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AN ANALYSIS OF THE REASONING SKILLS OF PRE-SERVICE TEACHERS IN THE CONTEXT OF MATHEMATICAL THINKING

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Abstract:
The aim of this study is to address and analyse pre-service teachers' mathematical reasoning skills in relation to mathematical thinking processes. For these purposes, pre-service teachers' mathematical reasoning skills namely generalising/abstraction/modelling, ratiocination, development and creative thinking skills and the relationships among these skills are examined. Apart from these, it is explored whether grade level and gender have an effect on the application of these skills. The study is based on a mixed method research design and is carried out with 197 pre-service teachers of different grade levels in the school of education of a public university. One of the data collection tools used in the study is mathematical thinking and reasoning skills test which was developed by Başaran (2011) and comprises 21 open-ended questions on real-life problems. The second one is the Mathematical Reasoning Skills and Indicators developed by the researchers in the light of a study by Alkan and Taşdan (2011). Content analysis is performed on the data gathered from the pilot study conducted as the first step of the data analysis and the content of the quantitative data analysis is defined. As the second step, some parametric and non-parametric tests are utilized using the SPSS 15.0 software. As a result of the study, it has been revealed that pre-service teachers' scores on generalising/abstraction/modelling and ratiocination skills are close to average whereas their scores on development and creative thinking skills are below average. It has also been concluded that all the relationships among pre-service teachers' reasoning skills are significant and that correlations among the skills which are associated with stages that follow one another are stronger than the others. Another result of the study is that, in relation to the gender variable, there is a significant difference among the scores concerning generalising/abstraction/modelling and ratiocination skills, yet there are not any significant differences among the development and creative thinking skills scores. In

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relation to the grade variable, on the other hand, there aren't any significant differences among the scores concerning generalising/abstraction/modelling (GAM) and reasoning (R) and improving (İ) skills, yet there are significant differences among the creative thinking skills scores of freshman and sophomore pre-service teachers and among those of the sophomore and junior pre-service teachers.Results of the present study are discussed in relation to the relevant literature and some recommendations for future studies pertaining to the subject and to learning environment quality are presented.

Keywords: mathematical reasoning, mathematical thinking skills, pre-service teachers

1. Introduction

Thinking is one of the most significant tools humans use to understand and control the world around them (Burton, 1984). This tool manifests itself particularly in individuals' attempts to enlighten or solve a problem or to explain emergent situations (Yıldırım, 2000). Therefore, thinking does not only comprise the skills we need to succeed in our working or education lives but also involves the basic skills we need in order to survive (Çelik, 2016). The current information age we live in aims to raise individuals who have due functional knowledge to survive and are able to use this knowledge efficiently. Some curricula prepared for these purposes are tailored so as to ensure individuals develop certain thinking skills. NCTM (1989) names the skills required for using established knowledge in life as problem solving, reasoning, communication, making connections and using representations and expresses that these are the skills that form the foundations of mathematical thinking. NCTM (1991) contends the goals of teaching mathematics are "*all students can learn to think mathematically*" (p. 21). In Turkey, it is stated in the mathematics curricula that the process of *exploration, recognising logical relationships and expressing in mathematical terms* is the basis of mathematical thinking. According to the curricula, it is also necessary to enhance students' skills in problem solving and making connections as well as their communication, mathematical modelling and reasoning skills in order to promote mathematical thinking (Ministry of National Education [MoNE], 2011).

As to the content of mathematical thinking, different studies suggest different mathematical skills and address different aspects of mathematical thinking process. Isoda and Katagiri (2012) group all these studies into two and report that education research on mathematical thinking has two different perspectives namely *mathematical processes* and *conceptual development.* According to this, researchers who handle the concept of mathematical thinking within the framework of mathematical processes focus largely on *how mathematical thinking is realized*. Polya (1945, 1957, 1962, 1965), one of the primary researchers who adopts this perspective, considers problem solving one of the fundamental components of mathematical thinking. Schoenfeld (1992), who holds a similar point of view regarding the relationship between mathematical thinking and problem solving, associates mathematical thinking with the concepts of disposition and metacognition and indicates learning to think mathematically means *(a) developing a mathematical point of view valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade and using those tools in the service of the goal of understanding structure mathematical sense making* (1994, p. 60). Mason, Burton, and Stacey (1982) sketched the dynamics of mathematical thinking as a helical model constituted of manipulating, getting a sense of pattern, and articulating that pattern symbolically. Burton (1984) described the framework of mathematical thinking in terms of operations, processes and dynamics which also includes both inductive and deductive learning. Stacey (2006) indicates mathematical thinking is an important objective of schooling and is an essential component in the process of teaching and learning mathematics. He adds that a student with high mathematical thinking skills will learn the areas of use of mathematics in daily life and will be able to use mathematics in their daily and working lives. Stacey (2006) defines the essential components of mathematical thinking as "*specializing and generalizing" and "conjecturing and convincing*."

The second approach which handles mathematical thinking in terms of conceptual development attempts, in broadest terms, to define mathematical thinking within the framework of how individuals construct mathematical concepts in their minds and of what processes occur during this construction process. According to Freudenthal mathematical thinking is a process of evolution that emerges from real experiences and culminates in mathematics. Throughout the process, mathematization is the most important component in moving from general thinking to mathematical thinking (Çelik, 2016). Tall uses the term *procept* to define conceptual development and defines mathematical thinking as a network of relationships among three mental worlds called *embodied world, symbolic world, and formal world* (Isoda and Katagiri, 2012). Dreyfus (2002) emphasizes the importance of abstraction and presentation in mathematical thinking indicating also discovering, defining, proving and other processes also occur in the construction of mathematical concepts.

Apart from the afore-mentioned studies, other studies on mathematical thinking mention various components and skills in relation to mathematical thinking processes. Alkan and Taşdan (2011) defined the stages of mathematical thinking processes as given in Figure 1.

