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THE DEVELOPMENT OF CONTENT KNOWLEDGE OF PROSPECTIVE MIDDLE SCHOOL MATHEMATICS TEACHERS ON ALGEBRA[i]

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Abstract:
Explanatory-confirmatory research design, one of the mixed methods research designs, was used in this study to investigate content knowledge (CK) developments of prospective teachers regarding algebra. Cross-sectional study method, as a type of descriptive research and one of the non-experimental research designs, was used to collect quantitative data in the study. In the qualitative part of the study, case study was used. The participants of the study were composed of 176 prospective teachers studying in the elementary mathematics education department of a university in Turkey, who were first, second, third, and fourth year students with equal numbers. Interview, observation and knowledge test for algebra were used as the instruments for the purpose of study. One way ANOVA test was used to compare the means of the total scores of the prospective teachers obtained from Algebra Content Knowledge Test (ACKT) since ACKT score are normally distributed. According to the results of the study, it was observed that knowledge levels of the prospective teachers have developed as directly proportional depending on the class level. This development continued during the passing from second year to third year while it decelerates. In spite of this, it was observed that the development of CK accelerated again in the fourth year. In addition, it was also observed that the knowledge of prospective teachers in terms of CK was not at the desired level.

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1. Introduction

Algebra is one of the most important learning domains of the mathematics lesson since it is a means of problem-solving, and a mechanism that ensures understanding and defining mathematical relations (Baş, Erbaş and Çetinkaya, 2011; Ususkin, 1995). Thanks to algebra, we can easily solve complex problem situations by establishing simple equations, express the relations between different amounts and variables, and make generalization (Ususkin, 1995). Algebra has introduced a new momentum and practicality to mathematics with the symbolic structure it has brought to mathematics. However, when the relevant literature is examined, it is observed that students encounter many difficulties because of the abstract structure of algebra in the teaching process of algebra, which is very important for mathematics, and algebra seems frightening to students (Akkaya and Durmuş, 2015; Dede and Argün, 2003; Philipp, 1992; Stacey and MacGregor, 1997; Şahin and Soylu, 2011). Teachers play the most important role in eliminating these difficulties and ensuring effective algebra teaching (Baş, Erbaş and Çetinkaya, 2011; Dede and Argün, 2003). Because the studies (Ball, Thames and Phelps, 2008; Kwong vd., 2007; Şahin vd., 2014) carried out show that the knowledge of teaching mathematics of teachers is effective in the learning of students. In other words, it is necessary for teachers to have the adequate pedagogical content knowledge in order for algebra, which is among the abstract and hard to understand learning domains of mathematics, to be understood by students.

Upon investigating teacher training programs, it is observed that there are constant new searches in the structure of teacher training programs and changes are frequently made. In this context, various teacher knowledge models were developed in order to ensure that teachers, who take a very important place in teaching activities, are better raised and developed, and the concept of pedagogical content knowledge comes to the forefront in these models developed (Ball et al., 2008; Grossmann, 1990; Ma, 1999; Marks, 1990; Shulman, 1986; Tamir, 1988). Teachers with the high level of pedagogical content knowledge can easily determine the errors and mistakes of students use the teaching strategies aimed at producing solutions to the problems that emerge during the teaching process, and explain the subject in accordance with the cognitive levels of students (Smith and Neale, 1989; Toluk-Uçar, 2011; Türnüklü, 2005). On the other hand, teachers with the inadequate pedagogical content knowledge encounter problems in understanding students and eliminating student problems (Bütün, 2012; Işıksal, 2006). In this context, no matter how adequate the content knowledge of teachers is, they need to have strong pedagogical content knowledge in order for their teaching activities to become successful in order for the educational activities to have the expected features and quality (Türnüklü, 2005).

The importance of teachers' content knowledge has been emphasized in many studies carried out on teacher training. However, the relationship between the content knowledge and pedagogical content knowledge was addressed in different ways in the models developed (Park and Oliver, 2008). Content knowledge was regarded as a separate category from pedagogical content knowledge in many teacher knowledge models (Grossman, 1990; Smith and Neale, 1989; Tamir, 1988) including that of Shuman (1987). Nevertheless, it was addressed as a sub-component of pedagogical content knowledge in certain teacher knowledge models (Hasweh, 2005; Loughran et al., 2006). The content knowledge, one of components of the pedagogical content knowledge that are strongly emphasized in the teacher knowledge models developed, consists of the theoretical knowledge teachers have in relation to the learning domain in which they teach (Shulman,1987). Shulman (1987) used two main structures while explaining the content knowledge. One of these structures is the means used for determining the accuracy and validity of the concepts and phenomena in the field (mathematics) (syntactic structure), while the second one consists of different ways used for creating content knowledge (substantive structure) (Dönmez, 2009). In the teacher knowledge model developed by Ball, Thames, and Phelps (2008) emphasized the concept of content knowledge more when compared to other models. In this model, content knowledge was divided into three different categories, being specialized content knowledge, common content knowledge, and horizon content knowledge. Among these categories, horizon content knowledge ensures that teachers transmit the concepts in a more intelligible way to students by ensuring that a relationship is established between the mathematical concept they will teach and the advanced forms of this concept.

Upon examining the literature related to pedagogical content knowledge, it is observed that the studies carried out focus more on teachers and pre-service teachers working in the field of science (Hashweh, 1987; Käpyla, Heikkinen and Asunta, 2009). However, there are not many studies Baki, 2012; Bütün, 2012; Jenkins, 2010; Kleickmann vd., 2013; Şahin vd. 2014) on the level and development of pedagogical content knowledge of mathematics teachers and pre-service teachers. Kleickmann et al. (2013) conducted a cross-sectional study in order to determine the effect of the teacher training programs in Germany on the PCK and content knowledge developments of pre-service teachers. A survey consisting of open-ended questions was used in order to determine the content and pedagogical content knowledge of teachers and pre-service teachers (first, third and fourth grades). Jenkins (2010) carried out a study with six pre-service teachers in order to determine the role of structured interviews for the pre-service teachers' development of the knowledge of understandings students. Şahin et al. (2014) carried out a cross-sectional study by applying a test consisting of open-ended questions to third and fourth-grade pre-service teachers and mathematics teachers in order to investigate the PCK development of primary school mathematics pre-service teachers in relation to numbers. Şahin et al. (2014) only examined the development of the sub-components of knowledge of understanding students and instructional strategies. Bütün (2012) carried out a longitudinal study in which third and fourth-grade pre-service teachers participated in order to examine the development of content knowledge for teaching mathematics (instructional explanation, teaching method, belief) of pre-service teachers of the enriched learning environments.

