European Journal of Education Studies
ISSN: 2501 - 1111
ISSN-L: 2501 - 1111
Available on-line at: www.oapub.org/edu

doi: 10.5281/zenodo.583168 Volume 3 │Issue 6 │2017

MATHEMATICAL CREATIVITY AMONG EXCELLENT 8TH GRADE PUPILS

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Abstract:
Creativity is finding a new way within a given frame and it is the ability to connect between segments of information, materials and experience, which exists in their own right, or were previously connected with different patterns. The current research engaged in solving problems that can be resolved in various ways or that they have several solutions, by excelling students and this in order to measure the level of mathematical creativity related to solving mathematical problems, by measuring the three elements of mathematical creativity: flexibility (multiple solutions), fluency (time needed to solve the problem), innovation (originality of ideas). It is important to emphasize that the way of solving tasks by using as many ways as possible, considered to be as one of the most efficient ways to the discovering and development of mathematical creativity. In the current research, 24 8th grade pupils from several schools have participated, every student received three diverse mathematical problems and was requested to suggest and raise as many solutions or ways to solve as possible for each problem without teacher intervention, while measuring the required time to solve every problem separately for every student by a stopwatch. Additionally, a semi-structured interview was held with each student separately regarding his or her approach to each problem, the interview focused on identifying the mathematical language, in the process of resolving and in the exposure of the pupil's mathematical thinking process until he or she reached the solution. The research findings indicate that there are differences between the pupils regarding flexibility, as multiple solutions were suggested by the pupils for each problem individually. Differences were also found in the level of innovation among the pupils, hence original solutions and new ideas to

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solve problems have been received. Moreover, a difference in fluency has been found so that pupils solved in a different time rates, this is despite the fact that all pupils have a very high level of achievement.

Keywords: mathematical creativity, mathematical content, excellent pupils

1. Introduction

The subject of Creativity is a wide subject that includes numerous fields in art, in thinking and in science, thus, creativity is a process in which meaningful ideas are born, and they become implementable. Well, creativity, to some extent, exists in every field and it can be nurtured and developed under the influence of the environment. In the current research, we will focus on creativity in the field of Mathematics.

Ervynck (1991), defines mathematical creativity as an ability to solve problems and develop a structured thinking, while referring to the logical-deductive nature of the field of knowledge, and to the alignment of the connections to the mathematical content.

Torrance (1996), has developed one of the many tests for testing creativity, the test has three components: *Fluency*- the ability to remember the information and gather ideas formed in a certain subject; *Flexibility*- multiple of the different approaches revealed by the ideas; *Novelty*- ability to respond in new and unconventional ways as well as addressing the originality of the formed ideas.

In context of the research held among excellent pupils, Leikin (2009) believes that the excelling pupils can reach high achievements when their potential is maximized. Moreover, they differ from others in level of creativity and ability to raise original ideas as well as flexibility and fluency. The current study engaged specifically with creativity that is revealed by problems that can be solved in different ways or by problems that have several solutions, among excelling pupils. The purpose of the research is to examine and measure the mathematical creativity level related to solving mathematical problems among excelling pupils of 8th grade via three parameters: Novelty, Fluency, and Flexibility. And to answer the question: is there creativity in the process of problem solving among excelling pupils?

It is important to stress that solving tasks in as many ways as possible, considered as one of the efficient ways to develop mathematical creativity. As an appropriate response for this issue of nurturing pupils’ mathematical creativity in solving problems, it can be found that mathematical creativity among excelling pupils is dependent on their prior experience.

2. Creativity – Definition

Creativity is a relatively new subject in the scientific research, and it is addressed as a feature that distinguishes a trifle of people that innovate novelties of great importance (Ziv, 1990).

In the general definition, as noted in the book "About Education" (1997), creativity is a process in which significant ideas are being born, and then making them practical as well as converting ideas from other fields to a new field. Definition from the "Lexicon of Education and Instruction" (1997): creativity is finding a new way within a given frame of work: the ability to connect between pieces of information, materials, and experience, existing each in itself or were connected in the past in different patterns (p 227).

Torrance (1969), defines creativity as a process that includes a sense of missing, interrupting elements, and the need to form ideas in relation to them. Creativity is the ability to create a new and original thing from existing elements.

In Torrance' (1966) test there are three components:

1. *Fluency*- the ability to remember the information and gather ideas formed about a certain subject.
2. *Flexibility*- Multiplicity of the different approaches revealed in the ideas. Having cognitive flexibility and capability to move rapidly from one idea to another.
3. *Novelty*- ability to response in new and unconventional ways and to relate to the originality of the formed ideas.