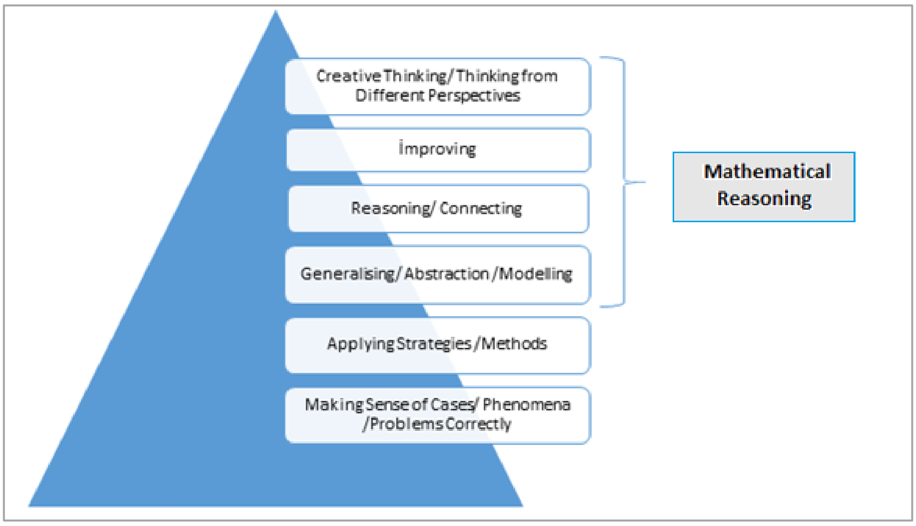

Figure 1: Stages of Mathematical Thinking (Alkan and Taşdan, 2011)

In the above-mentioned study it is explained that there are studies which handle mathematical thinking in terms of such different levels as advanced and elementary (Edwards, Dubinsky and McDonald, 2005; Harel and Sowder, 2005; Tall, 1995); however, the same paper also notes that there are some other studies which indicate it is not possible to clearly distinguish between the elementary and advanced levels of mathematical thinking and which argue that mathematical thinking is a developmental and multistage process. Alkan and Taşdan (2011), who are in favour of the second approach, report that individuals may have different levels of mathematical thinking based on their prior knowledge, experience and lives and that mathematical thinking is a six-staged process which requires reasoning and knowledge of strategies/methods. These stages are presented in Figure 1. The last four stages represent mathematical reasoning processes. Stacey (2006) expresses mathematical thinking in four main processes. They are specialising (trying special cases, looking at examples), generalising (looking for patterns and relationships), conjecturing (predicting relationships and results), convincing (finding and communicating reasons why something is true). At the same time, the author states that mathematical thinking and using this skill for problem-solving are important objectives of schooling and associates this skill with the concept of literacy which emphasizes individuals' ability to use the knowledge of mathematics they gain at schools in their daily lives. According to Stacey (2006) mathematical literacy involves many components of mathematical thinking, including reasoning, modelling and making connections between ideas. In a similar vein, Schoenfeld (1992) defines mathematical thinking skill as an auxiliary skill that helps apply acquired knowledge in real-life situations. In their study, Alkan and Bukova-Güzel (2005) define mathematical thinking as a useful type of thinking since it is used in meeting needs and in ensuring productivity in problem solving. They suggest the following scheme given in Figure 2 to explain the working of mathematical thinking.

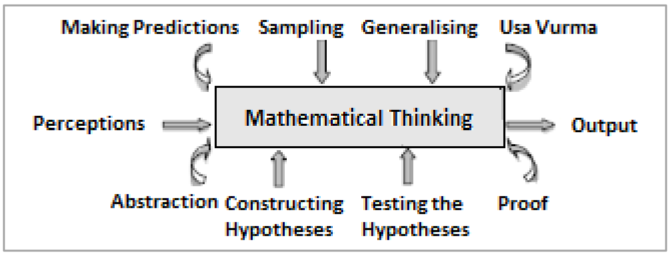

Figure 2: The Working of Mathematical Thinking (Alkan and Bukova-Güzel, 2005)

Liu and Niess (2006) define mathematical thinking as a combination of complex processes that involve *"guessing, induction, deduction, specification, generalization, analogy, reasoning and verification"*; Dreyfus (1990), on the other hand, defines mathematical thinking as abstracting, proving and finally reasoning from hypotheses. Mason, Burton and Stacey (1985) and Çelik (2016) handle mathematical thinking in terms of specializing, generalizing, and conjecturing, justifying and convincing skills. Dunlop (2001) is among the researchers who study mathematical thinking together with problem solving and emphasizes the necessity of mathematical thinking in the process of solving non-routine problems. Henderson (2002) defines mathematical thinking as the application of mathematical techniques, concepts and processes either explicitly or implicitly in the process of solving problems. Burton (1984) expresses that mathematical thinking would be used when trying to solve problems in any area of application. Therefore, it can be concluded from the afore-mentioned studies that mathematical reasoning is an important component of mathematical thinking (Alkan and Taşdan, 2011) and that mathematical thinking skill is an important skill that is used in problem solving processes. Pollack (1997) reports that mathematical reasoning plays an important role in students' attempts to solve open-ended questions stating also that students convey this into tasks of real-life situations (cited in Jbeili, 2003).

Reasoning or, alternatively, ratiocination or intellection is the process of thinking and achieving a reasonable outcome by taking all the elements into consideration. Those who can engage in reasoning in relation to a given subject are knowledgeable enough about that subject and analyse, explore, make reasonable estimations and assumptions, justify their opinions, can come up with conclusions, explain and defend their conclusions (Umay, 2003). Mandacı-Şahin (2007) notes the main indicator of reasoning skills is the ability to use these skills for solving a problem and for seeing the relationship between mathematical subjects. Hence, one of the assessment-related aspects of reasoning skills is problem solving processes. In a study by TIMSS (2003), *non-routine problem solving* is considered to be an indicator of reasoning skills (Beaton, Mullis, Martin, Gonzalez, Kelly and Smith, 2003). Similarly, PISA study which emphasizes the use of mathematics in daily life handles reasoning as a skill for using mathematics and indicates that reasoning skills in particular constitutes the core of problem solving skills. PISA (2003) proves the necessity of using reasoning skills at each stage of problem solving process as follows; for example, an individual should be able to distinguish between the facts and opinions at the problem comprehension stage. They should be able to recognize the relationships among variables at the stage of finding a solution and they should also be able to consider cause and effect. As to the stage of discussing the results, they should be able to organize information in a logical manner (OECD, 2004, p.158).