In his study, Bütün (2012) used scenarios, observations, lesson plans and self-assessment forms as data collection tools. In the studies carried out on the PCK development of pre-service teachers in the literature (Baki, 2012; Bütün, 2012; Jenkins, 2010; Kleickmann vd., 2013; Şahin vd. 2014), it is generally observed that third and fourth-grade pre-service teachers make up the participants of the study together with teachers. Nevertheless, no study was encountered in which the PCK development of first, second, third and fourth-grade primary school mathematics pre-service teachers was examined together (Aydın and Boz, 2012; Depaepe, Verschaffel ve Kelchtermans, 2013). In addition to these, only qualitative, only quantitative methods or mixed research methods in which only qualitative methods gained weight were generally preferred in the studies (Baki, 2012; Bütün, 2012; Jenkins, 2010; Kleickmann et al., 2013; Şahin et al. 2014) carried out (Depaepe, Verschaffel and Kelchtermans, 2013). Nevertheless, it was observed that the PCK development of pre-service teachers was not examined with the mixed research designs in which quantitative research methods gained weight. In this context, in this study, it was aimed to investigate the development of pedagogical content knowledge of middle school mathematics pre-service teachers in algebra using quantitative and qualitative research methods together. In this study, the PCK development of pre-service teachers was examined in the context of the content knowledge given as a sub-component in many PCK models (Grossman 1990; Hasweh 2005; Shulman, 1987). Therefore, it is aimed to contribute to the literature on the development of pedagogical content knowledge of mathematics teachers by determining how the content knowledge of teachers in algebra develops. Furthermore, with this study, it is expected to get an idea regarding what level the content knowledge of pre-service teachers in algebra at the beginning of their undergraduate education will reach at the end of their undergraduate education. In addition to these, it will also be seen whether the content knowledge levels achieved by pre-service teachers at fourth grade fit the expectations. Thus, it is expected for the results of this study to provide researchers, experts, managers and politicians with the ideas about the effectiveness of middle school mathematics teaching undergraduate program.

2. Method

In this study, the *explanatory-confirmatory research design*, one of *mixed research* designs, has been used to investigate content knowledge regarding algebra developments of prospective mathematics teachers. In the explanatory-confirmatory research design, firstly quantitative data is collected and analyzed. After quantitative data are analyzed, participants are selected for the qualitative part of the study based on quantitative data, and qualitative data is obtained from these participants. The data obtained from the qualitative part of the study is used to make sense of the quantitative findings of the study. In other words, the general framework of study established by using quantitative data is made more understandable with the support of qualitative data (Creswell, 2011; McMillan and Schumacher, 2010). *The cross-sectional comparative study* method, one of the non-experimental research designs, was used in the process of obtaining quantitative data. The selection of the cross-sectional comparative study can be justified by the fact that it is difficult to collect data from a group for a long time in the longitudinal research method. In a cross-sectional study, different groups of subjects can be studied at the same time-for example, in this study first, second, third and fourth grade teacher candidate groups, all of whom was surveyed at the same year (McMillan and Schumacher, 2010). The case study method was used in the qualitative section in this study. Case study method gives researchers a chance to examine an event, a situation, a relationship or a process in depth with the help of a limited number of samples (Denscombe, 2010. In this study, qualitative data was collected by different methods such as interview, observation (data triangulation) to support quantitative data.

2.1 Participants

In the qualitative part of the study, case study was used. The participants of the study were composed of 176 prospective teachers studying in the elementary mathematics education department of a university in Turkey, who were first, second, third, and fourth year students with equal numbers. Since the cross-sectional study method is employed in the study, it gains importance for preservice teachers studying at the same university to be selected with regard to rendering the groups are close to each other.

The participants were determined by a convenience sampling method from non-random sampling methods. In convenience sampling method, a group of subjects can be selected on the basis of accessible and expedient (McMillian and Schumacher, 2010). Because the researchers have the opportunity to study with people who are easy to reach (Yıldırım and Şimşek, 2011). The real names of the preservice teachers was not used for research ethics. The first class who participated in the study were assigned with the codes from *1S1* to *1S44*, the second class from *2S1* to *2S44*, the third from *3S1* to *3S44*, the fourth from *4S1* to *4S44.* Following table shows the demographic information of preservice teacher which consists of gender, socio-economic levels, secondary schooling, parents' education, and preference order of faculty of education in university entrance exam, reason of being teacher. It has been taken into consideration that the teacher candidates participating the study in each grade level are close to each other numerically, the reasons for section preference are similar, the number of male and female are equal, the education of parents are similar so as to form groups equivalent to each other.

Table 1: Demographic characteristics of preservice teacher

Grade Level / **Demographic Characteristics**		**First Grade**	**Second Grade**	**Third Grade**	**Fourth Grade**
Gender	Male	13	13	13	13
	Female	31	31	31	31
Socio-Economic Levels	High	6	5	6	8
	Mediocre	36	38	37	36
	Low	2	1	1	-
Secondary Schooling	Teacher Training High School	24	15	11	8
	Others	20	29	33	36
Father's Education	Not Go To School	-	-	-	2
	Primary School	15	15	12	17
	Middle School	6	4	6	9
	High School	10	15	10	9
	University	13	10	16	7
Mother's Education	Not Go To School	1	1	6	8
	Primary School	29	25	29	23
	Middle School	2	7	4	5
	High School	10	8	5	6
	University	2	3	-	2
Order Of Preference	1-5	29	28	31	35
	5-10	10	9	6	4

	10-15	4	7	3	4
	15-20	1	-	1	1
Reason For Preference	Idealism	16	20	18	16
	Easy To Get A Job	23	18	25	20
	Family Pressure	5	6	1	8

It has been taken into consideration that teacher candidates who will participate in the study are numerically equal to each other in terms of gender variable. Because, gender variable have been taken into consideration in many studies on the professional development of teachers (Gökbulut, 2010; Kleickmann et al., 2013; Stephens, 2006) while selection of participants. The participants of the research made by Gökbulut (2010) consisted of 4 teachers, 2 female and 2 male. Stephens (2006) stated that the study consisted of a total of 30 teacher candidates, 28 girls and 2 boys, and that this sample represented the population of study perfectly. The participants of the study consist of 44 pre-service teachers for each grade level, 31 of them being female, 13 male students studying at the department of elementary school mathematics education of a university in Turkey. As seen Table 1, preservice teachers in each grade level are numerically close to each other in terms of variables such as gender, parents' education, secondary schooling, socio-economic level, order of preference and reason for preference. After forming equivalent groups accordance with grade level, data collection tools developed by researchers was applied to participants.

2.2. Data Collection Tools

In this study, interview, observation and Algebra Content Knowledge Test (ACKT) have been used as data collection tools to examine the development of content knowledge regarding algebra.

2.2.1. Algebra Content Knowledge Test (ACKT)

The researcher first formed a question pool consisting of twenty-five questions using the Teacher's Guide Book prepared by Ministry of Education and the related literature (Altun, 2013) when preparing ACKT (APP-1). Then the number of questions in ACKT was reduced to fourteen in the direction of expert opinions. The ACKT consisting of fourteen questions was applied to 65 pre-service teachers in the framework of a pilot study by removing the questions not serving the aim of the study and measuring similar skills from the test. After the pilot study, the ACKT consisting of thirteen questions was formed after making the necessary corrections by totally removing question six of the first form and two options of the fourth question of the ACKT from the test in line with expert opinions. The attainments in the Secondary School Mathematics Curriculum were taken into consideration when creating the ACKT. The ACKT consists of open-ended questions that measure the pre-service teachers' skills of defining mathematical concepts, explaining operations and mathematical justifications behind the operations, and their generalization and modeling skills.

