacognitive strategies are effective in facilitating learning and problem-solving in geometry (Efklides, 2008). These strategies enhance mathematical thinking by promoting cognitive flexibility, self-regulation, and reflection on problem-solving approaches (Lestari & Jailani, 2018; Park *et al.*, 2014; Vorholter, 2023). Zhong *et al.* (2019) found that students who engage in metacognitive processes demonstrate improved proficiency in recognizing spatial patterns, visualizing complex geometric configurations, and solving spatial problems. The effect of employing metacognitive strategies to improve geometry learning and understanding has been demonstrated to be positive and significant in studies (e.g., Sahin & Kendir, 2013; Yang, 2012; Altan, 2023; Anif *et al.*, 2019; Nahmias & Teicher, 2021; Naufal *et al.*, 2021; Aydin & Ubuz, 2010).

Senior high school graduates' proficiency in core mathematics is a requirement to gain admission into higher education institutions (Yarkwah *et al.*, 2020). However, mathematics is often perceived as a negative, challenging, and abstract subject (Aboagye *et al.*, 2021), which causes most students to complete school with poor grades. This perception extends to geometry, where many students face difficulties (Kyeremeh *et al.*, 2023). The West African Examinations Council (WAEC) has consistently reported that students perform poorly in geometry-related questions in the West African Senior School Certificate Examinations (WASSCE) across multiple years. In 2017, it was reported that students' performance on geometry-related questions was below average (WAEC, 2017). Many students demonstrated weakness in tackling questions in geometric constructions, circle theorems, and their applications (WAEC, 2017). Per the Chief Examiner's reports (2021), the issue persisted, and issues have been identified. In 2021, a notable problem was students' frequent misinterpretation of geometry questions, leading to incorrect answers. Common errors included confusion between different types of angles and inaccuracies in geometric diagrams, particularly in questions requiring precise constructions or shape representations (WAEC, 2021). Similarly, the 2023 Chief Examiner reported weaknesses in students' geometry performance. A key concern was students' difficulty in applying procedural knowledge, such as the correct steps needed to prove geometric theorems or solve problems involving multiple geometric properties (WAEC, 2023). Specific challenges included problems related to circle theorems, mensuration, and cyclic quadrilaterals, where many students struggled with geometric transformations, angle properties, and calculations of areas and volumes (WAEC, 2023). Moreover, research also indicates that Ghanaian students consistently underperform in geometry compared to other mathematical areas. Many students find it challenging to identify geometric properties due to a poor grasp of formal geometric concepts. This issue is further exacerbated by the abstract nature of geometry- a sentiment echoed by both teachers and students, and deficiencies in teaching methods (Oladosu, 2014; Asemani *et al.*, 2017). Furthermore, Mereku and Baffoe (2011) found that a significant percentage of students struggled with geometric concepts, with only 1% reaching level 3 of understanding. Luneta (2015) argued that students lacking adequate geometric thinking skills struggle to solve geometry problems effectively. Purnomo and Machromah (2018) further linked poor visualization of solid geometry to students' difficulties. They noted that traditional teaching methods often fail to engage students or address their individual needs in geometric visualization (Purnomo & Machromah, 2018; Luneta, 2015).

Practical approaches to address these issues require innovative geometry instructions that enhance students' spatial visualization abilities and foster a deeper understanding of geometric concepts (Arici & Asian-Tutak, 2015; Hidayah & Istiandaru, 2018). Consequently, studies have focused on techniques aimed at developing students' geometry visualization skills. Especially the use of manipulatives (origami) in geometry instruction has received much support and has been documented in a plethora of studies (e.g., Garrity, 1998; Strtchens *et al.*, 2001; Enki, 2014; Hwang *et al.*, 2009; Jones, 2010; Arici & Asian-Tutak, 2015). Manipulatives are physical objects used in teaching to help students understand geometric concepts by making abstract ideas more tangible when integrated with written and oral questions (Hidayah *et al.*, 2018). Similarly, Dynamic Geometry Software like GeoGebra and Cabri 3D has been studied to offer interactive environments for students to manipulate geometric shapes and observe the effects in real-time (Baki *et al.*, 2011; Hodiyanto & Santoso, 2020; Ilma & Lestari, 2016). Computer-aided instructions using DGS have also been shown to enhance visualization abilities (Mjenda *et al.*, 2023). Furthermore, studies have emphasized virtual and augmented reality provide immersive experiences that allows interaction with virtual three-dimensional geometric shapes (Gulkilik, 2016), virtual manipulatives (Steen *et al.*, 2006; Lee & Chen, Leopold, 2005), freehand sketching combined with computer graphics applications (Contero *et al.*, 2005), computer simulations and animations (Rafi *et al.*, 2008), and interactive learning strategies, such as the use of SketchUp Make (Wahab *et al.*, 2018). Despite this, minimal efforts have been directed toward the use of metacognitive instructional techniques. The purpose of this study is to investigate the impact of metacognitive strategies on the development of spatial visualization among senior high school students in geometry. The study seeks to achieve the following objectives: (i) identify the gender differences in spatial visualization performance after students had been taught geometry using metacognitive strategies, (ii) determine the effects of metacognitive instruction on spatial visualization performance among senior high school students, and (iii) find out the difference among the metacognitive strategies used in the geometry teaching.