When the above-mentioned studies that address mathematical thinking in general and specifically reasoning skills are examined, it is seen that only few studies attempt to investigate mathematical reasoning skills which play an important role in the learning and teaching of mathematics as well as in the application of mathematics knowledge to solve real-life problems. Başaran (2011, p.2) writes "*In spite of the fact that mathematical thinking and reasoning are significant in the teaching and learning of mathematics; the system of mathematics education is still to embrace a more global perspective so as to further mathematical thinking and reasoning that fit in with the learning and studying experiences of students.*" In this regard, analysing the mathematical reasoning skills of pre-service teachers who get into university after completing their basic education and have the prospect of raising the next generation is important for two reasons. The present study is expected to contribute to the evaluation of current curricula which aim to enhance mathematical thinking and mathematical reasoning skills of students. Furthermore, it may also offer an insight into the future teaching and learning environments with the observations it provides concerning the skills level of individuals who are the prospective implementers of the curricula and are expected to teach these skills to their students in the future. In view of these, investigating the mathematical reasoning skills of pre-service teachers can be considered as the primary objective of the study.

The study investigates the answers to the following questions:

- What are the generalising/abstraction/modelling, reasoning/connecting, improving and creative thinking skills levels of pre-service teachers?
- Are there any significant relationships among the mathematical reasoning skills of pre-service teachers?
- Are there any significant differences among the pre-service teachers' levels of mathematical reasoning skills based on gender and grade level?

2. Method

The research methodology used in this study is a mixed methods approach. Greene, Caracelli and Graham (1989) define mixed methods designs as research designs with at least one qualitative and one quantitative method. *Main characteristics of mixed methods research involve merging qualitative and quantitative data and offering a better understanding* *of the research problem than would do any individual research method* (Creswell & Clark, 2007, p.5). Mixed methods research helps answer questions that are otherwise would not be answered via qualitative or quantitative research alone (Creswell & Clark, 2014). Many individual research questions or sets of questions would be fully answered thanks to the solutions mixed methods approaches offer (Johnson and Onwuegbuzie, 2004). The reason for preferring mixed methods approach in the resent study is the nature of the research questions which are not answerable by one single research method.

Of the mixed methods research designs, exploratory sequential design was selected for the present study. Exploratory designs begin with and prioritize the collection and analysis of qualitative data and based on the exploratory results, researchers begin applying the second that is the quantitative phase (Creswell & Clark, 2014). The aim of the two-phase exploratory design is to ensure that data from the first or in other words from the qualitative phase help build and enhance data in the second that is in the quantitative phase (Greene et al., 1989). In the first phase of the present study (qualitative phase), a content analysis was applied to each question in the data collection tool in relation to the sub-dimensions of mathematical reasoning skills and their indicators. In the second phase (quantitative phase), numeric data were built based on the data obtained as a result of the content analysis and they were interpreted within the context of the research questions.

2.1 Study Population

Study population consisted of 197 pre-service teachers enrolled in the department of elementary mathematics education at a public university. Participants of the study were defined via typical case sampling, a purposeful sampling approach that helps explore situations with abundant information. Studies that employ typical case sampling do not aim to generalize to the universe by selecting a typical case but to gain an insight into a given field or to inform those who are not familiar with the given field, subject, practice or innovation (Patton, 1987).Details of the study population are presented in Table 1.

Table 1: Characteristics of the Students in the Study Population

Grade Level	Number of Female Students	Number of Male Students	Total
1	38	9	47
2	40	12	52
3	33	17	50
4	29	19	48
Total	140	57	197

2.2 Data Collection Tools

Mathematical Thinking and Reasoning Skills Test (MTR), which was developed by Başaran (2011), was used for evaluating participant pre-service teachers' mathematical reasoning skills. MTR is a tool developed for undergraduate students from different departments and contains a range of mathematical thinking processes in relation to real-life problems. Questions in MTR cover the sub-dimensions of mathematical thinking and mathematical reasoning skills in the relevant literature, in addition to real-life mathematical processes and skills that are related to mathematical literacy and are emphasized in studies by PISA 2003, TIMSS 2003, NAEP 2009 and Epp (2003). Questions number 1, 4, 5, 8, 14 and 21 were developed by Başaran (2011). The study by Durand-Guerrier (2003) was used for preparing question number 9. As to the 10th and 11th questions, they were derived from a study by Wason (1968) and by PISA 2006, respectively. Finally, other questions were prepared in the light of D'Angelo and West's study (2000).

Another data collection tool used in the study was *Mathematical Reasoning Skills and Indicators* (MRS). MRS was prepared by the research team in the light of a study by Alkan and Taşdan (2011). The resultant MRS differs from the original one by Alkan and Taşdan (2011) only slightly. It is believed that these slight differences do not have a significant effect on the definition and evaluation of the subject skills. One of these differences is the indicators "*being curious" and "exploring the whys and hows*" used at the development stage of mathematical reasoning described by Alkan and Taşdan (2011). The subject indicators were not used in the MRS used by this study since the data collection tool did not involve any questions or situations with such an indicator. All the main indicators adopted both by the original and present studies for the observation of development skills are believed to be sufficient for the observation and evaluation of the each question of the study. This assumption aims to make the mathematical reasoning process easy to study; however, this does not mean that other indicators are rejected. Mathematical reasoning skills and their indicators (MRS) created by the researchers of this study are given in Table 2.