2.2.2. Interviews

In this study, semi-structured interviews were held in order to assess the picture emerging as a result of ACKT applied to pre-service teachers and learn the opinions of pre-service teachers. The aim of the interviews held with pre-service teachers is to make the answers of pre-service teachers to the knowledge test more intelligible. In other words, the need to interview these pre-service teachers emerged since the answers of certain pre-service teachers to the knowledge test were not sufficiently clear and intelligible. The interviews were mostly held with pre-service teachers who answered in *"partially correct b"* and *"wrong answer"* categories. In other words, the interviews focused on the questions in which pre-service teachers had difficulty in reaching the correct answer. The answer to the question why pre-service teachers had difficulty in these questions was sought as a result of the interviews.

2.2.3. Observation

The observations was used as a data collection tool in this study in order to determine the extent to which pre-service teachers could apply the things they knew and the difference between what they aimed and what they actually did. The structured field study method was preferred in this study among the types of observation. The researcher performed observations using a structured observation form in the structured field study method (Yıldırım and Şimşek, 2011). Furthermore, the researcher took notes in observation forms when making an observation.

2.3. Data Analysis

Qualitative and quantitative data analysis techniques were used together in analyzing collected data. Wolcott (1994) state that one of the most preferred methods in qualitative data analysis is to transfer the data (interview, observation) to the research report descriptively through direct quotations without disrupting the originality of the data (Yıldırım and Şimşek, 2011). In this study, teacher candidates' responses to interviews observations were reported as direct quotations.

The scoring categories (Kwong et al., 2007; Şahin et al., 2014) regarding the answers of the students to the content knowledge test take place in Table 2 below. Also, analysis framework of first question was given in APP-2 to make data anlaysis process more understandable.

Table 2: The scoring categories of ACKT

Categories	Completely true	Partially True (a)	Partially True (b)	Wrong Answer	No Answer
Score	4	3	2	1	0

- **Completely true:** This category is the case when the preservice teachers answer a question precisely and completely.
- **Partially True (a):** This category is the case when the preservice teachers cannot give a precise, in other words, complete answer but the answer to the question is very close to the correct and contains minor mistakes.
- **Partially True (b):** This category is the case when the preservice teachers do not give a completely wrong answer, and their answers contain correct expressions though little when compared to the wrong.
- **Wrong Answer:** This category is the case when the answers of the preservice teachers are completely wrong.
- **No Answer:** This category is the case when the preservice teachers cannot give any answer to the question.

As seen in Table 3, since $p> .05$, it can be stated that the scores of the content knowledge test of the preservice teachers are distributed normally. Thus, One Way ANOVA was used to compare the means of prospective teachers' Algebra Content Knowledge Test (ACKT) scores since these scores are normally distributed.

Table 3: The results of Kolmogorov-Simirnov test

	Statistics	df	P(sig)
ACKT	0.056	176	.200

2.4 Validity and Reliability

To ensure the validity of the ACKT, it was first examined by a linguist. Later ACKT was examined by three experts studying on mathematics education. Before the opinions of the experts were taken, they were provided with the use of the statement table regarding ACKT. The final form of the ACKT was given by making necessary corrections in line with the expert opinions. ACKT was applied to 65 teacher candidates to control whether it serve the purpose of the study. At the end of the pilot study, the 6th question in the ACKT was removed from the test in accordance with expert opinions. Also, two line graph was removed from question four in ACKT. After the pilot study, the opinions of the experts were taken again and the final version was given to the ACKT.

In this study, the *intraclass correlation coefficient*, which examines the correlation among the scorings different raters make, was benefited from in order to ensure the reliability of the curriculum knowledge test. Initially, a new data set was created randomly in a way that it would represent at least 10% (Cleophas and Zwinderman, 2015) of the total data for each test with the help of SPSS program. The data in this data set were graded by two researchers with the help of the answer key which was previously prepared and organized in accordance with the expert opinions. Afterwards, the correlation coefficient between these two scorings was calculated by SPSS program. The intraclass reliability coefficient of the content knowledge test was included in Table 4 below.

Table 4: The intraclass correlation coefficient of ACKT

Number of Researchers	İntraclass Correlation Coefficient	df1	df2	sig
2	.979	19	19	.000

As seen in Table 4, the intraclass correlation coefficient of the content knowledge test is quite high and significant. In other words, it can be stated that the data obtained in this study are highly reliable.

The studies to ensure the validity and reliability of the qualitative part of this study are given in Table 5.

Table 5: Validity and reliability of qualitative part of study

Validity	Reliability
Literature Review	Data Analysis Based on Conceptual Framework
External Audits (Experts on mathematics education)	Triangulation
Pilot Study	Pilot Study
Peer examination	Peer Review
	Using video camera and recorder

3. Findings

In this part of the study, the results of the analysis of the data obtained from the interviews, observations and ACKT applied to the prospective teachers in order to determine the content knowledge developments related to algebra were presented. In addition, the frequency-percentage distribution table for each question in the ACKT is given in APP- 3. In the findings of first question of ACKT, it was seen that most of the prospective teachers had difficulty in explaining why they had to change the sign of constant term when it was going to the opposite side of the equation in the case of solving equations. Because most of the prospective teachers stated that it was just a rule of mathematics. For example, the phrase "*It is a rule of mathematics"* used by 1S10 teacher candidate is a statement that supports this result. During the observation of lectures of prospective teachers, it has been seen that the prospective teachers had difficulty in explaining the rational reasons behind the mathematical operations.

2S21 prospective teacher tried to teach subtraction in algebraic expressions as follows: *"When performing subtraction in algebraic expressions, subtraction is firstly turned into addition, and then addition is performed. (One of the students asked what? since he/she did not understand this). In other words, if we apply it in an example (3x+ 5)-(2x+3), we should distribute the minus inside since we first turn subtraction into addition. 3x plus 5, plus, negative 2x and negative 3. It becomes negative 2x negative 3 since we have distributed the minus inside. We would perform addition, then, we would add the variables among themselves, and the constants among themselves. Consequently, we find the expression 3x minus 2x is equal to x, and 5 minus 3 is equal to plus 2."*

It can be concluded from the instructional explanations of the 2S21 teacher candidate, the prospective teacher preferred to give a rule directly, without explaining the logical justifications of the mathematical rules.

Pre-service teacher 3S39 wrote the expression 4.a-1 on the board as the algebraic expression equivalent of the verbal expression "*The age of Kerem's father is 1 less than 4 times the age of Kerem.*" Whereupon, one of the students asked: "*Shouldn't it be a.4-1?*" The pre-service teacher answered this question as "*That is also possible.* We can also write it as: *a.4-1.*" As can be seen from this example case, the pre-service teacher did not explain to students why the expressions of *4.a-1* and *a.4-1* were equal by using their mathematical justifications, i.e., the commutative property of multiplication.

4S34 asked the example in Figure 1 to students. He/she found the relationship between the number of pencil cases and the number of pencils as "*n.5*" by solving this question together with students. Then, the pre-service teacher wrote the result as "5.n" by saying "*We do not write this as n times 5. We are replacing it. This is actually 5 times n*". The following dialogues took place during the solution of this problem.