2. Method

2.1 Research Design and Participants

This study employed a randomized controlled trial (RCT) design, specifically a 'post-test only control and experimental group design' (Cohen *et al.*, 2018). An equal number of participants were randomly assigned to the control group and the experimental group, with no pre-test administered to establish baseline measurements. Each experimental group received specific metacognitive instruction aimed at enhancing spatial visualization learning strategies, while the control group did not receive any intervention. Following the intervention, all groups completed a post-test to evaluate the effectiveness of the treatment (metacognitive strategies). This design allows for a robust comparison of the outcomes across groups, as illustrated in Figure 2, which outlines the structure of the study, including the control group and the three distinct experimental groups. By focusing solely on post-test results, this design minimizes potential biases related to pre-existing knowledge and provides clearer insights into the impact of the metacognitive strategies employed.

Figure 2: Study Design

Experimental	R_1	X_1	O_1
Experimental	R_2	X_2	O_2
Experimental	R_3	X_3	O_3
Control	R_4		O_4

Source: Author's Construct (2024).

The study comprised 353 convenience samples of senior high school students studying core mathematics in their second semester. The sample size was computed using the Yamane formula as shown below:

$$n = \frac{3014}{1 + 3014(0.05)^2}$$

$$n = 353$$

The sample was divided into four groups: a control group (CG) that received no instruction, a group taught with cognitive modelling (CM), a group taught with self-questioning (SQ), and a group taught with reciprocal teaching (RT). The groups were equivalent and each comprised 88 participants. The CG group did not receive any instruction, but they were given their tests straight away; however, the CM, SQ, and RT groups each took their tests after undergoing tutelage by the researcher himself. Both the administration of standardised tests was personally undertaken by the researcher to ensure consistency and adherence to research protocols. A total of 353 test scripts were printed according to the determined sample size, each carefully prepared to evaluate students' spatial visualization skills following instruction with metacognitive techniques. Before the commencement of data collection, formal consent was obtained from the school authorities. This involved submitting an introductory letter outlining the research objectives and seeking permission to administer questionnaires and conduct tests among the students. Upon receiving approval, a specific date was scheduled for the data collection process to commence, ensuring minimal disruption to the school's academic activities.

During the administration phase, participants were provided with the test scripts after receiving instruction in geometry utilizing the outlined metacognitive strategies. They were given adequate time to independently complete the test items based on their understanding and interpretation of the subject matter. The researcher ensured a conducive testing environment and adhered to a strict 30-minute duration for completing the tests, achieving a commendable 100% response rate from all participants. To uphold ethical standards, the researcher prioritized data protection, confidentiality, and anonymity throughout the study. Participants' responses were treated with utmost confidentiality, with identifiers removed to safeguard their privacy.

2.2 Instruments

A standardised spatial visualization test was used to gather data. The spatial visualization test was administered to participants across three groups after they had been taught the metacognitive strategies. This test consisted of objective items designed to assess students' spatial visualization performance. The items were adapted from established measures used in previous research and tailored to suit the specific objectives of this study (Jones, 2016). Reliability analysis was computed to determine the consistency and internal consistency of the spatial visualization test. The reliability analysis helped to assess if the spatial visualization test measured what it was designed for. The Cronbach's alpha coefficient was used to quantify the internal consistency of the test (Cronbach, 1951). Cronbach's alpha was used to estimate the test's internal consistency. It evaluated how well various items on the test measured the same variable. The interpretation of the coefficient was based on its magnitude. A value between 0 and 1, with higher values closer to 1 indicating a good measure of reliability; however, a value of 0.70 or higher is generally deemed acceptable. Cronbach's alpha was computed in SPSS (version 25), and the results showed that the test had a CA coefficient of 0.69 (closer to

0.7), indicating that there is good reliability. Similar CA results have been confirmed in previous studies (Bandoh et al., 2024; Koomson et al., 2024; Lotey et al., 2025), demonstrating acceptable criteria.