Many researchers discussed the issue of creativity and addressed several essential aspects to the completion of the Creativity act and assist in understanding it, such as: the creative approach, the creative process, and the features of the creative person.

Since the research of creativity grew from the research of intelligence, Landau (1990) pointed out in her book "The Courage to be Talented" several points that differentiate between the two:

a) While intelligence, as it is measured by IQ tests, is the ability to store and remember information, while the creativity is the ability to form new connections between the stored items of information.
b) Intelligence tests examine focused thinking – each problem has only one solution; whereas creativity is aided by branching thinking- each problem can have multiple solutions created due to the fluency, flexibility and originality of the mind (see p 4)

c) While the "intelligent" answer is correct- since it can be compared to a previous answer or solution, the "creative" answer is suitable- due to its novelty and lack of any kind of previous reference frame.
d) While the "intelligent" solution is finished, usually with a dot, the "creative" solution holds a question that may lead to additional problems and solutions, and thus it ends with a question mark.
e) As it is measured, intelligence allows the individual to adapt into new situations with what he learned earlier, while creativity enables the individual to reach self-realization in a new situation.

The creative individual does not satisfy with adapting or aligning himself, but rather he tries to use what he has, his potentials, in view of the new situation.

3. Mathematical Creativity

Creativity in mathematics is expressed by independent formulation of uncomplicated mathematical problems, finding ways and means to solve these problems, and finding original methods to solve unusual problems. One of the ways to create evoking-creativity thinking situations is by presenting open problems that have multiple solutions rather than one unambiguous solution (Gazit and Patkin, 2009).

Pupils who excel in mathematics are different from all pupils in several characteristics, the notion stemming from it is that the personality built is a leading role in the development and actualization of their potential, thus, when planning activities for excelling pupils, teachers should take the pupils' special characteristics under consideration, as a complex task.

Guberman (2010) points out in her essay the importance of the main massages the mathematics teacher should deliver to his or her pupils, and states that mathematics is not a collection of facts and methods of operation that should be followed accordingly, but rather it entails imagination, creativity and raising original ideas, and it is very important to allow the pupils to search for their own ways to deal with different tasks.

Leiken (2009) suggests and stress the possible option to distinct between two kinds of mathematical creativity:

1. Absolute creativity- refers the mathematical discoveries historically, of great impact on the development of mathematics as science.
2. Relative creativity- refers to the day-to-day discoveries that can occur in every classroom.

Ervynck (1991), claims that mathematical creativity enable to build a scheme from which a person can produce an infinite number of connections that constitute problem solving in various ways.

Though the new ideas may lead to novelty, they can also lead to error. Pupils struggle to accept the possibility of making a mistake in the stage of mathematical thinking, since failure can bring to the end of a creative process, however, it can also be used as a new starting point (Babaeva, 1999; Ervynck, 1991).

It is possible to find the connection between mathematical creativity and solving mathematical problems in silver's (1997) and Ervynck's (1991) definition. Silver (1997) claims that mathematical creativity can be developed among all pupils by presenting suitable mathematical activities. According to him, dealing with open-ended problems strengthens the main qualities of creativity; fluency, flexibility and novelty.

In contrast, Ervynck (1991), defines mathematical creativity as an ability to solve problems and develop a constructed thinking, while addressing the logical-deductive nature of knowledge, and matching the connections to the mathematical content.

3.1 Solving problems in different ways

The standards of the National Council of Teachers of Mathematics (2000), mention four kinds of mathematical connections that enable to solve problems in different ways:

1. Between different presentations of a certain mathematical concept;
2. Between different concepts, between different meanings of the same concept or between different procedures in a specific mathematical field;
3. Between the various branches of Mathematics;
4. Between Mathematics and different fields of knowledge.

In the same publications of the NCTM (2000), appears that constructing those connections contributes principally and fundamentally to the development of a profound mathematical understanding.

The method of solving tasks in many ways as possible is considered as one of the efficient ways for the development of mathematical creativity, and this is because solving problems in different ways develop connectivity of mathematical knowledge, flexibility and intellectual fluency (Leikin, 2009).

Leikin (2006), suggests that the difference between the different solutions can be expressed in the use of different representations, use of tools, definitions, qualities, different concepts or various theorems from the same mathematical field or from a different mathematical field. Leikin (2006) demonstrated the types of the existing differences between the solutions, by presenting several mathematical problems and their solutions.