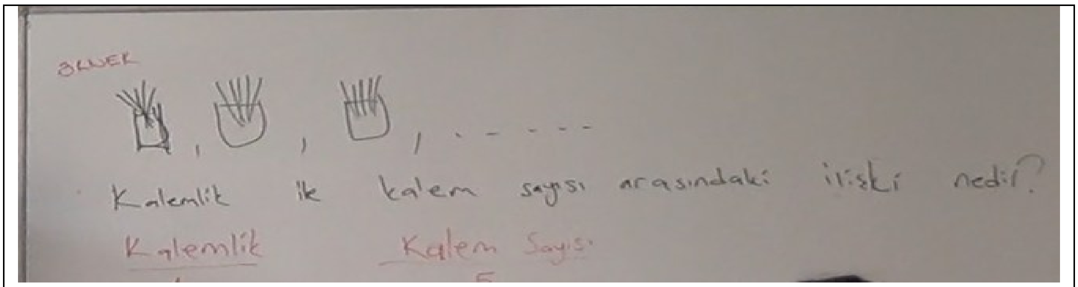


Figure 1: The question asked by 4s34 pre-service teacher

4S34: *How many pencil would I have had, if I had x number of penholder?*
Student1: *Multiply x by 5.*
4S34: That is 5 times x. *What is the meaning of ''x'' and ''n'' in this situation?*
Student1: *Unknown.*
4S34: *It is also called variable.*
Student 2: *It represents the numbers.*
4S34: *Yes, you are right. Because ''n'' represents lots of numbers.*

Pre-service teacher 4S34 used a different letter in order to prevent students from focusing on one letter. The pre-service teacher found the result as n times 5 and then stated that it could not be written as n.5 and it should be written as 5.n. However, the pre-service teacher did not explain why the expressions of n.5 and 5.n should be equal by using mathematical arguments. From the classroom observations of pre-service teachers 2S21, 3S39 and 4S34, it was observed that the pre-service teachers had difficulty in explaining the justifications underlying mathematical operations.

Starting from the fact that a very small part of the answers given by pre-service teachers to the second question of the ACKT is categorized as correct and partially correct A, it can be said that the levels of pre-service teachers' knowledge of the concept of algebraic expressions are not at the required level. It is observed that pre-service teachers fail to differentiate the concepts of algebraic expression and equation from each other in general. It was observed that pre-service teachers sometimes named equations as algebraic expressions, and sometimes they named algebraic expressions as equations. Furthermore, it was observed that the explanations of pre-service teachers on why an expression was or was not an algebraic expression did not match the algebraic expressions. For example, pre-service teacher 2S4 gave the justification of "*There must be an unknown term in algebraic expressions*" for an expression to be an algebraic expression. While the pre-service teacher should have specified the expressions "$A = \pi r^2$,*x-5, E=mc*^{2} and *x+2=5"*as algebraic expressions according to the definition, the pre-service teacher stated that only the expressions of "*x-5* and *x+2=5"* were algebraic expressions.

From the written and verbal answer of pre-service teacher 3S34 to the second question, he/she answered in partially correct B category by not being able to distinguish the concepts of algebraic expression and equation from each other. While the pre-service teacher correctly expressed that there must be a variable and operation in the algebraic expression, he/she stated it incorrectly that there must be an equation. Furthermore, the pre-service teacher stated that only the expression x+2=5 was an algebraic expression while he/she should have stated that the expressions $\mathrm{A} = \pi \mathrm{r}^2$ and $\mathrm{E} = \mathrm{mc}^2$ were algebraic expressions according to the definition he/she made. In other words, the pre-service teacher both made an incomplete definition and failed to choose the examples matching the definition.

Table 6: Answer of preservice teacher coded with 3S34 to item 2 ACKT

<table>
<tr><th>Participant</th><th>Answer to
Item 2</th><th>Translation of
Quotation</th></tr>
<tr><td>3S34</td><td>x+2=5 bir cebirsel ifadedir Çünkü cebirsel ifadeler bilinmeyen, eşitlik ve işlemden oluşur</td><td>x+2=5 is a algebraic expression. Because algebraic expressions include variable and equality.</td></tr>
<tr><td colspan="3">Researcher: In this question, why did you call only x+2=5 as a algebraic expression?
3S34: $A = \pi r^2$ is a formula. $2(3^2-7)+(2+9^5)$ is a just operation with number. I said that algebraic expressions had to include variable, operation and equality while answering question 2 in ACKT. There is a is a just operation in expression of $2(3^2-7)+(2+9^5)$. E=mc^2 is a only formula in physics. x+ 2 = 5 fits my definition of algebraic expression. This expression can be solved as an equation.
R: What is the difference between an equation and an algebraic expression?
3S34: Actually, I considered algebraic expression as an equation. The expression of “x-5” does have just variable but not have operation and equality.
R: There is a subtraction in x-5 expression, isn’t it?
3S34: Operation? We can find a result If we add -2 to both side of equation. There has to be a result. I can say that algebraic expressions include variable, operation and equality.
R: Do you mean the equation is the same as the algebraic expression?
3S34: Yes.</td></tr>
</table>

From the written and verbal answers given by pre-service teachers to the third question of the ACKT, it is observed that most pre-service teachers make various mistakes such as the fact that the degree of the variable in an expression should be at least two for it to be an identity, to be a full square, the use of parenthesis or an expression must have an expansion. Some pre-service teachers failed to explain the justifications although they could detect the identities and equations in the expressions given.

It has emerged from the findings of fourth question of ACKT that the vast majority of prospective teachers wrote the equations directly without doing any mathematical operations needed. During the interview process, 2S14 preservice teacher stated that I wrote the equations of a line by *"trial and error method"* after question of *“how to write a equation of a line given”* asked. Nevertheless, a very small number of pre-service teachers could write the appropriate line equations to the lines given by determining the points through which the lines passed, considering the angles of the lines to the x-axis and performing the necessary mathematical operations in writing the appropriate line equation for a given line. That is, most of preservice teacher did not use mathematical notations such as $\frac{x}{a}+\frac{y}{b}=1, y=mx+n, ax+by+c=0\ or\ \frac{y_2-y_1}{x_2-x_1}=\frac{y-y_1}{x-x_1}$ while writing equations.

The findings of fifth question of ACKT shows that preservice teachers have difficulty in defining the concepts related to algebra, although they have knowledge about concepts, their representations and their daily life examples. It was found out that pre-service teachers failed to define the concepts of slope and pattern, despite not having had much difficulty in defining the concepts of the variable and linear equation. Furthermore, pre-service teachers used incorrect definitions such as "*the angle of the line to the x-axis*" or "*the value of the positive angle of the line to the x-axis*" instead of the fact that slope is "*the tangent value of the positive angle of a line to the x-axis*" when defining the slope. It is observed that pre-service teachers focus on the fact that the pattern may be formed from numbers and shapes when defining the concept of pattern. Nevertheless, it can be said that pre-service teachers neglect the fact that patterns may consist of different elements such as voices, objects or symbols. It was observed that first and second-grade pre-service teachers had problems in defining the algebraic concepts while teaching, just as in their written and verbal answers. Pre-service teacher 1S1 incompletely defined the algebraic expression as "*The expressions that contain at least one variable are algebraic expressions*" when teaching because an expression must contain at least one variable and operation to be an algebraic expression. Similarly, pre-service teacher 2S2 made an incomplete definition as "*The numbers next to unknown numbers are our coefficients.*" The observations of the pre-service teachers' teaching processes support the finding that pre-service teachers have difficulty in defining algebraic concepts. Pre-service teacher 2S2 used the algebraic expression of *2a+3b+4c-3* when explaining the concept of the coefficient and made the definition that "*The numbers next to unknown numbers are our coefficients.*" Upon examining the definition of coefficient made by the pre-service teacher, it is observed that the pre-service teacher has not emphasized that the coefficient is in the form of multiplication, and he/she has pointed to an uncertain situation instead by saying the number next to it. The fact that one of the students said 9 when the pre-service teacher asked the coefficient of the expression *6y+3* later in the lesson is an indicator of this. The fact that pre-service teacher 3S39 only mentioned the numbers in his/her definition of the pattern as "*Yes, the number or the series of numbers expanding according to a particular rule is called pattern*" shows that he/she defined the concept of pattern incompletely.