3. Results

3.1 Demographic Information of Participants

This study comprised 353 second-year students selected from four senior high schools in two regions in Ghana. The background information of the participants is summarized in Table 2. Table 2 shows that participants comprised exactly 152 males, accounting for 43.2%, and 200 females, accounting for 56.8%. Looking at the percentages, it can be inferred that females participated more in the study than males. However, the factors that accounted for this disproportion could not be identified. The number of students aged 15 or younger was 64 (18.2%). The majority (272), corresponding to 77.3% of participants, are aged between 16 and 20 years. This means that most senior high school students are teenagers aged in this range. In addition, 16 students were aged between 21 and 25 years. This number also accounted for 4.5% of the total number of participants (see Table 2).

Table 1: Profile of Participants

Demographics	Frequency (N)	Percentages (%)
Gender	**352**	**100.0**
Male	152	43.2
Female	200	56.8
Age	**352**	**100.0**
15 years and below	64	18.2
16-20 years	272	77.3
21-25 years	16	4.5

Source: Field test (2024).

3.2 Descriptive Statistics

This study comprised three groups that received metacognitive instructions, namely cognitive modelling (CM), self-questioning (SQ), and reciprocal teaching (RT). The effects of these metacognitive strategies on spatial visualization performance were assessed through spatial visualization tests. The study also comprised a control group (CG) that did not receive instructions under any of the metacognitive instructional techniques. The control group served as the baseline for measuring the spatial visualization performance of students in the metacognitive groups. The performance of students in each group was recorded, and the results of descriptive analysis are presented in Table 3. The descriptive statistics show that the lowest median was scored among students in the control group (median = 3.00). The mean and standard deviation of this group were 3.14 and 1.13, respectively. A higher median was recorded among the CM, SQ, and RT groups. The CM group recorded a mean of 6.34 and a standard deviation of 1.90. The other groups taught by SQ and RT had means of 5.95 and 5.68 and standard deviations of 1.24 and 1.41 in that order. Comparing the median of the control group to the groups taught through metacognitive strategies, it is reasonable to conclude that the use of metacognitive techniques in geometry teaching and learning affects spatial visualization performance because there was a higher mean recorded. However, additional research is required to validate these descriptive findings.

Table 2: Descriptive Statistics of Study Groups

Group	Median	Mean	Std. Deviation
CG	3.00	3.14	1.13
SQ	6.00	5.95	1.24
CM	6.00	6.34	1.90
RT	6.00	5.68	1.41

Source: Field test (2024).