Table 2: Mathematical Reasoning Skills and Indicators (MRS)

Mathematical Reasoning Skills (MRS)	**Codes**	**Indicators**
Generalising/Abstraction/Modelling (GAM)	GAM1	Estimating possible contingencies
	GAM2	Using intuition (for the solution)
	GAM3	Conjecturing
	GAM4	Defining the constraints
	GAM5	Justifying opinions
	GAM6	Building accurate relations with the present and the desired thing
	GAM7	Sub-modelling

	GAM8	Modelling
	GAM9	Testing the working and applicability of the model
	GAM10	Drawing conclusions and explaining these conclusions
	GAM11	Justifying
	GAM12	Proving
Reasoning /Connecting (R)	R1	Making inferences
	R2	Thinking critically
	R3	Rational/Logical/ Formal or informal reasoning
	R4	Finding out the meaning of stages/parts for the whole and their contributions to the whole/Making analyses
İmproving (İ)	İ1	Assessing the present situation/phenomena in the context of different circumstances
	İ2	Questioning
	İ3	Using intuition (for development)
	İ4	Answering such questions as "If...happened"
Creative Thinking/ Thinking from Different Perspectives (CT)	CT1	Seeing beyond the present situation
	CT2	Thinking outside the box
	CT3	Thinking flexibly
	CT4	Describing the case/phenomena in a creative way
	CT5	Generating practical ideas

2.4 Analysis of the Data

2.4.1 Pilot Study and Qualitative Data Analysis

As the first step of the data analysis, it was explored which reasoning skills and indicators MTR questions involved and a pilot study was conducted for these purposes. Eight pre-service teachers studying at the same university, who were not included in the real study, were selected for the pilot study group. The group was formed so as to include equal number of pre-service teachers from different grade levels and particular attention was paid to ensure that their GPAs (grand point average) were at medium level. Response options to the questions in MTR, as opposed to the real study, were excluded. It was administered to pre-service teachers as such and they were expected to indicate and explain their responses. No time limit was imposed. After the end of this process, clinical interviews were conducted with each pre-service teacher and the interview process was video-recorded. Interview durations ranged from 60 to 90 minutes. After the interview process ended, researchers watched all the records and tried to define mathematical reasoning skills of pre-service teachers in relation to each question, taking also the previously defined indicators. A sample content analysis process is as follows:

A Sample Content Analysis for Question 3

Question 3- *Hale encounters three people on the street. Each of these three people either always tells the truth or lies all the time. What they say to Hale is given below:*

A: "We all are lying."

B: "Only two of us are lying."

C: "The other two except me are lying."

According to this information, who tells the truth?

Solution Process followed by Pre-service Teacher 1 and the Codes Assigned by the Researchers

Assume that Respondent A tells the truth.– *(GAM3)Conjecturing*

In this case, the statement "We all are lying" will turn out to be true. – *(R3) Logical ratiocination*

This, however, means that Respondent A is also lying and therefore contradicts with our initial assumption. Under the circumstances, Respondent A is lying. – *(GAM5) Justifying opinions*; (*R1) Making inferences; (İ3) Using intuition; (D4) Answering such questions as "If...happened"*

In that case, the statement "We all are lying" will turn out to be false; and hence, the argument "At least one of us tells the truth" will be true. – *(GAM1)Estimating possible contingencies*; (*R1)Making inferences; (İ1) Assessing the present situation/phenomena in the context of different circumstances*

Since we assumed initially that Respondent A is lying, either Respondent B or Respondent C should be telling the truth. Under the circumstances, there are two possibilities:

i. B is true, C is false
ii. C is true, B is false.

- (GAM1) Estimating possible contingencies; (*R3)Logical ratiocination; (R1)Making inferences; (GAM4)Defining the constraints*

In view of the above, the statement of Respondent B namely the one reading "Only two of us are lying." is true both for the case i and case ii. Accordingly, Respondent B is telling the truth.

- (GAM5)Justifying opinions; (*GAM6)Building accurate relations with the present and the desired thing; (R1)Making inferences; (R3)Logical ratiocination; (İ2)Questioning*

Under the circumstances, it is certain that Respondent A is lying and Respondent B is telling the truth. – *(GAM4) Defining the constraints*

The statement by Respondent C reading *"The other two except me are lying."* is false. That is Respondent C is lying. – *(GAM6) Building accurate relations with the present and the desired thing; (R1) Making inferences*

According to the above, research questions and the reasoning skills and respective indicators they are associated with are presented in Table 3 together with their reliability coefficients.

Table 3: Reasoning Skills and Indicators of the Questions in the Data Collection Tool

Question No	Reasoning Skill	Indicator	Percentage of Agreement
	R	R3	19÷(19+6)= 0.76;
1,2	D	D1, D3	18÷(18+7)=0.72
	CT	CT1, CT5	
3	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM10, GAM11, GAM12	
	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	20÷(20+5)=0.80
	CT	CT2, CT4, CT8	
4, 5	GAM	GAM6, GAM7, GAM8, GAM10	21÷(21+4)= 0.84
	R	R3, R4, R5	
6	R	R1, R4	21÷(21+4)= 0.84
7	GAM	GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10	22÷ (22+3) =0.88
	R	R1, R3, R4	
8	GAM	GAM5, GAM6, GAM7, GAM8, GAM9, GAM10	21÷(21+4)= 0.84
	R	R1, R3, R4	
	İ	İ1, İ4	
9i	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	19÷(19+6)= 0.76;
	R	R1, R2, R3, R4	
	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	19÷ (19+6)= 0.76;
9İi, 9iii	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	
	CT	CT2, CT4, CT8	
9iv, 9v	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11	19÷ (19+6)= 0.76;
	R	R1, R2, R3, R4	
9vi	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10	19÷(19+6)= 0.76;
	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	
	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	18÷(18+7)=0.72
10	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	
	CT	CT1, CT2, CT4, CT8	
	GAM	GAM5, GAM6, GAM7, GAM8, GAM10	19÷(19+6)= 0.76
11	R	R1, R2, R3, R4	
	İ	İ1, İ2	