Findings on the sixth question of the ACKT show that pre-service teachers have difficulty in explaining different purposes of use of variables. Many pre-service teachers stated that variables are used only in order to find the result of unknown expressions. Pre-service teachers could make partially sufficient explanations in expressing the purposes of use of the variables in the expressions *"if a+5=8, a=? and a+b=10 and a<7"*. Nevertheless, it was observed that pre-service teachers had more difficulty in explaining the role of the variables in the expressions *"if a+b=25, a+b+3=? and add 4 to a+5"*. For example, pre-service teachers gave incomplete answers such as "*The value of the equation changes according to the value of the variable*", "*The sum of a variable and a constant number*", and "*To show that we can add numbers only to numbers*" for the expression of "*add 4 to a+5*" in which the value of the variable is not important, i.e., the letter represents an uncertain situation. From the answers of pre-service teachers, it can be said that they failed to explain the purpose of use of the variable by using the mathematics language correctly. The deficiencies of pre-service teachers in using the mathematics language also reflected on the lesson practices. For example, pre-service teacher 1S1 made the definition that "*Each expression separated with a plus and minus is called a term*" when defining the concept of the term. The pre-service teacher focused on plus and minus signs while he/she was supposed to emphasize the operations of addition and subtraction. Consequently, it can be said that pre-service teachers have difficulty in identifying the purpose of using variables in a given mathematical expression.

It is observed that many pre-service teachers mostly prefer to pose number problems when posing suitable problems for given expressions. For example, it is observed that they prefer to pose problems for the algebraic expression of 2s+5 as *"5 plus two times a number"*, for the algebraic expression *if* a+b=25 a+b+3=? as*"If the sum of two numbers is 25…"*, and "*Which number is equal to 18 when it is multiplied by two and one is subtracted?*" for the equation of 2(x-1)=18. Nevertheless, a very small number of pre-service teachers could pose problems in different themes. For example, a pre-service teacher posed a problem as "*5 plus the sum of the ages of two twins*" for the algebraic expression of 2s+5. Furthermore, it can be said that pre-service teachers are generally successful in posing a problem that fits the algebraic expressions and equations given. However, it was observed that first and second-grade pre-service teachers had difficulty in posing problems suitable for the algebraic expression.

Upon examining the interview held with pre-service teacher 1S1, it is observed that the pre-service teacher has experienced serious problems in posing a problem suitable for the algebraic expression despite not having difficulty in posing problems suitable for the equations. The justification behind this difficulty is the thought that no problem can be posed if an expression is not an equation, i.e., if there is no equality and this is not equal to a result.

Table 7: Answer of preservice teacher coded with 1S1 to item 7 ACKT

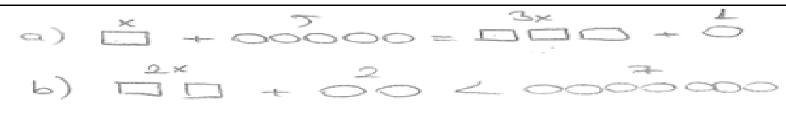
<table>
<tr><th>Participant</th><th>Answer to
Item 7</th><th>Translation of
Quotation</th></tr>
<tr><td>1S1</td><td>a)
b) Ahmet ile Veli'nin toplam 25 kalemi vardır. Bunların 3 kalemi daha olsaydı toplam kaç kalemleri olurdu?
c) Hangi sayının iki fazlasının yarısı 10'dur</td><td>a)
b) Ahmet and Veli had totally 25 pencils. If they buy 3 pencils how many pencils will they have?
c) What number do you get 10 ıf you add 2 to half of this number?</td></tr>
<tr><td colspan="3">A: As seen in your answer to question 7, you posed a problem for expression of "What is the result of a+b+3 If a+b=25" and "2(x-1)=18". But you did not pose a problem for expression of 2s+5. Could you please explain your answer?
1S1: There has to be a result to pose a problem. So I did not pose a problem for a expression of 2s+5.
A: How do you call 2s+5 mathematically?
1S1: I cannot call it equation because it doesn't have a equality.
A: Do you mean that there has to be equation to pose a problem?
1S1: Yes, I meant that. There has to be equality. So I easily posed a problem for expression of " What is the result of a+b+3 If a+b=25" and "2(x-1)=18"
A: Is this problem "What is the result of adding 5 to two times a number? Not appropriate for expression of 2s+5.
1S1: No. Because there is not any result. There is not equality.</td></tr>
</table>

It was observed that the problems used by pre-service teachers in lesson applications contain certain mistakes. For example, when pre-service teacher 4S34 wrote on the board the following question "*A pedestrian and a bicycle rider start to move at the same time. The average hourly speed of the bicycle rider is 2 plus 3 times the average speed of the pedestrian per hour. Accordingly, examine the distance that the pedestrian and bicycle rider went.*", one of the students asked the question, "*Teacher, I would like to ask something. There is something weird in this question. If you ride a bicycle, you can accelerate, but don't you get exhausted after some time?*" The pre-service teacher tried to persuade the student by saying "*But we assume that he does not get exhausted. He does not go forever; he goes for 3 hours or 5 hours.*" However, the student said "*I do not know a single person who can run for 3 hours*" by not becoming satisfied with the answer of the pre-service teacher. Starting from this case study, it can be said that the pre-service teacher has experienced problems since he/she has not paid attention to the suitability of the example to real life in the selection of examples.

In the eighth question of the ACKT, many pre-service teachers failed to reach generalization with the help of mathematical notations although they achieved the correct result numerically. In other words, it can be said that pre-service teachers experience problems in explaining an event or situation by using mathematical symbols. Furthermore, many pre-service teachers reached an incorrect result in this question since they did not divide the algebraic expressions or numerical expressions found into two.

It was observed that many pre-service teachers failed to model these equation and inequality although they correctly found the solution set of the equation and inequality. In the modeling they performed, pre-service teachers did not make any modeling for addition, equality and inequality situations while preferring various concrete objects to model the *x* variable and numbers. The pre-service teachers who answered the ninth question of the ACKT correctly generally preferred to use the scales model. A very few number of pre-service teachers used the side length of geometric shapes in order to model the equation and inequality. Similarly, in the eleventh question of the ACKT, it is observed that the skills of modeling an expression in the form of "x^2+5x+6" using geometric shapes vary in direct proportion to the grade level. Although second-grade pre-service teachers were more successful than third-grade pre-service teachers in the visualization of equations and inequalities, third-grade pre-service teachers became more successful than second-grade pre-service teachers in the modeling of an algebraic expression in the form of "x^2+5x+6". While it was observed that first and second-grade pre-service teachers made mistakes by separately modeling each expression in the expression of "x^2+5x+6", it was observed that third and fourth-grade pre-service teachers preferred to model this expression correctly as a whole.
The interview with 2S21 preservice teacher was given in Table 8 below.