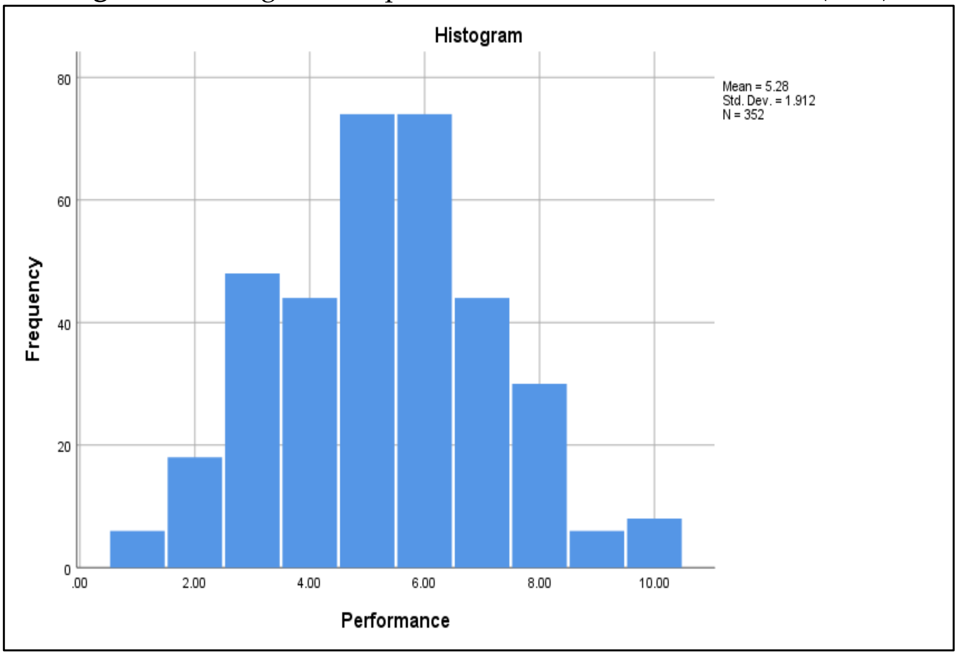
3.3 Normality Analysis

Normality of any distribution is crucial for deciding whether to employ parametric or non-parametric data analysis procedures (Ghosh & Mitra, 2020). To assess the normality of the spatial visualization performance (SVP) data, histograms (see Figure 4) and box plots (see Figure 5) were created to graphically illustrate the properties of the distribution. When examining the histogram for the SVP data, it did not exhibit the characteristic bell shape associated with a normal distribution; instead, it appeared skewed, indicating a departure from normality. Additionally, a normally distributed dataset should display a single peak at the centre, but the histogram for the SVP data showed two peaks (bimodal), suggesting that the data may not follow a normal distribution pattern. Furthermore, normal distributions feature tails that taper off symmetrically, while the histogram displayed heavy or asymmetric tails, further indicating potential deviations from normality in the SVP data. The box plot analysis reinforced these findings; in a normally distributed dataset, the median should be approximately centred within the box, but in the case of the SVP data, the median was closer to either the lower quartile, suggesting potential skewness. The whiskers in the box plot extend from the box to the minimum and maximum values within 1.5 times the interquartile range (IQR), and for normal distributions, the lengths of the whiskers should be roughly equal. However, the box plot for the SVP data revealed unequal whiskers, indicating skewness and supporting the conclusion that the data deviates from normality. The presence of points plotted beyond the whiskers indicated potential outliers in the SVP data; a normal distribution typically has fewer outliers, but the box plot displayed a notable number of outliers, suggesting that the distribution is not normal. Additionally, skewness and kurtosis were computed, with skewness values ranging from -0.513 to 0.100 and kurtosis values ranging from -1.945 to 1.135. According to Hair *et al.* (2010), data are considered normally distributed if skewness ranges from -2 to +2 and kurtosis ranges from -7 to +7. However, these values alone provide little evidence to support that SVP was normally distributed.

To further investigate the normality of the data, the Shapiro-Wilk and Kolmogorov-Smirnov tests were conducted. The results presented in Table 5 indicate that the distribution of SVP was not normally distributed. Both tests yielded statistically significant results, leading to the rejection of the null hypothesis (H0), which states that the distribution is normally distributed. This finding supports the conclusion that the distribution of SVP significantly deviates from normality (see Table 5). Specifically, if the p-values from these tests are less than the chosen significance level (typically 0.05), the null hypothesis of normality is rejected, indicating that the distribution is not normal.

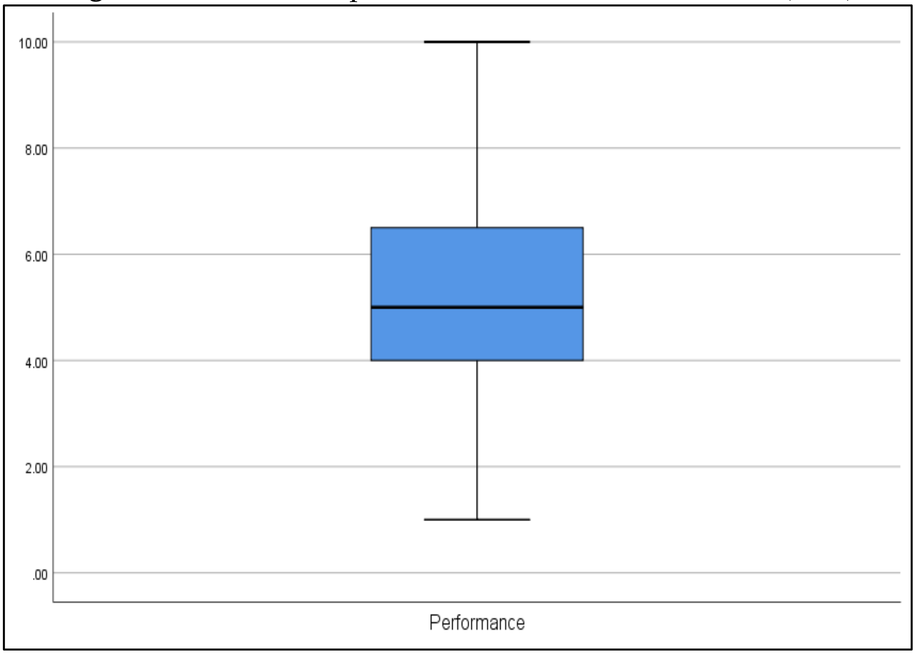
Given this lack of normality, the use of parametric tests would be methodologically inappropriate. Therefore, this study employed non-parametric alternatives, specifically the Mann-Whitney U test, to compare the two gender categories and to conduct pairwise comparisons of the metacognitive strategies. Additionally, the Kruskal-Wallis H test was used to compare the performance of the experimental groups (CM, SQ, and RT) against the control group (CG).