	CT	CT1, CT2	
12	GAM	GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	21÷(21+4)= 0,84
	R	R1, R2, R3, R4	
13	GAM	GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	
	R	R1, R2, R3, R4	22÷(22+3) =0.88
	İ	İ1, İ2, İ4	
14	GAM	GAM6, GAM10, GAM11	20÷(20+5) =0.80
	R	R1, R2, R3, R4	
	İ	İ1, İ2	
15	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11	19÷(19+6)= 0.76
	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	
	CT	CT1, CT2, CT4, CT8	
16, 18	GAM	GAM6, GAM7, GAM8, GAM10, GAM11	18÷(18+7)=0.72;
	R	R1, R2, R3, R4	20÷(20+5)=0.80
17	GAM	GAM1, GAM2, GAM5, GAM6, GAM7, GAM8, GAM10	19÷(19+6)= 0.76
	R	R1, R2, R3, R4	
	İ	İ2, İ3	
	CT	CT2, CT8	
19	GAM	GAM1, GAM2, GAM5, GAM6, GAM7, GAM8, GAM10	19÷(19+6)= 0.76
	R	R1, R2, R3, R4	
20	GAM	GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, GAM7, GAM8, GAM9, GAM10, GAM11, GAM12	18÷ (18+7)=0.72
	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ3, İ4	
	CT	CT2, CT3	
21	GAM	GAM5, GAM6, GAM7, GAM8, GAM10, GAM11	19÷(19+6)=0.76
	R	R1, R2, R3, R4	
	İ	İ1, İ2, İ4	

2.4.2 Quantitative Data Analysis

As the first step into the digitisation of the study data, responses to each question in the data collection tool were converted into quantitative data as true (1) and false (0). Table 4 was utilized for calculating the scores on reasoning skills as separate from each other. Responses to questions which included the indicators of the subject skill were analysed whereas responses to other questions were omitted. In this way, four types of scores were acquired for the reasoning skills of each pre-service teacher. Data converted into quantitative scores were analyzed using parametric and non-parametric tests. Of the chosen tests, independent samples t-test and ANOVA are parametric while Mann Whitney U test and Kruskall Wallis H test are non-parametric. In this analysis SPSS 15.0 software was used and the analysis was conducted in a way that helped answer the sub-questions of the study. Besides, $\boldsymbol{r} = \frac{\boldsymbol{Z-score}}{\sqrt{\boldsymbol{n}}}$ formula was used for the analysis of the practical effect of the significant differences among the defined groups (Pallant, 2011).

2.5 Validity and Reliability

Cronbach Alpha reliability coefficient of the reliability of the data collection tool used in the study was calculated to be 0.76 in Başaran's (2011) study while it was found to be 0.62 for the present study. In the association of the questions in MT with the indicators in MRS, classifications by the researchers and mathematics educations specialists (3 academics) were calculated via the percentage of agreement according to the [Disagreement / (Disagreement + Agreement) X 100] formula. Reliability coefficients acquired as a result of the analyses conducted for each question are presented in Table 4. It is seen from Table 4 that as a result of the process undertaken for defining the reliability of the data analysis, reliability values were found to be higher than 0.70 for each category. This indicates that researchers' classifications are reliable (Yıldırım and Şimşek, 2006).

3. Fındıngs

Skewness and kurtosis values were calculated in order to check whether study data followed a normal distribution. Skewness and kurtosis values for the reasoning skills involved in the study are given in Table 4.

Table 4: Skewness and Kurtosis Values for Reasoning Skills

Reasoning Skill	Skewness	Kurtosis
GAM	-.379	.292
R	-.363	.565
İ	.206	.-475
CT	.375	.-500
Total	-.363	.565

It can be said that data on the reasoning skills followed a normal distribution as the calculated values ranged from -1 to +1 (George and Mallery, 2003). Another assumption of parametric tests is that variances are homogenous. Homogeneity of the study groups were analysed via Levene test. As a result, it was seen that variances of female and male pre-service teachers groups regarding the improving (İ) and creative thinking (CT) variables were homogenous (p>.05) while those regarding the generalising/abstraction/modelling (GAM), reasoning/connecting(R) and total reasoning skills (total) variables were not homogenous (p<.05). When the homogeneity of the groups formed according to grade levels was examined, on the other hand, variances regarding generalising/abstraction/modelling (GAM), reasoning/connecting (R) and improving (İ) were homogenous (p>.05) while those regarding the variable creative thinking (CT) were not homogenous (p<.05).

3.1 Findings concerning the Reasoning Skills

In the present study, pre-service teachers' reasoning skills were examined under four different categories namely generalising/abstraction/modelling (GAM), reasoning/connecting (R), improving (İ) and creative thinking (CT). Findings concerning the levels of reasoning skills are presented in Table 5.

Table 5: Pre-service Teachers' Scores on Reasoning Skills

	N	The Highest and Lowest Possible Scores	X	SD
GAM		0-22	13.522	2.623
R	197	0-26	14.751	2.836
İ		0-15	6.101	1.905
CT		0-10	3.096	1.423

According to the data in Table 5, pre-service teachers' scores on *generalising/abstraction/modelling and reasoning* skills are close to average while their scores on *improving* and *creative thinking* skills are below average. Coefficients of the Pearson correlation analysis which was conducted to test the significance of the correlations among the reasoning skills of the participant pre-service teachers are given in Figure 3.

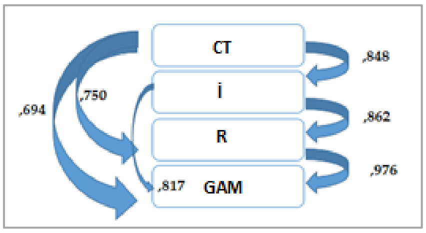

Figure 3: Pearson Correlation Coefficients for the Reasoning Skills (P<.01)

According to the data in Figure 3, all the correlations among the types of reasoning skills are significant and that correlations among the types of skills which are associated with stages that follow one another appear to be stronger than the others.

Results of the independent samples t-test and Mann Whitney U tests conducted for the purposes of analysing pre-service teachers' reasoning skills in relation to gender variable are given in Table 6 and Table 7.