Table 8: Answer of preservice teacher coded with 2S21 to item 9 ACKT

<table>
<tr><th>Participant</th><th>Answer to
Item 9</th></tr>
<tr><td>2S21</td><td>
</td></tr>
<tr><td colspan="2">R: Could you please expand your model?
2S21: I used square to model variable of x. So I used two square to model 2x(x+x). I used circles to model numbers.
R: You did not use any model for equality and plus sign?
2S21: Yes.
R: Can you model this expression with a more concrete model from everyday life?
2S21: I can use pear instead of x and apple for number.
R: Don't you know anything about equation and inequalities from daily life?
2S21: No, I don't know anything.</td></tr>
</table>

Pre-service teacher 2S21 answered this question in the partially correct A category. Upon examining the written and verbal answers of the pre-service teacher, it is observed that he/she did not have any problem in the modeling of x variable and numbers when modeling the equation and inequalities given, but he/she had problems regarding which concrete elements to use in order to model the equality and inequality situations because the pre-service teacher did not use any concrete element for the modeling of the addition, equality and inequality expressions while he/she used various concrete elements for the modeling of x variable and numbers in the modeling of the equation and inequalities.

Upon examining the frequency-percentage distribution table of the ACKT tenth question, it is observed that pre-service teachers have had difficulty in explaining the concepts related to the concept of algebraic expression and these relations because it can be said that pre-service teachers have experienced difficulty in incorporating the concept of algebra sufficiently in the concept maps they have created, and explaining the relationship of these concepts to each other. For example, pre-service teacher 2S15 included only the concept of the unknown in the concept map he/she created. Moreover, it is observed that the skills of pre-service teachers in relation to this question vary in direct proportion to the grade level.

In the twelfth question of the ACKT, in which it is required to take into consideration all possibilities between the number of equations and the number of unknown, it is observed that pre-service teachers have only focused on the situations when the number of unknown and the number of equations is equal. It is observed that pre-service teachers have difficulty in expressing although they have correct knowledge about the solution of the equation in cases when the number of equations is less than the number of unknown, and the number of equations is more than the number of unknown. For example, pre-service teacher 3S32 answered this question incompletely as "*The number of equations and the number of unknown must be equal.*" However, the fact that the pre-service teacher answered the question "*What will happen if the number of equations is less than the number of unknown?*" as "*We cannot find the unknown if it is less.*" and the question "*What if the number of equations is higher than the number of unknown?*" as "*It can be solved.*" in the interview supports this finding. In the thirteenth question of the ACKT, in which the operational elements of pre-service teachers gain weight, it is observed that the conceptional elements gain weight, and pre-service teachers are quite successful when compared to other questions because a major part of pre-service teachers could achieve the correct result by performing the necessary operations in this question.

The descriptive statistics of the Algebra Content Knowledge Test are given in Table 9 below.

Table 9: The descriptive statistics of ACKT

ACKT	N	$\overline{X}$	Std. Deviation	Std. Error	Min.	Max.
First Grade	44	24.23	3.319	.500	17	30
Second Grade	44	28.39	4.677	.705	18	40
Third Grade	44	31.64	2.771	.418	26	37
Fourth Grade	44	37.82	3.848	.580	29	45
Total	176	30.52	6.198	.467	17	45

When the means of the total scores in Table 9 which elementary school mathematics preservice teachers obtained from ACKT are examined, the content knowledge levels of the preservice teachers has developed as directly proportional depending on the class level. In other words, whereas the highest content knowledge test mean belongs to the preservice teachers studying in the fourth grade, the lowest mean belongs to the preservice teachers studying in the first grade. As seen in the graphic below and in Table 10, the algebra content knowledge mean which is 24,23 in the 1st grade has reached 37,82 in the 4th grade. Furthermore, in the Algebra Content Knowledge Test from which maximum 52 scores can be obtained, the mean of total scores of preservice teachers is 30,52.

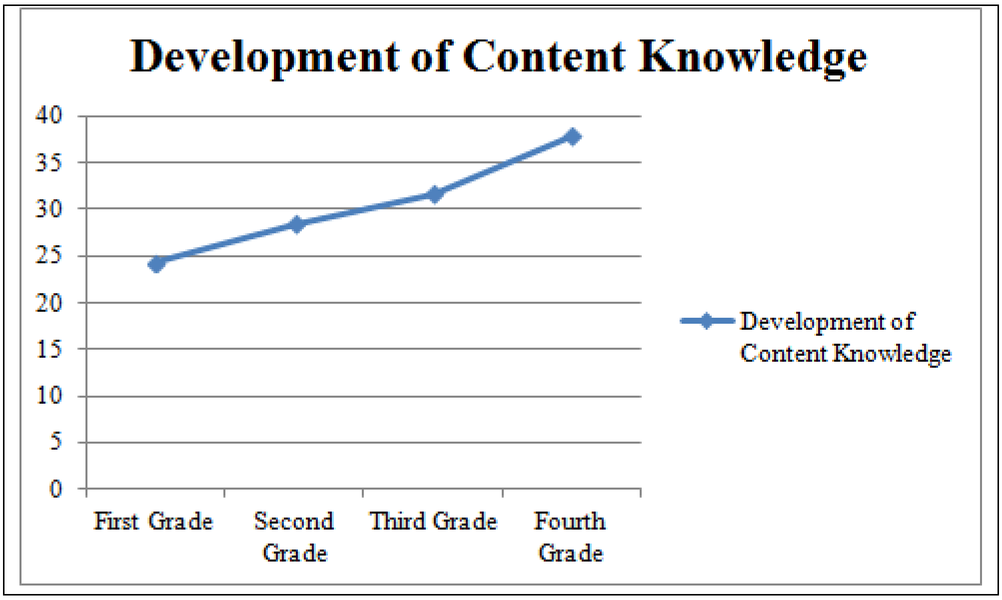


Figure 2: The development of content knowledge

Findings in Table 9 and Figure 2, it is seen that improvement occurs in the content knowledge levels of elementary school mathematics preservice teachers regarding algebra at each grade level throughout their undergraduate education. Moreover, the content knowledge improvements of preservice teachers display a significant increase in the second grade when they take teaching vocational courses. Although the content knowledge improvement continues in the third, the improvement rate decelerates. The content knowledge improvement continues in the fourth grade, the improvement rate increases again. However, these data are not sufficient to understand whether there is a statistically significant difference among the groups. The One Way ANOVA test among parametric tests was used in order to understand whether there is a significant difference among the grade levels. The ANOVA results of the Algebra Content Knowledge Test are given in Table 10 below.