Hodgson (1995), presents the learning of mathematics as an active process. The pupil construct the mathematics for himself in accordance to the activity in the class, some pupils perceive mathematics as a collection of rules and isolated facts, while some perceive it as a network of ideas and connections. The use of connections strengthens the ability to solve problems.

Coxford (1995) notes various kinds of mathematical connections: connections between mathematics and other fields of knowledge, connections between the different fields of mathematics and connections between different representations to the same concept. In his view, these connections can be developed via solving problems in different ways, and it does depend on the type of problem.

Leikin (2007) encourages solving problems in different ways, and concur with the other researchers and adds that developing a habit of solving problems in different ways advances high levels of mathematical thinking, and argues that it is possible to use various solutions in order to connect knowledge in different fields of knowledge in mathematics and thus bettering the knowledge in each of these fields.

Tzahor (2011) adds and underlines in his study that the use of alternative tools, not merely open to the pupil additional solution options, but also enable him or her to grasp a wider and more general image, of how different branches of mathematics complete each other. Tzahor (2011) further claims that there is no need for a special chapter for teaching solution with alternative tools, rather, throughout the entire instruction of Geometry, one should think how and where alternative proofs can be combined. Because the difficulty is more for teachers than for students, since they are those who need to change their old habits.

Levav-Waynberg & Leikin (2012), who have examined in a study about solving geometric problems in different ways, found that a systematical implementation of instruction that encourage finding multiple solutions for geometric problems contribute both to the mathematical knowledge connectivity, and to the pupil's creativity expressed by the fluency of ideas and the extent of his or her intellectual flexibility when solving problems in geometry.

4. Methodological Chapter

4.1 Participants

In the current research, 24 8th grade pupils, defined as excelling students from four schools in the northern district of Israel, have participated. The pupils were selected as excelling students according to the classroom teachers and mathematics teachers who teach them at the schools, and in accordance to the schools' standards. The choice to investigate mathematical creativity among 8th grade pupils stemmed from the fact that these students are ready and prepared for different mathematical tasks, furthermore, they have acquired all kinds of skills in mathematics, geometry and other fields.

4.2 Research Tools

In the current research, there was a use of three tools, that were used in Najar's (2014) research as well, and they are:

i) 3 mathematical problems that have different ways for solution:

Problem 1:

Solve the following system of equations, solve in many different ways as possible:

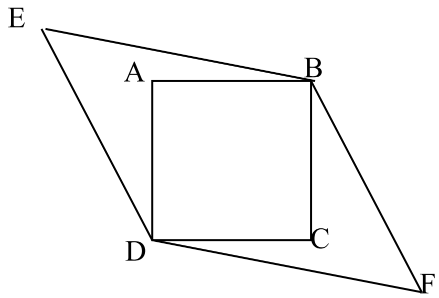
$$\begin{cases} 4x + 3y = 14 \\ 3x + 4y = 14 \end{cases}$$

Problem 2:

It is given that ABCD is a square.

E and F are on the continuation of the AC diameter

So, AE = CF.



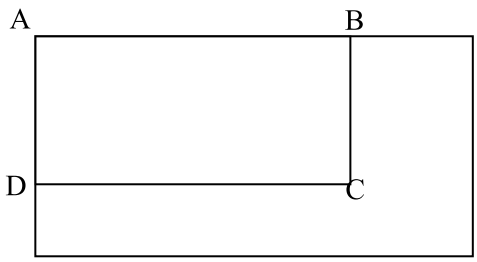
It was proven that the square EBFD is a rhombus; bring the proof in as many different ways as possible.

Problem 3:

The ABCD rectangle has increased its length and width by 2 cm.

The area of the additional part equals 24 cm. Find the perimeter of the original rectangle.

Find as many different solutions as possible.



ii) There was a use in observation as well in order to measure the time it takes to solve every task, via stop watch, the measurement began from the moment of receiving the problem until the pupil reached the final solution, including all of his dilemmas risen during the act of solving.

iii) Moreover, there was a use in a semi-structured interview with each student separately. The purpose of the interview was to assist in the analysis of the mathematical language of the pupil in terms of usage in mathematical concepts, as well as describing the thinking process conceived in his mind while solving the problems. The questions of the interview address to the elements of creativity: flexibility, fluency, novelty, description of the solving process and assist with describing the personal traits of the excellent pupil in the process of solving a problem.