Table 6: Mann Whitney U Test Results for Female and Male Students

		n	Mean Rank	Sum of Ranks	u	p	$r = Z/\sqrt{n}$
GAM	**Female**	140	92.67	13066.50	3055.50	0.013	-0.177
	Male	57	114.94	6436.50			
R	**Female**	140	93.84	13232	3221	0.043	-0.144
	Male	57	111.98	6271			
Total	**Female**	140	93.84	13232	3221	0.043	-0.144
	Male	57	111.98	6271			

Table 7: Independent Samples t-test Results for Female and Male Students

	Gender						
	Male (n:56)		**Female (n:141)**		**SD**	**t**	**P**
	$\overline{X}$	**SD**	$\overline{X}$	**SD**			
İ	6.42	1.79	5.97	1.93	195	-1.52	0.129
CT	3.35	1.51	2.99	1.37		-1.62	0.105

According to Table 6 and Table 7, of the overall reasoning skills, there is a significant difference between the scores on generalising/abstraction/modelling and reasoning/connecting skills of female and male students based on the gender variable (U_1=3055.50; U_2= 3221, p<.05); however, no significant difference appears between those regarding development and creative thinking (p>.05). Pre-service teachers' scores on mathematical reasoning skills differ significantly according to the gender variable (U= 3221, p<.05). As the size effect (r) values for these analyses are lower than .1 (Cohen, 1988), gender can be said to have a relatively low effect on the subject skills.

Results of the ANOVA test conducted for the purposes of defining whether there were significant differences among pre-service teachers' generalising/abstraction/modelling (GAM) and reasoning (R) and improving (İ) skills based on the grade level variable are given in Table 8 and Table 9.

Table 8: Descriptive Results of the ANOVA Test based on the Grade Level Variable

Reasoning Skill	Grade level	n	Mean	SD
	1	47	13.70	2.25
GAM	2	52	13.94	2.36
	3	50	12.86	2.74
	4	48	13.58	3.02
	1	47	14.76	2.31
R	2	52	15.38	2.70
	3	50	13.98	2.96
	4	48	14.85	3.18
	1	47	5.72	1.59
İ	2	52	6.63	2.08
	3	50	5.60	1.84
	4	48	6.41	1.88

Table 9: Variance Analysis Table for the Grade Level Variable

		Sum of Squares	SD	Mean Square	F	P
GAM	Between Groups	32.804	3	10.935	1.603	0.190
	Within Groups	1316.343	193	6.820		
	Total	1349.147	196			
R	Between Groups	51.120	3	17.040	2.156	0.95
	Within Groups	1525.692	193	7.905		
	Total	1576.812	196			
İ	Between Groups	38.841	3	12.947	3.712	0.13
	Within Groups	673.129	193	3.488		
	Total	711.970	196			

As a result of the variance analysis, no significant differences were found among the generalising/abstraction/modelling (GAM), reasoning/connecting (R) and improving (İ) skills of pre-service teachers of different grade levels (F = 1.603; 2.156; 3.712, p>.05). Results of the Kruskal Wallis test conducted for the purposes of defining whether there were significant differences among pre-service teachers' creative thinking (CT) skills based on the grade level variable are given in Table 10.

Table 10: Kruskall Wallis Test Results

	Grade Level	N	Mean Rank	SD	X^2	P	Significant Difference	r=Z/ √n
CT	1	47	89.61					
	2	52	117.65	3	13.688	0.003	1-2	-0.232
	3	50	80.53				2-3	-0.189
	4	48	107.23					

According to the data provided in Table 10, significant differences were observed among the creative thinking skills of pre-service teachers' based on the grade level variable (χ^2(sd=3, n=197) =13.688, p< .05). Mann Whitney U test was administered in order to understand which differences in scores among grade levels were significant. As a result, significant differences were detected among the creative thinking skills of freshman and sophomore pre-service teachers (U=823.00, p< .05) and among those of the sophomore and junior pre-service teachers (U=888.00, p< .05). As size effect (r) values for these analyses are lower than .1 (Cohen, 1988), grade level can be said to have a relatively lower effect on the subject skills.

4. Conclusıons and Dıscussıon

The present study has attempted to identify pre-service teachers' mathematical reasoning skills via open-ended real-life problems and to define whether these skills demonstrate significant differences in accordance with gender and grade level. Within the scope of the study, mathematical reasoning skill were taken as a multistage process and these stages were specified as generalising/abstraction/modelling, reasoning/connecting, improving and creative thinking skills. As a result of the study, it was found out mathematical reasoning skills scores of pre-service teachers who were students at the department of elementary mathematics education were above average at the *generalising/abstraction/modelling* and *reasoning/connecting* stages while their scores were below average at the stages of *improving* and *creative thinking.* These results indicate that pre-service teachers were more competent at the initial stages of mathematical reasoning process and that their levels of achievement fell at the following stages. The initial stages of the mathematical reasoning process as handled in the present study namely *generalising/abstraction/modelling* and *reasoning/connecting* stages require one to study the present situation. As to the *improving* and *creative thinking* stages, they require one to go and see beyond the present situation, assess and handle the present situation in relation to different circumstances, see the results, and suggest new and original ideas using the current ones. Therefore, the last two stages demand higher order skills; indeed, the pyramid given in the introduction part emphasizes the multistage nature of the model adopted for the study purposes.