Figure 1: Histogram of Spatial Visualization Performance (SVP)



Source: Field test (2024).

Figure 2: Box Plot of Spatial Visualization Performance (SVP)



Source: Field test (2024).

Table 1: Kurtosis and Skewness Statistics

Skewness		Kurtosis	
Statistic	**Std. Error**	**Statistic**	**Std. Error**
.100	.183	-.261	.364

Source: Field test (2024).

Table 2: Normality Tests

Kolmogorov-Smirnov[a]			Shapiro-Wilk		
Statistic	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
.113	352	.000	.970	352	.000

a. Lilliefors Significance Correction
Source: Field test (2024).

3.4 Gender Differences in Spatial Visualization Performance among the Metacognitive Instructional Strategies

Given that both the Kolmogorov–Smirnov test and the Shapiro-Wilk test demonstrated that the spatial visualization performance (SVP) distributions were not normally distributed ($p \leq 0.05$), the Mann–Whitney U test was utilized to compare the results across gender categories. The findings are detailed in Table 6. For the CM group, the descriptive statistics indicated that SVP scores for males (median = 6.00, mean rank = 47.34) were comparable to those of females (median = 6.00, mean rank = 40.76). The Mann–Whitney U test results for the CM group revealed no statistically significant differences between the genders ($U = 808.00$, $Z = –1.251$, $p > 0.05$). The observed difference in SVP performance between males and females was minimal ($r = -0.133$), suggesting an insignificant effect size. In the case of the SQ group, the descriptive statistics similarly reflected comparable SVP scores for males (median = 6.00; mean rank = 47.15) and females (median = 6.00; mean rank = 41.60). The Mann–Whitney U test results (U = 844.00, Z = -1.058, $p > 0.05$) confirmed that there was no statistically significant difference in SQ scores between genders, with a small effect size observed (r = -0.113).

For the RT group, the descriptive statistics indicated that males (median = 6.00; mean rank = 41.07) and females (median = 6.00; mean rank = 46.28) had similar SVP scores. The results from the Mann–Whitney U test (U = 767.00, Z = -0.927, $p > 0.05$) further suggested that there were no significant differences in RT between males and females, once again reflecting a small difference (r = -0.099). In conclusion, the findings indicate that there are no significant differences in spatial visualization performance based on gender for any of the assessed components (CM, SQ, and RT). This suggests that gender does not have a meaningful impact on SVP in this study.

Table 3: Mann-Whitney Tests for Gender Difference in SVP across Groups

Group	Category	N	Mean Rank	Sum of Ranks	U	Z	P-value
CM	Male	50	47.34	2367.00	808.00	-1.251	0.211
	Female	38	40.76	1549.00			
SQ	Male	46	47.15	2169.00	844.00	-1.058	0.290
	Female	42	41.60	1747.00			
RT	Male	30	41.07	1232.00	767.00	-0.927	0.354
	Female	58	46.28	2684.00			

Source: Field test (2024).