5. Findings

First Problem - Solving a System of Equations

Flexibility: the flexibility in the method of solving a mathematical problem among excellent students was examined by measuring the number of the solution the students suggested for every problem. The first problem received between two to seven different solutions, when two solutions were suggested by four pupils, while seven solutions were suggested by a single pupil. That is, the minimal number of solutions is two and the maximal number of solution is seven. The differences between the students in the number of solutions examined by a single-sample t test.

Table 1: Average and standard deviation for the number of solutions to the first problem suggested by the students, the value for a single-sample t test (N = 24)

t	Standard Deviation	Average number of solutions	Max number of solutions	Min number of solutions
13.86***	1.41	4.00	7	2

***$p<0.001$

The findings represented in the tabled above indicate that the number of solutions suggested by the students to the first problem variates between 2 to 7 solutions, with an average of 4 solutions, it was also found that there is a distinct difference in the number of solutions the pupils suggested to the first problem ($t=13.86$, $p<0.001$), meaning, the pupils suggested a different and diverse number of solutions, and there is no cohesiveness in the number of solutions suggested by the pupils to the first problem.

Conclusion: excellent pupils suggest a multiple, different and diverse number of solutions, and they vary in the number of solutions and that indicates on a difference in the flexibility level among the students regarding the first problem.

Fluency: the fluency in the method of solving the mathematic problem among excellent pupils was examined by measuring the time the pupils assigned to solve the problem, planning time, execution time and overall time. In order to test if there are differences between the students in the times of problem solving process, a single-sample t test has been held.

Tablet 2: Averages and Time Standard Deviations (in seconds) Planning, execution and overall time to solve the first problem, values for a single-sample t test (N = 24)

t	Standard Deviation	Average time in seconds	Max time	Min time	Time
11.43***	13.50	31.50	64.60	14.40	Planning
13.71***	101.40	283.70	451.30	121.20	Execution
15.11***	95.60	294.80	468.00	143.10	Overall

***$p<0.001$

The findings presented in the tablet above indicate that the time allocated by the students for planning the solution of the second problem ranges between 27.00 seconds to 81.00 seconds with in average of 57.34 seconds and a standard deviation of 12.72 seconds. In addition, a distinct difference was found in the time of planning the solution of the second problem ($t=22.08$, $p<0.001$), meaning, the students assigned a different time to the planning of the solution of the third problem, and there is no unity in the times that the pupils assigned for planning the solution of the second problem.

The findings suggest that the time allocated by the pupils for the solution execution of the second problem ranges from 193 seconds to 461 seconds with the average of 301.65 seconds and a standard deviation of 98.24 seconds, additionally, a distinct difference was found in the time of the solution execution of the second problem ($t=15.04$, $p<0.001$), meaning, the pupils assigned a different time to the solution execution of the second problem, and there is no unity in the times allocated by the students for the solution execution of the second problem.

An additional finding is that overall time allocated by the pupils for the solution of the third problem ranges from 231.00 seconds to 572 seconds with the average of 366.80 seconds and a standard deviation of 111.18 seconds. There is also a distinct difference in the time it took to solve the second problem ($t=16.16$, $p<0.001$), meaning, the pupils allocated a different time to solve the second problem, and there is no cohesiveness in the times assigned by the students to solve the second problem.

Conclusion: the findings indicate a fluency in problem solution among excellent pupils which was expressed in the planning and execution time and the overall time to solve the problems, nonetheless, a difference and variance was also found between the students in the time they assigned to planning, execution and overall time to solve the second problem.

Innovation: The analysis of the solutions presented by the pupils was done by examining the method and the ability to respond in new ways, which may indicate the level of originality and unconventional thinking.

An analysis of the solutions revealed that in the first stage, the students chose the way in which they knew the characteristics of the square whose sides were equal to each other and that it was a rhombus, this is the more convenient way to solve, it seems that there is a sense of confidence to choose the way they know and perhaps it saves them time.

On the second stage the pupils presented ways that require higher thinking skills which base on prior knowledge, since the pupil needs to prove that the square is parallelogram first, and only afterwards the pupil is required to find a connection between the parallelogram and the rhombus. The methods require and force the pupil to reach the stage of making conclusions as well as a successful connection with the prior knowledge, in conclusion, "*when is the parallelogram considered a rhombus?*" at this level the pupils express a different level of creativity than that discovered in the classroom, and they do express an ability to response in unusual ways and demonstrate the originality of the ideas. In the third stage, the students presented solutions that required higher thinking skills based on prior knowledge, here the students demonstrated creativity that was different from the lessons in the class and even higher. At this level, the students relied on the characteristics of the Dalton and the Equilateral Triangle, hence they received a square whose entire sides were equal and thus they reached the rhombus.