In the relevant literature, there is a variety of theoretical and applied studies of different scope and extent which address mathematical reasoning skills; however, only few studies attempt to investigate mathematical thinking and hence mathematical reasoning skills thoroughly. Such a study is the one by Arslan and Yıldız (2010). In their study, mathematical thinking process was handled as a multistage process and these stages were specified as specialising, generalising, conjecturing and proof. As a result of that study, it was revealed that achievement levels of 24 students at the 11th grade declined beginning from the specialisation stage of mathematical thinking towards the proof stage. In view of this, results of the present study can be said to correspond to those of the afore-mentioned study. In his successive and complementary studies, Lithner (1998, 2000a, 2000b) aimed to reveal difficulties students experience in situations that require mathematical reasoning in school environment. In those studies, he addressed reasoning types as plausible reasoning and superficial reasoning based on established experiences. In plausible reasoning, reasoning about a given situation is highly probable but not certain. Superficial reasoning based on established experiences, on the other hand, means that one selects and adopts a reasoning that they used in a similar situation before although its accuracy is not guaranteed. Lithner reported in the afore-mentioned studies that students had a disposition to use these two reasoning types in school environments. In another study he co-authored with (Palm, Boesen and Lithner, 2006), he formalized and developed this idea and suggested a framework for mathematical reasoning skills. The framework comprises rich problem solving (in terms of creative mathematically founded reasoning) and a family of reasoning types characterised by a strive for a recall of algorithms or facts (in terms of imitative reasoning). According to Palm, Boesen and Lithner (2006), one reason why students fail to reach the desired level of achievement is that they usually adopt uncreative and superficial reasoning types in their mathematics classes. In the same study, it is suggested that students struggle to remember familiar past experiences and the reasoning means they adopted in those cases rather than trying to focus on the features of mathematical objects and using these features (Bergqvist, Hiebert, 2003; Lithner and Sumpter, 2003; Lithner, 2000a, 2003, 2004; Lithner and Långström, 2008; McGinty, Van Beynen, and Zalewski, 1986; Schoenfeld, 1985). These statements are consistent with the conclusions of the present study. In that, this study has indicated that pre-service teachers usually fail in problem situations which require one to move beyond the present mathematical situation and generate their own ideas and which involve creative thinking. In addition, it has also been concluded that students are more competent in problems which relate to the initial stages of mathematical reasoning skills and which are similar to the problems they previously encountered in their mathematics classes. In a similar vein, Akkuş-Çıkla and Duatepe (2002) explored the proportional reasoning skills of prospective elementary mathematics teachers and posed ratio and proportion problems to students. According to the results of the study, prospective teachers had the knowledge of arithmetic operations; however, they did not have sufficient conceptual knowledge. That study also revealed that pre-service teachers with higher order operational skills failed to comprehend the conceptual basis of the subject and did the operations by rote.

Umay and Kaf (2005) aimed to explore what kinds of flawed reasoning elementary school students engaged in. They conducted the study with 90 students from 6th, 7th and 8th grade and revealed that students tended to use commonplace solutions and therefore the rate of flawed reasoning was higher than that of correct reasoning. Bergqvist, Lithner and Sumpter (2008) aimed to identify mathematical reasoning styles of high school students. As a consequence of the qualitative analyses, it was discovered that students tried to propose solutions using mostly superficial ideas. It was also pointed out that the most frequently exhibited mathematical reasoning type was algorithm-based mathematical reasoning and that although there were many questions that could be solved via creativity-based reasoning, the number of such cases were few. Boesen, Lithner and Palm (2010) attempted to investigate the relation between types of tasks and mathematical reasoning used by students. The study was carried out with 8 high school students in Sweden. 107 questions classified in accordance with the defined task classification style were posed to students and the responses were analysed. According to the results of that analysis, when students were given questions similar to those in their textbooks, they mostly adopted imitative reasoning type. In other words, they reached a solution without constructing new reasoning or without considering mathematical properties in depth. Çiftçi (2015), in his study on mathematical reasoning skills of pre-service mathematics teachers administered a data collection tool with six problems to ten pre-service teachers of secondary school mathematics. As a result of the study, it was concluded that pre-service secondary mathematics teachers exhibited superficial reasoning structures when confronted with problem situations and preferred mathematical reasoning types based on imitation. Due to the preference of memorisation and algorithm-based mathematical reasoning types over others, pre-service teachers were not able to use their conceptual knowledge on the issue and their power of thinking totally. Pre-service teachers preferring mathematical reasoning based on creativity in the same problem situations, on the other hand, were found to have a better command of mathematical concepts. It is clear that conclusions of all the afore-mentioned studies emphasize similar points and that the present study is similar to them due to the above-mentioned and explained reasons.

Another conclusion of the current study is about the correlations among mathematical reasoning skills. Correlations among the mathematical reasoning skills used at the successive stages of mathematical reasoning were stronger than the others. This in a way proves the developmental and multi-stage nature of mathematical reasoning process, because successive processes comprise skills that follow upon and complement one another and thus it is normal for these skills to exhibit a higher correlation. In the relevant literature, there are studies handling mathematical thinking skills in a multistage framework, yet there aren't any studies that attempt to investigate mathematical reasoning skills as a multistage and developmental process.

Another conclusion drawn from the study is that, in relation the gender variable, there was a significant difference among the scores on *generalising/abstraction/modelling* and *reasoning/connecting* skills in favour of males while there weren't significant differences in terms of the scores on *improving* and *creative thinking* skills. There was a significant difference among the pre-service teachers' scores on total mathematical reasoning skill in favour of males. Furthermore, all the scores on mathematical reasoning skill were slightly higher in favour of male participants. In the literature on the subject, there are many studies that address reasoning skills in relation to gender variable. Of them, the most comprehensive one is the study by Benbow and Stanley (1980). That study comprises the results of six successive studies conducted on the same sample in 1972, 1973, 1974, 1976, 1978 and 1979. A total of 9927 students aged between 12 and 15 made up the study population. The first three studies involved 7th, 8th, 9th and 10th grade students while the last three studies were conducted on 7th grade students only. In all the six studies, it was found out that male students were able to use mathematical reasoning skills more efficiently than did female students. The difference between male and female students was roughly valued at a standard deviation of 0.40.

The same study was repeated with 40,000 high school students between 1966 and 1997 and it yielded similar results (Benbow and Stanley, 1983).