Table 10: ANOVA results of ACKT

ACKT	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	4341,063	3	1447,021	104,536	,000
Within Groups	2380,886	172	13,842		
Total	6721,949	175			

As seen from Table 10, it is observed that there is a statistically significant difference among the content knowledge levels of preservice elementary school mathematics teachers regarding algebra depending on the grade level [$F_{(3,172)}$= 104,536, p<.05]. Thus, Posthoc test was used in order to determine among which groups there are significant differences. Homogeneity of variances was taken into consideration to determine which one of posthoc test must be used. As can be seen from Table 11 below, the variances are not distributed homogenously (p<0.05). Dunnett's T3 test was used among PostHoc tests since the variances were not distributed homogeneously.

Table 11: The results of homogeneity of variances

Test of Homogeneity of Variances				
ACKT	**Levene Statistic**	**df 1**	**df 2**	**Sig.**
	4,626	3	172	.004

Results belong to Dunnett's T3 test was given in Table 12 below to determine among which groups there are significant differences.

Table 12: The Dunnett's T3 results of ACKT

(I) Grade	(J) Grade f	Mean Difference (I-J)	Std. Error	Sig.
First Grade	Second Grade	-4.159*	.865	.000
	Third Grade	-7.409*	.652	.000
	Fourth Grade	-13.591*	.766	.000
Second Grade	First Grade	4.159*	.865	.000
	Third Grade	-3.250*	.819	.001
	Fourth Grade	-9.432*	.913	.000
Third Grade	First Grade	7.409*	.652	.000
	Second Grade	3.250*	.819	.001
	Fourth Grade	-6.182*	.715	.000
Fourth Grade	First Grade	13.591*	.766	.000
	Second Grade	9.432*	.913	.000
	Third Grade	6.182*	.715	.000

* The mean difference is significant at the 0.05 level.

Upon investigating the findings in Table 12, it is observed that the average ACKT scores of pre-service teachers vary by the grade level. In other words, their content knowledge scores increase as the grade level increases. Nevertheless, the fact that the average score of the fourth-grade pre-service teachers in the ACKT, in which the maximum score that can be taken is 52, is 37.82 shows that the content knowledge of pre-service teachers of algebra at the end of their undergraduate education does not reach a required level. In other words, it can be said that the content knowledge of pre-service teachers of algebra develops statistically significantly, but this development is not sufficient. Furthermore, the written and verbal answers given by pre-service teachers to the ACKT show that pre-service teachers have incorrect knowledge about certain concepts related to algebra and these mistakes are encountered throughout all grade levels. In addition to these, it was observed that pre-service teachers were more successful in mainly operational questions when compared to the questions in which conceptual elements gained weight. Indeed, the finding that pre-service teachers have difficulty in defining the concepts and effectively using the mathematics language found in the observations made during lessons is an indicator that they have deficiencies in their conceptual knowledge.

4. Discussion and Conclusions

The content knowledge of preservice teacher was examined in the context of operational knowledge, conceptual knowledge, generalization skill, problem-posing, definition of concepts. Furthermore, in the Algebra Content Knowledge Test from which maximum 52 scores can be obtained, the averages of fourth-grade preservice teachers are 37,82. When the fact that partially A and partially B answers are dominant in the answers which fourth-grade preservice teachers give to ACKT is considered, it can be said that the content knowledge of fourth-grade preservice teachers of algebra does not reach a desired level at the end of the undergraduate education because it was seen that fourth-grade preservice teachers had trouble in reaching the correct answers to many questions. After all, in many studies in the literature, it was seen that the content knowledge level of teachers and preservice teachers regarding many mathematical concepts was not at the desired level (Borko et al., 1992; Gökkurt et al., 2015; Hacıömeroğlu, 2005). It is also often stated that there is a positive relationship between the content knowledge of teachers and student achievement (Baumert et al., 2010; Welder, 2007). In other words, content knowledge has a significant role in effective teaching (Shulman, 1986). Goulding, Rowland and Barber (2002) stated that teacher who had inadequate content knowledge encountered difficulties while responding unexpected question of students. Similarly, Stacey et al. (2001) indicated that the content knowledge has an important role in understanding the learning difficulties that students have. Based on results of study, it can be said that it is not possible for the teacher candidates whose content knowledge is not at the desired level to successfully teach algebraic subjects to the students.

As can be seen from the responses of the prospective teachers to the questions in the ACKT, mistakes about some concepts related to algebra were found at all grade levels. Similar to this research, Tanışlı and Köse (2013) stated that teacher candidates had misconceptions about concepts related to algebra. Stacey et al. (2001) stated that pre-service teachers transfer their misconceptions to students. In addition to these, it was observed that pre-service teachers were more successful in mainly operational questions when compared to the questions in which conceptual elements gained weight. Indeed, the fact that pre-service teachers have become quite successful in the thirteenth question of the ACKT that contains operation-based questions in this study also supports this result. Similarly, in his study, Black (2007) stated that high school teachers make many conceptual mistakes although they do not make many operational mistakes. Menon (2009) stated in his study that pre-service teachers are quite successful in performing mathematical operations, while they have difficulty in justifying these operations.

At the end of the study, the mean score of the first grade preservice teachers in ACKT was 24.23, the mean score of the second grade preservice teachers in ACKT was 28.39, the mean score of the third grade preservice teachers in ACKT was 31.64, while the mean score of the fourth grade preservice teachers in ACKT was 37.82. It is observed that there is a statistically significant difference among the content knowledge levels of preservice elementary school mathematics teachers regarding algebra depending on the grade level [$F_{(3,172)}$= 104,536, p<.05]. In other words, the content knowledge levels of the preservice teachers have developed as directly proportional depending on the class level. As seen from the means of ACKT scores of preservice teachers, the greatest development was in the fourth grade, where courses such as "*School Experience*" and "*Teaching Profession Practice*" took place. As known, in these courses prospective teachers go to internship schools, interact with students, and have experience regarding teaching profession. The content knowledge development of preservie teachers continued during the passing from second year to third year while it decelerates. In the third grade, preservice teachers get courses such as "*Special Teaching Methods I*" and "Special *Teaching Methods II* ".

In his study, Eraslan (2009) stated that pre-service mathematics teachers do not have much opportunity to apply the theoretical knowledge they have learned, they receive inadequate feedback about their practices, and they cannot associate the mathematics lessons they have learned at university with mathematics at school. In this context, it can be said that practices performed with students one-to-one and taking place in the real classroom environment have an important place in the development of content knowledge. Seviş (2008) stated that *Mathematics Teaching Methods* course had an important effect on the development of the content knowledge of preservice elementary mathematics teachers. Wright (2009) concluded that the lesson study contributed to teachers' content knowledge development. If we compare the teacher's training process at the faculties of education to a pyramid, practice-oriented lessons increase as we go down in this pyramid, i.e., as the grade level increases. The content knowledge develops in the same proportion to the increase in practice lessons. Indeed, a directly proportional success to the grade level was observed in teaching practices performed by pre-service teachers.

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Appendix 1: Algebra Content Knowledge Test

S.1. What is the reason of the change the sign of constant term when it is going to the opposite side of the equation in the case of solving equations?

S.2. Which one of expression given below are algebraic expression. Justify your answer.