We may conclude that the level of novelty revealed in relation to the second problem describes two aspects, the first includes ordinary ways that the pupils practice usually in math class, relatively convenient and fast methods, whereas the second aspect includes a variety of new methods and they do express an ability to response in unconventional manners and reflect originality of ideas.

Interviews with the pupils after finishing to solve the second problem:

We can conclude the main characteristics that stood out and came up from the interviews with the pupils, as follows:

1. The pupils demonstrated intellectual flexibility by presenting multiple solutions.
2. The students demonstrated importance to intellectual fluency by using the time in the process of solving the problem.

In regard to the creative personality characteristics that arose from the interviews, we found: Mastering the subject matter and correct identification of the main points of the problem; Systematic and organized hierarchically work built from a set of qualities; Ability to make conclusions.

Third Problem - Find the perimeter of the original rectangle:

The problem reveals the creativity among the students by solving an integrated geometry problem with algebraic technique different from the presented problem formulations during math lessons, and it includes the elements of flexibility, fluency and novelty as well.

Flexibility- the flexibility of excelling students in their method of solving a mathematical problem examined by measuring the number of solutions the students suggested to the problem.

Tablet 5: Averages and standard deviations for the number of solutions that students proposed for the third problem, values of a single-sample t test (N = 24)

t	Standard Deviation	Average number of solutions	Max number of solutions	Min number of solutions
10.46***	5.90	12.60 (without “a great many” and “infinite”)	A great many to infinite	4

***p<0.001

The findings presented in the tablet above, indicate the notion that the number of solutions the students suggested to the third problem ranges from 4 to them writing “a great many” or “infinite” solutions, while the average of the suggested solutions (without “a great many” or “infinite”) is 12.60 solutions, we also found that there is a distinct difference between the students in relation to the number of solutions they proposed to the problem ($t=10.46$, $p<0.001$), meaning, the students offered a different number of solutions, and there is no cohesiveness in the number of solutions the students offered to the problem.

The multiplicity of the proposed solutions indicates the novelty level among the pupils, it indicates dependency between the level of novelty, intellectual skills and the ability to make conclusions, that is, if students made a well-organized order in their thinking and gathered confidence in developing targets, then they can reach a correct solution in various ways as well as finding additional possible solutions for this problem especially, the pupils are required to search for possible solutions to the same existing formula for the problem.

Fluency: the fluency in solving the mathematic problem among students has been examined by measuring the time allocated by the pupils to solve the problem, whether it is planning time, execution time, and overall time.

Tablet 6: Averages and Standard Deviations for Planning Time, Execution and Overall Time to Solve the Third Problem, Values of Singular-Samples t Test (N = 24)

t	Standard Deviation	Average Time in Seconds	Max Time	Min Time	Time
9.62***	36.42	71.52	134.00	22.00	Planning
22.05***	107.15	482.28	678.00	335.00	Execution
20.32***	130.84	542.73	755.00	381.00	Overall

***p<0.001

The findings presented in the tablet above indicate that the time the students assigned to plan the solution of the third problem ranges from 22 seconds to 134 seconds with an average of 71.52 seconds and a standard deviation of 36.42 seconds, we have also found that there is a distinct difference in the solution planning time of the third problem ($t=9.62$, $p<0.001$), meaning, the students allocated a different time to the solution planning of the third problem, and there is no unity in the time assigned by the pupils to the solution planning of the third problem.

The findings show that the time the pupils assigned to the solution execution of the third problem ranges from 335.00 seconds to 678.00 seconds with an average of 482.28 seconds and a standard deviation of 107.15 seconds, additionally, a distinct difference was found in time of the solution execution of the third problem ($t=22.05$, $p<0.001$), meaning, the pupils allocated a different time to the solution execution of the third problem, and there is no cohesiveness in the time assigned by the students to the solution execution of the third problem.

Furthermore, the research suggest that the time assigned by the students to solve the third problem ranges from 381.00 second to 755.00 seconds with an average of 542.73 and a standard deviation of 130.84 seconds, in addition, there is a distinct difference in the time of solving the third problem ($t=20.32$, $p<0.001$), meaning, the students allocated a different time to solve the third problem, and thus there is no cohesiveness in the time allocated by the pupils to solving the third problem.