Although there are studies suggesting that gender has significant effect on mathematical reasoning skills (Aiken, 1986, 1987; Başaran, 2011; Benbow 1988, 1990; Çoban, 2010; Dougherty et al. 1980; Friedman 1989; Hedges and Nowell, 1995; Jensen 1980, 1988; Maccoby and Jacklin 1974; Marshall and Smith, 1987; Meehan, 1984; Mills, Ablard and Stumph, 1993; Stanley, 1993, 1994) there are some studies that have contradictory findings (Karakoca, 2011; Lamprianou and Lamprianou, 2003; Leahey and Guo; 2001; Pallas and Alexander, 1983; Sprigler and Alsup, 2003;). Geary (1994) points out males outperform females in mathematical reasoning rather than mathematical computations as well as in geometry rather than algebra (Harnisch et al. 1986; Lummis & Stevenson 1990). Leahey and Guo (2001), argue that studies on male and female students' mathematical reasoning skills yield different results because of the differences among student groups and sample sizes. In the same study, it is suggested that as sample size grows, gender-based differences will decline among students. For this purpose, results of the National Longitudinal Study of Youth (NLSY: 1979) and the National Educational Longitudinal Study (NELS: 1988) are utilized. NLSY is administered to children aged between 4 and 13 (NELS: 1988) contains data on students in 8th, 10th, and 12th grades (roughly ages 14 through 18). The total number of observations for NLSY analyses is 12.159 and the total number of observations for NELS analyses is 6253. Leahey and Guo (2001) report that in the subject studies, no significant differences were found in student performances and only among high school students there were slight gender differences. Therefore, it is seen that researchers' conclusions are consistent with their hypothesis. Leahey and Guo (2001) argue that drawing judgements from the past studies concerning the effect of gender on mathematical performance will not offer reliable conclusions.

Some researchers (Feingold, 1988; Leahey and Guo, 2001), on the other hand, indicate that a literature review shows the difference between male and female students' performances decline year by year while it persists among high school students. An idea supported in the studies by Benbow and Stanley (1980) and Leahey and Guo (2001) is that the size of gender-related differences in the reasoning skills of students with notable academic achievement levels is always greater than the size of overall differences in the society. If the results of the present study are analysed in the light of the studies referred to up to this point, it is significant that scores on total reasoning skills are in favour of male students. Notwithstanding, this might stem from the fact that the study was conducted on a random sample and the sample size was relatively small. Gender differences decline towards the top of the reasoning pyramid. This can be taken to mean that gender factor has an effect on student performance in cases when it is the main determinant of reasoning skills, yet it is not a significant effect.

This inference, however, is valid only for this study and cannot be generalized to other studies.

As to the findings of the study in relation to the grade level variable, no differences were observed in the *generalising/abstraction/modelling*, *reasoning/connecting* and *improving* skills of pre-service teachers of different grade levels whereas there were significant differences among their creative thinking skills. In this study, creative thinking skills represent the top of the mathematical reasoning pyramid. Thus, the grade level variable is more influential at the later stages of mathematical reasoning skills although it is small. Of the earlier relevant studies in the literature, Tourniaire and Pulos' (1985) study on proportional reasoning, a study with the characteristics of a literature review, it is indicated that students' proportional reasoning performance improves considerably with age, up to adulthood. Offenbach (1965) conducted a study on pre-school and 4th grade elementary school students in order to define how students used reasoning in a game designed in probability subject and found out that older students could use probability reasoning more aptly than did younger students. Similarly, Kitchener and King (1981) studied with students of different age groups (high school, undergraduate and graduate) in their study on reasoning styles of young people and observed considerable differences among the reasoning competences of students.

Başaran (2011), in her study on university students' mathematical thinking and reasoning skills, revealed significant differences among grade levels. In Başaran's (2011) study, 4th and 5th grade students were found to use their reasoning skills more efficiently than did other students while 1st grade students were better than 2nd and 3rd grade students. Arslan (2007) investigated the development of elementary 6th, 7th and 8th grade students' ideas on mathematical reasoning and proof with 679 elementary school students from seven different schools. As a result, it was seen that as students' grade levels increased so did their probability of finding the correct answers to some questions in the data collection tool. In some other questions, however, no significant correlation was found between grade level and student performance. Verhaegen and Salthouse (1997), in their meta-analysis on the conclusions of 91 studies, revealed that such mental skills as reasoning, quick thinking and episodic memory reached their peak between 20 and 30 years of age and deteriorated with time. In view of these studies, it is seen that reasoning skills are in general enhanced with age until 20s although some studies have different results. Considering that factors other than age such as educational background might have an effect on the development of reasoning skills (Benbow, Lubinski, Shea and Eftekhari-Sanjani, 2000; Park, Lubinski and Benbow, 2007; Steen, 1999; Tourniaire and Pulos, 1985) different conclusions reached by these studies become meaningful. In the present study, for example, except for the creative thinking skills no significant differences were detected among grade levels in terms of students' other reasoning skills. This might stem from the fact that individuals in the sample had similar academic performances and were around the same age. After all, they all were placed in their departments in accordance with certain academic performance scores. As to creative thinking skills, sophomore pre-service teachers were more successful than did all the other participants. This is as interesting as Başaran's (2011) findings. In that, senior students were expected to outperform when compared to their peers in previous grades due to their academic backgrounds; however, the result was the opposite. Possible results of this can be educational backgrounds and past experiences of the sophomorepre-service teachers.

5. Recommendations

In this study, pre-service teachers' mathematical reasoning skills were explored and assessed in relation to different variables. Although it used mixed methodology, its qualitative aspect outweighed the other aspects. There is a need for both theoretical and applied studies that handle reasoning skills via descriptive methods as these methods might offer a better insight into the subject skills considering also the number of studies of that kind is quite limited. Future studies should address and analyse reasoning skills in relation to specific cases and disclose what weaknesses exist in the use of the subject skills. It is also suggested that such studies also establish the ways to overcome these weaknesses. In this regard, the concept of mathematical reasoning can be defined in detail from a theoretical aspect and a new framework that would form the foundations of this concept can be devised; because, there is a need for such theoretical framework for the examination of the subject skills in future studies. Apart from these, other mathematical skills that are closely related to mathematical reasoning can be identified and it can be ensured that all these skills are used in learning settings for the enhancement of reasoning skills.

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