3.5 Effects of Metacognitive Instructional Strategies on Spatial Visualization Performance

Both the Kolmogorov–Smirnov and Shapiro-Wilk tests indicated that the spatial visualization performance (SVP) data were not normally distributed across all groups ($p \leq 0.05$). Consequently, the Kruskal-Wallis H test was conducted to identify potential differences in SVP among the groups in this study (see Table 7). The findings from the Kruskal-Wallis test revealed significant variations in the mean ranks of SVP among the four groups ($\chi^2 = 173.227$, H = 153.23, $p \leq 0.05$, df = 3). To explore differences specifically between the experimental groups—CM, SQ, and RT—and the control group (CG), pairwise Mann-Whitney U tests were performed, utilizing Bonferroni correction to mitigate the risk of Type I errors resulting from multiple comparisons. The Kruskal-Wallis test highlighted the necessity for post hoc analyses to assess differences between individual group pairs. The ensuing comparisons uncovered statistically significant differences ($p < 0.05$), as outlined in Table 8. The results in Table 8 demonstrated that the SVP for the CM group (median = 6.00; mean rank = 126.84) was significantly greater than that of the CG group (median = 3.00; mean rank = 50.16). The Mann-Whitney U test confirmed a significant difference between the CG and CM groups (U = 498.00, Z = -10.098, $p < 0.05$). Moreover, the SVP for the SQ group (median = 6.00; mean rank = 127.86) was also found to be higher than that of the CG group (median = 3.00; mean rank = 49.14), with the Mann-Whitney U test results indicating a significant difference between the SQ and CG groups (U = 408.00, Z = -10.413, $p < 0.05$). Additionally, the SVP for the RT group (median = 6.00; mean rank = 124.59) surpassed that of the CG group (median = 3.00; mean rank = 52.41). The Mann-Whitney U test results corroborated a significant difference between the RT and CG groups concerning SVP ($U = 696.00$, $Z = -9.53$, $p < 0.05$).

Table 4: Kruskal-Wallis H Output

Group	Mean Ranks	H	df	p
CG	62.70	153.23	3	0.000
CM	225.20			
SQ	216.95			
RT	201.14			

Source: Field test (2024).

Table 5: Post-hoc Pair-wise Comparison of Experimental Groups to Control Group

Groups	N	Mean Rank	Sum of Ranks	U	Z	P-value
CG	88	50.16	4414.00	498.00	-10.09	0.00
CM	88	126.84	11162.0			
CG	88	49.14	4324.00	408.00	-10.41	0.00
SQ	88	127.86	11252.0			
CG	88	52.41	4612.00	696.00	-9.53	0.00
RT	88	124.59	10964.0			

Source: Field test (2024).

3.6 Difference in Spatial Visualization Performance among Metacognitive Instructional Strategies

The Kruskal-Wallis H test, suitable for analysing non-parametric data (Kruskal & Wallis, 1952), was utilized to investigate differences in spatial visualization performance (SVP) among the experimental groups (CM, SQ, RT) at a significance level of 0.05 (refer to Table 9). This test assesses variations in mean ranks throughout the dataset to evaluate the null hypothesis (H_0), which posits that all medians are equal. The results are expressed as H, a statistic that approximates chi-square (χ^2) with k - 1 degrees of freedom (df). A higher H value indicates a more substantial difference between the medians. The Kruskal-Wallis test results revealed that the mean ranks for SVP across the three metacognitive groups were not significantly different, yielding a low H value ($H = 3.78$, $p > 0.05$, $df = 2$). Nevertheless, since the Kruskal-Wallis test does not provide insights into the differences between individual group pairs, further post hoc analysis was conducted using the Mann-Whitney U test (Mann & Whitney, 1947). For the pairwise comparisons, a series of Mann-Whitney U tests were performed (see Table 10), as outlined in the second objective. To mitigate Type I statistical errors (the incorrect rejection of a true null hypothesis), the Bonferroni correction was applied. This correction involved dividing the a priori significance level ($p = 0.05$) by the number of tests conducted on the dataset ($n = 3$), establishing a significance threshold of $p < 0.0167$ ($p = 0.05/3$). The results from the Mann-Whitney U tests indicated that the SVP of the CM group (median = 6.00, mean rank = 91.70) did not differ significantly from that of the SQ group (median = 6.00, mean rank = 85.30), as the statistical results were not significant ($U = 3590.00$, $Z = -0.854$, $p > 0.0167$).

Similarly, the comparison between the CM group (median = 6.00, mean rank = 95.66) and the RT group (median = 6.00, mean rank = 81.34) showed no significant difference, as supported by the Mann-Whitney U test results ($U = 3242.00$, $Z = -1.896$, $p > 0.0167$). Lastly, no significant difference was found in SVP between the SQ group (median = 6.00, mean rank = 92.80) and the RT group (median = 6.00, mean rank = 84.20), with Mann-Whitney U test results indicating ($U = 3494.00$, $Z = -1.148$, $p > 0.0167$).