Conclusion: the findings indicate an existence of fluency in problem solution among excelling students which came to expression in the planning and execution time and in the overall time to solve the problems, it was also found that there was a difference and variance between the students in the time they assigned to planning, execution and overall time to solve the third problem.

Novelty- analyzing the solutions of the pupils to expose the novelty level made by testing the method and the ability to response in new and unconventional ways, which indicates level of originality.

We can see in the novelty analysis of the solutions suggested by the students that they offered 3 different equations in their form but each gives the same algebraic meaning, thus, every student chose the convenient equation for him or her.

Ten pupils suggested the four possible solutions at the first stage, it can be presumed that everyone can identify that they need two numbers whose sum is equal to 10, and then it seems that they reached to the self-confidence level and chose to replace the presented values. A higher level of novelty appeared in one student that proposed 16 possible solutions and in a student who proposed 10 possible solutions and another pupil that offered 13 possible solutions, this indicates organized work but the stage of making conclusions is missing here. And finally, a higher level of novelty appeared among six pupils who reached a generalization and wrote that there are infinite solutions and one of them wrote that there are numerous solutions for that problem, which affirm organized and sophisticated work and a high capability in relation to the other students in the group in the level of making conclusions and also showed intellectual order and tenacity to continue to solve and it came orderly across by the actions that direct the processes of the solution.

Interviews with the pupils after finishing solving the third problem: the interviews assist in analyzing the mathematic language among the students in terms of using terms as well as in terms of the way of expression, moreover, the interviews reflect the thinking process came up in the students' heads from the moment they began the task until the point of executing the solution including all the dilemmas they encountered on the way.

The main characteristics that stood out in the sections of the interviews with the students after finishing solving the third problem, can be concluded as follows:

1. The pupils demonstrated flexibility by suggesting and bringing a large number of solutions.
2. The pupils demonstrated an intellectual fluency.

3. The pupils demonstrated awareness to novelty by offering new unconventional ways.

In addition to the creativity characteristics of problem solution, a number of creative personality traits were observed among the pupils, such as:

1. Ability to make conclusions.
2. Reflection- assessment and review of solution processes.
3. Mathematical capability and self-confidence.

6. Discussion

The main goal of the current research was to examine the level of mathematical creativity in relation to solving various and classified mathematical problems in different ways among excellent pupils in 8th grade on the grounds of the three components of Torrance's (1996) creativity test, which are: *Fluency*, *Flexibility*, and *Novelty*.

The *flexibility* level of the solutions was measured by analyzing the number of solutions the students suggested for every problem: the *fluency* of the solutions was determined by measuring the time each pupil gave to every problem separately, and the *novelty* of the solutions was examined by categories of solutions and adjustment in accordance to the level of originality the student relied on.

The main hypothesis of the current research held that there is creativity in the process of problem-solving among excellent students, that is to say, there is a solutions multiplicity to a mathematical problem given to the excelling pupils who learn in the same class; there is a difference in the time of executing the problem solution among excellent pupils who study in the same class; and there is novelty in the method of solving mathematical problems among excellent pupils who study in the same classroom.

The research findings indicate that excelling students nurtured high thinking skills on the ground of the mathematical knowledge they have acquired throughout their math studies at the school, and it stood out via their ability to explain their answers successfully from a profound understanding of the studied subject matter in the class and/or from educational knowledge they studied previously. The research findings confirmed the research hypothesizes, on the grounds of the problems given to the excellent pupils a clear image arose that the mathematical knowledge acquired by the students did affect their creativity and among others improved their ability to solve problems in various ways and/or approaches, Liekin (2006) explained this connection by claiming that the difference between the various solutions may expressed by a use of different presentations, a use of various tools, definitions, qualities, different terms or theorems taken from the same mathematical field or taken from a different mathematical field. Leikin (2006) demonstrated the kinds of the existing differences between solutions, through representing several mathematical problems and their solutions.

The findings indicated *flexibility* in problem-solving among excellent pupils that expressed by the number of solutions suggested by the students, but it was also found that there is a variance and a difference in the level of flexibility between the students regarding the three problems, additionally, the interviews held in the current research brought up that the students addressed clearly to flexibility that came across in their ability to report different solutions to the problem they received, moreover, they have demonstrated mastering the subject matter as well as a correct identification of the problem's central points, and it stood out in their branching thinking, which strives for multiple possibilities of a solution along with classifying and organizing ideas.

These findings support Leikin (2007) claims that encouraged problem-solving in different methods, and argued that developing a habit to solve problem in various ways promotes mathematical thinking in high levels and thus bettering the knowledge in each of these fields.

In addition to Leikin's claim, the NTCM's (2000) publications presented that one of the important destinations of the mathematical education is teaching and learning math via constructing mathematical connections of different kinds that enable to solve problems in various ways, well, the current research raise that the excelling students proposed a different number of solutions, and there is no unity in the number of solutions the pupils offered for the three problems, this emphasizes variance in the flexibility level among the pupils.

Furthermore, we can also say that the findings show that the solutions flexibility among the excelling students is depended on the kind of the problem, this stood out in the number of solutions offered by the pupils to each problem, especially in the lesser number of ways proposed to the geometric problem in comparison to the number of ways suggested for the solution of the two other problems; the problem is geometric, hence, it requires reasoning and proofs, perhaps the students have higher thinking skills in problems of algebraic technique.

The findings support Coxford's (1995) arguments, who notes different kinds of mathematical connections: connections between mathematics and other fields of knowledge, connection between the various fields of mathematics, and connections between different presentations to the same concept. In his opinion, it is possible to develop these connections via solving problems in different ways, and it does depend on the kind of the problem.

Examining the planning time and execution of the solution has been performed in the light of many researchers who practiced the same field, Schoenfeld (1992), for instance, detailed the problem understanding phase in three sub-steps: reading the problem, data analysis and investigation, he used a graphic description to describe the steps on a timeline.

In view of the results, the findings pointed out on fluency in problem-solving among excelling students that expressed in the planning and execution time and overall time to solve the problem, but it was also found that there is a difference and a variance in the time the students allocated to planning, execution and time in total to the solution of each of the three problems.

This function, consist, to a certain extent, with the claims of the great mathematician and educator, Poye (1961), who proposed strategies for solving a problem, that include four steps: understanding the problem; conceiving a plan for the solution; performing the plan and review backwards.

Poye (1961), explains that in order to solve problems a based knowledge of the subject is required, as well as the ability to conceive a good idea. Good ideas are based on past experience, and knowledge acquired earlier.

Arbel (1990) mentions that solving a problem means to find a series of steps, starting from the given status (the problem) to the desired target, so that each step is received from its former through a logical action, and it is indeed expressed in the research in the time division to three stages: planning, executing and time in total.

The difference in the time allocated by the pupils to every problem solution in the current research underlining and reaffirms the sayings of Patkin and Gazith (2009) that the process of problem-solving requires a factual and classifying reading of the problem data, an appropriate choice of a strategy suitable to the combination of data in the problem, solving the problem, and eventually, reviewing the solution and *it varies indeed from pupil to pupil*.

We can add that these findings also show that the solutions fluency among excelling students who learn in the same class, depends on the problem's kind, this stood out in the time differences that the students gave and we can see that the overall time average of all the students varies from problem to problem. Well, according to these time data we can see that the pupils in the group solved the algebraic problem faster than the other two problems. This task belongs to the field of Algebra, even though the task is in the school's version like it appears in the middle school curriculum, and does not require high skills of mathematical thinking nor binding conclusion making, on the contrary, the problem requires from the pupils some skills to solve on the base of their general knowledge that they learned together in the class with the subject's teacher, therefore, they knew how to solve the same problem in several ways and in a short period of time.

The findings of the algebraic problem show that the students' relation to the problem consists with the claims of Sowder (1985) who has examined and held that when the pupils learn an algorithm to solve a task in the class, the task is no longer a problem, it becomes an exercise, the task is to remember the algorithm in order to use it, hence, the overall time averages we have received regarding the three problems are reasonable.