Table 6: Kruskal-Wallis Output for Difference in Metacognitive Groups

Group	Mean Ranks	H	Df	P
CM	142.86	3.776	2	0.151
SQ	133.59			
RT	121.05			

Table 7: Pairwise Comparisons of Metacognitive Groups

Groups	N	Mean Rank	Sum of Ranks	U	Z	P-value
CM	88	91.70	8070.00	3590.00	-0.854	0.393
SQ	88	85.30	7506.00			
CM	88	95.66	8418.00	3242.00	-1.896	0.058
RT	88	81.34	7158.00			
SQ	88	92.80	8166.00	3494.00	-1.148	0.251
RT	88	84.20	7410.00			

Source: Field test (2024).

4. Discussion

4.1 Effects of Gender on Spatial Visualization Performance

The study's finding that gender has no significant effect on spatial visualization challenges long-standing stereotypes and preconceived notions about inherent gender differences in cognitive abilities. Spatial visualization, the ability to manipulate, rotate, and transform visual and spatial information, is a critical skill in various fields (Voyer *et al.*, 1995); however, historically, research has suggested that males generally outperform females in spatial tasks (Linn & Petersen, 1985; Voyer, Voyer, & Bryden, 1995). These findings have often been used to explain the gender disparity in STEM (science, technology, engineering, and mathematics) fields. However, the current study's results align with more recent research that questions these broad generalizations and highlights the role of socio-cultural factors and individual differences over biological determinism (Hyde, 2005). The findings of this study were generally affected by the current methodology. Unlike some previous studies that utilized tasks with significant male-centric biases or tasks influenced by social and cultural conditioning, this study employed a diverse set of spatial tasks designed to be gender neutral and equally accessible to all participants (Spelke, 2005). This methodological rigor ensured that the results were not skewed by extraneous variables, providing a more accurate measure of innate spatial visualization ability. The absence of a significant gender effect in spatial visualization supports the broader argument for gender equity in educational and professional opportunities. It underscores the need to dismantle gender stereotypes that can discourage females from pursuing careers in spatially demanding fields. Encouragingly, initiatives aimed at promoting female participation in STEM should continue to emphasize that cognitive abilities are not inherently gendered but are instead influenced by a variety of factors, including socialization, education, and personal interest (Ceci & Williams, 2010). The finding has important implications for education and policy. Previous research has shown that with appropriate training and practice, individuals can significantly improve their spatial skills regardless of gender (Uttal *et al.*, 2013). This suggests that educational interventions focused on enhancing spatial reasoning skills can be equally effective for both males and females, thereby helping to close the gender gap in STEM participation (Newcombe, 2010).

4.2 Effects of Metacognitive Instructions on Spatial Visualization Performance

The current finding marks a significant contribution to the fields of educational psychology and cognitive development. Metacognition, defined as awareness and regulation of one's cognitive processes, is very important in enhancing learning outcomes. Research by Schraw and Dennison (1994) highlighted that metacognitive strategies like self-monitoring, self-regulation, and reflective thinking improve problem-solving skills and academic performance. Building on this foundation, the current study reveals that metacognitive instruction can significantly enhance spatial visualization skills. Metacognitive instruction involves teaching students to think about their thinking, allowing them to gain deeper insights into spatial tasks, identify errors, and develop effective strategies for problem-solving. The mechanisms through which metacognitive instruction impacts spatial visualization are multifaceted. Flavell (1979) posited that metacognitive strategies encourage learners to engage in deeper cognitive processing of spatial tasks. By actively monitoring and regulating their cognitive processes, learners can identify and correct errors, leading to improved performance. Furthermore, Zimmerman (2002) suggested that metacognitive instruction aids learners in developing effective strategies for approaching spatial tasks, such as breaking down complex visualizations into manageable steps. This approach helps learners visualize rotations and transformations more accurately and efficiently. The findings align with prior research demonstrating the benefits of metacognitive instruction across various cognitive domains. For instance, Mevarech and Kramarski (2003) found that metacognitive training improves mathematical problem-solving skills by enhancing students' ability to plan, monitor, and evaluate their problem-solving processes. Similarly, Pressley and Afflerbach (1995) reported that metacognitive strategies improve reading comprehension by helping students better understand and retain textual information. The implications of this study for educational practice are profound. Incorporating metacognitive instruction into curricula can significantly enhance students' spatial visualization abilities, which are critical for success in STEM fields (Newcombe, 2010). Educators can implement metacognitive strategies through explicit instruction, modelling, and guided practice. Paris and Winograd (1990) emphasized that fostering metacognitive awareness can empower students to become more effective and independent learners. The integration of metacognitive instruction into their teaching will enable educators to help students develop skills necessary to monitor and regulate their cognitive processes effectively.

4.3 Differences in Metacognitive Instructional Strategies

The current study's finding that there are no significant differences in the effectiveness of various metacognitive instructional strategies on spatial visualization performance is an intriguing result. Previous research has extensively documented the benefits of metacognitive strategies on various cognitive domains. For instance, Schraw and Dennison (1994) identified that metacognitive strategies, such as self-monitoring and self-regulation, enhance academic performance across various subjects. Similarly, Mevarech and Kramarski (2003) demonstrated that metacognitive training significantly improves mathematical problem-solving skills, while Pressley and Afflerbach (1995) found it beneficial for reading comprehension. Metacognitive strategies work by encouraging learners to reflect on their thinking processes, leading to better self-regulation and error correction (Flavell, 1979). These strategies typically involve planning, monitoring, and evaluating one's cognitive processes, which can lead to improved performance in tasks requiring spatial visualization (Zimmerman, 2002). The current study's finding that no single metacognitive instructional strategy significantly outperforms others in enhancing spatial visualization performance suggests that the underlying mechanisms of these strategies may be equally effective. This lack of difference might be attributed to the fundamental nature of metacognitive strategies, which universally encourage deeper cognitive engagement and reflection regardless of the specific approach used. While previous studies have highlighted the effectiveness of specific metacognitive strategies in various cognitive domains, they have not always compared these strategies against each other in terms of spatial visualization performance. The finding that no significant differences exist among metacognitive strategies in this context aligns with the broader understanding that the core components of metacognition—self-monitoring, self-regulation, and reflective thinking—are universally beneficial. For instance, studies by Schraw and Dennison (1994) and Mevarech and Kramarski (2003) emphasized the overall benefits of metacognitive strategies without distinguishing between specific methods. Similarly, Pressley and Afflerbach (1995) focused on the general impact of metacognitive strategies on reading comprehension, not comparing individual strategies. Therefore, the current study's findings suggest that all metacognitive strategies may share a common efficacy due to their foundational emphasis on cognitive reflection and regulation. These findings imply that educators can have flexibility in choosing which metacognitive strategies to implement, knowing that any well-structured metacognitive approach is likely to enhance spatial visualization skills. This flexibility can allow educators to tailor their instruction to the needs and preferences of their students without worrying about the relative effectiveness of different metacognitive strategies.

5. Recommendations

Educators should design and implement spatial visualization tasks that are gender-neutral and accessible to all students. Through this strategy, biases in task selection can be avoided, helping teachers to create a more equitable learning environment that allows both male and female students to demonstrate their spatial abilities without the influence of preconceived notions about gender differences.

Senior high and technical schools should incorporate metacognitive strategies into their teaching practices to enhance students' spatial visualization skills. Teachers can provide explicit instruction on self-monitoring, self-regulation, and reflective thinking, equipping students with tools to analyze and improve their cognitive processes. This could involve structured activities that encourage students to think critically about their approaches to spatial tasks in geometry.

5.1 Future Research Direction

Future research could investigate the socio-cultural factors that impact spatial visualization performance across different demographics. This could involve qualitative studies that examine how cultural background, educational experiences, and parental influences shape students' spatial abilities and attitudes toward senior high and technical schools.

6. Conclusions

This study reveals crucial insights into spatial visualization performance (SVP) in geometry teaching and learning, challenging stereotypes about gender differences by demonstrating no significant disparities between males and females. It highlights that cognitive skills are shaped more by teaching strategies than by gender alone. The findings also emphasize the effectiveness of metacognitive instruction in enhancing spatial visualization performance, indicating that such strategies benefit all learners and can help close the gender gap in senior high technical schools. Furthermore, the study found no significant differences among various metacognitive strategies, suggesting that their core strategies are universally beneficial. Overall, the research underscores the importance of educational reforms that promote gender equity and integrate metacognitive instruction into curricula, fostering an inclusive environment that develops the spatial visualization skills essential for success in geometry.

Conflict of Interest Statement

The author(s) declare no conflicts of interest.

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