The novelty level of the solutions suggested by the pupils to the algebraic problem was higher than the novelty level appeared in the other two problems, we may conclude that the problem is algebraic and is a common version of the school, as it appeared in the curriculum hence the students are trained to solve this kind of problems and therefore they offered somewhat different ways of what appears in their lessons and did not stick only to one way or two, and everyone present the ways that require thinking skills in addition to being based on prior knowledge they studied before. Moreover, we can highlight that in the geometric problem there were different solutions from the first solution that is meant to be the direct and quick solution that darting the proof, and the rest of the solutions bind two level of thinking, the first is proving that the square is a parallelogram and the other way is to prove that a parallelogram is a rhombus, which require from the pupil to connect between the parallelogram's and the rhombus's traits, the pupils who reached the second level witness the ability to response in different ways that can be attached to the originality of the conceived ideas, these results consist to a certain extent with Tzahor's (2011) claims, who emphasized in his research that the use in alternative tools not only opens the student to additional solution options, but also enables the pupil a wider and more general image perception, how different math branches complete one another. Tzahor (2011) added that there is no need for a special chapter for teaching solution via alternative tools, but rather the pupils should exercise throughout the Geometry studies how and where alternative proofs can be integrated. Because the difficulty is less for the students and more for the teachers who are required to change their habits. The findings of the geometric problem show that the students' approach toward the problem consist with the claims of Levav-Waynberg & Leikin (2012), who have conducted a study about solving geometric problem in different ways, and found that a systematic use of teaching that encourages multiple solutions to geometric problems contribute both to connectivity of mathematical knowledge, and to the creativity of the student expressed by the fluency of ideas and the level of intellectual flexibility while solving geometric problems. It is important to mention that in the Geometry standard curriculum we can find numerous problems with different geometric solutions, that base on various auxiliary structures or different theorems.

We should mention that Falk (2010) stated in his research that the Infinity term consist until this day a big challenge for many researchers who try to search for the difficulties behind its terminology. Furthermore, in their study, Avitan and Nesher (2012) add and confirm the difficulties created when the students attempt to understand the Infinity concept, and here they showed the existence of the intuitive thinking that pulsing in the students and affects their comprehension of the Infinity concept, the pupils' intuitive responses, their failures in comprehending the concept of Infinity, including the creation of cognitive conflicts, and the And the developmental line (by age) of the degree of intuitiveness in the comprehension of the Infinity concept, should bring us to look straight at the intuition, and be prepared to deal with it through having additional studies, that would shed light on issues of difficulty and will extend the existing corpus of knowledge.

The multiplicity of solutions offered by the student is depended on his or her level of novelty, which guaranty co-dependence between the novelty level and the thinking skills and the ability to make conclusions of each student, meaning, if the pupil has a well-organized thinking and he formed confidence in developing targets then he can achieve a correct solution in different ways.

Additionally, in a review conducted by Martindale (1989), which focused on in the stages of the creative process and this raises the question of what creativity is expressed in the student's answers, and can we find and integrate it in solving math problems? For instance, when a student is given with an unusual mathematical task that does not rely on an immediate algorithm, therefore, in the solution stage, it is impossible to rely only on a regular association, but rather a more complex process is needed, that connects between the components of the mathematical problem in a way that was not previously known- this combination allows to achieve creative solutions. The interviews' findings raised a clear image that reflects the students' approach toward the three elements of creativity, which are: *creative flexibility* which expressed in their ability to solve the problem with various approaches and/or methods; *intellectual fluency* when solving the problem, which stood especially out in their relation to solution planning and execution time; and novelty that relies on originality in the curriculum, which stood especially out in the solutions category of each problem separately.

These findings, which arose from the interviews with the pupils reflects in the characterization of the creative person, since Martindale (1989) and Landau (1990) concluded dozens of studies that engaged in the characterization of the creative person and mentioned several traits and tendencies that were found as related to the issue, among them: the ability to connect elements that seem unrelated, extensive and deep knowledge in the content of the work, interest in broader areas, flexibility and originality in the functions of classification and reorganization, proper identification of important problems and internal motivation with openness to new ideas, are more imaginative.

Guberman (2010) pointed out the importance of the math teacher's central messages that he wishes to deliver to his pupils, and notes that the mathematics is not a collection of facts and rules of action that according to it one should act, but rather it has a place for imagination, creating and raising original ideas, and it is very important to allow the pupils to search for their own methods to deal with different tasks.

In view of these findings, raised in the current research, it is possible to refer to novelty as a basic element in problem-solving, everything depends on methods of teaching bestowed upon the shoulders of those middle school teachers, and here is where the educational massage that characterizes the creative learning stands out. And it is extremely important to not stick to a single teaching method and deliver the subject matter in various possible approaches in order to develop creative thinking in the student basing on the originality of the formed ideas. Stegner (2011) adds and recommends that teachers of heterogenic classrooms should receive training in teaching mathematics to excellent students in regular classes and the awareness to the importance of the issue should be raised. The mathematical language among the pupils which has been expressed in the interviews characterized by the pupils' use in terms that referred to the thinking process that has occurred in the pupil's mind from the moment he or she grasped the task and began to plan the solution and then moved to the solution execution stage with all the dilemmas he encountered on the way to the final solution.

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