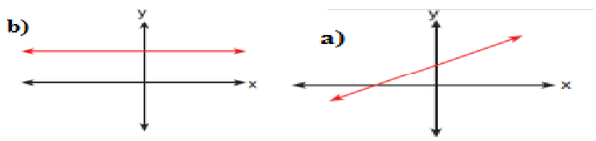
a) $A = \pi r^2$ **b)** x-5 **c)** $2(3^2-7)+(2+9^5)$ **d)** $E=mc^2$ **e)** x+2=5

S.3. What is the difference between equations and identities? Find the equation and identity among the expression given below. Also Justify your answer.

a) 4x-7=2x+1 **b)** 4x+8= 4(x+2) **c)** $a^2-6a+9= (a-3)^2$

d) 2(m-2)+m+1= 3(m-1) **e)** 3(a+3)+4(a-1)=2(a-2)+2(2a-1)

S.4 To form an equation for each line graph given.



S.5. Describe the following concepts and give examples for each of them.

a) Variable **b)** Pattern **c)** Linear equation **d)** Slope

S. 6. For what purposes variables are used in each expression given below?

a) If a+5=8, a=? **b)** If a+b=25, a+b+3=? **c)** Add 4 to a+5. **d)** Find ''b'' If a+b=10 and a<7.

S.7. To pose a problem for each algebraic expression and equatin given.

a) 2s+5 **b)** a+b=25 ise a+b+3=? **c)** 2(x-1)=18

S.8. Everyone is shaking hands at a meeting of 20 people. Find the total of handshakes? Is there a generalization for this situation or not?

S.9 Model the following statements.

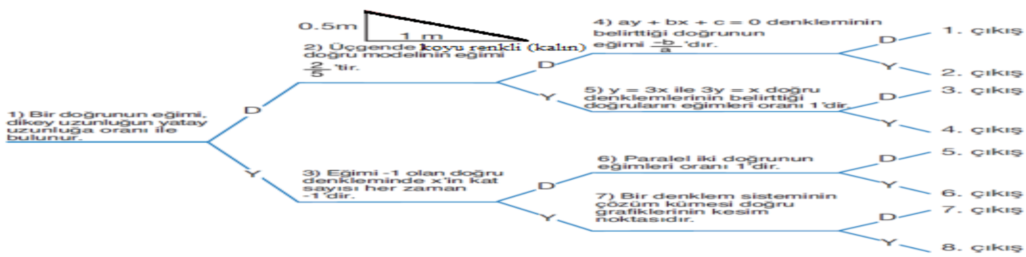
a) x+5=3x+1 **b)** 2x+2<7

S.10. Construct a concept map about the concept of algebraic expression for the Algebra Course in the middle school.

S.11. Model the x^2+5x+6 with geometric shapes.

S.12. What kind of relation between the number of equations and the number of variables in a equation has to be in order to be solved.

S.13. The correct and incorrect statements are given in the Diagnostic Branched Tree below. To decide which statements are correct and which ones are incorrect. Each correct /incorrect answer leads you to different exits. Reach one of the exits. Buna



Appendix 2: The Framework of Data Analysis (The first Question)

Data Analysis Framework of First Question of ACKT

Scoring Categories	**Scoring Criteria**	**Sample of Teacher Candidates' Answers**
COMPLETELY TRUE	*The mathematical reasoning behind the sign of the the constant term changing as passing through to the other side of equation is to add the inverse of the constant term to the both sides of the equation.* *For example;* *x+2=5* *x+2-2=5-2* x=3 As seen from the example above, "0" is obtained on the left side of the equation after adding the inverse of constant term according to addition operation to constant term. Since we do not need to write this "0" , we are practically switching the sign of the constant term as passing through to the other side of equation. If the teacher candidates answer this question as given above, the answers are evaluated in the ***"completely true"*** category.	Eşitlikteki bilinmeyeni bulabilmek için bu bilinmeyeni yalnız bırakmalıyız. Yanındaki ifadeler eşitliğin diğer tarafında olmalıdır. Burayada işaretini ters çevirerek geçer. Bizim değişkenimizin yanındaki sayı-dan kurtulmak istiyoruz. Bu sebeple eşitliğimizin her iki tarafını bu terimin ters işaretlisiyle işleme sokulur. Değişkenin yanındaki terimimizde bu şekilde yok olmuş olur. ÖRNEK: $x+5=13$ $\quad$ $x-2=12$ $x+5-5=13-5$ $\quad$ $x-2+2=12+2$ $x=13-5$ $\quad$ $x=12+2$ $x=8$ $\quad$ $x=14$ As seen from the answer of 4S22 teacher candidate, he or she answered this question correctly by explaining clearly why he or she should change the sign of the constant term.

PARTİALLY TRUE (A)	If the answers of the teacher candidates are incomplete according to the correct answer, the answers are evaluated in the ***"partially true (a)"*** category.	Eşitliğin bozulmaması için denklemin her iki tarafa aynı sayıyı ekleyip yada çıkardığımızdan dolayı işaret değişir. **(Because we add or subtract the same number on the both sides of the equation so that equality is not changed.)** The 3S2 teacher candidate gave close answer to completely true. However, since the teacher candidate did not emphasize the fact that zero is obtained next to the variable while solving equation. So this answer was evaluated in the ***"partially true (a)"*** category.
PARTİALLY TRUE (B)	In this case answers of teacher candidates are evaluated in the ***"partially true (b)"*** category. Because they cannot explain the reason for changing the sign of the constant term by passing to the opposite side of equation. But they just stated that it is a rule of math.	Denklemin sağlanması amaçlı olarak ifadenin işareti değişir. İfadenin işaret değiştirmesi matematiğin bir kuralıdır. **(The sign of the constant term changes so that equation can be solved. It is a rule of mathematics.)** 1S10 teacher candidate's answer to fist question of ACKT was evaluated in the ***"partially true (b)"*** category.
WRONG ANSWER	If the answers of the teacher candidates are completely wrong or irrelevant, their answers are evaluated in the ***"wrong answer"*** category.	Mutlak değerden dolayıdır. **(Because of absolute value.** The answer was evaluated incorrect because 1S17 teacher gave an irrelevant answer to the first question of ACKT.
NO ANSWER	*If teacher candidates do not answer this question, their answers are evaluated in the **"no answer"** category.*	--

Appendix 3: Frequency-percentage distribution of each question

Number of Items	Scoring Categories	Completely true		Partially True (a)		Partially True (b)		Wrong Answer		No Answer	
		f	%	f	%	f	%	f	%	f	%
Q1	First Grade	3	6.8	3	6.8	19	43.2	11	25	8	18.2
	Second Grade	9	20.5	17	38.6	10	22.7	5	11.4	3	6.8
	Third Grade	9	20.5	23	52.3	6	13.6	6	13.6	-	-
	Fourth Grade	18	40.9	17	38.6	9	20.5	-	-	-	-
Q2	First Grade	-	-	5	11.4	22	50	12	27.3	5	11.4
	Second Grade	-	-	11	25	19	43.2	11	25	3	6.8
	Third Grade	-	-	11	25	25	56.8	8	18.2	-	-
	Fourth Grade	3	6.8	19	43.2	14	31.8	8	18.2	-	-
Q3	First Grade	2	4.5	26	59.1	16	36.4	-	-	-	-
	Second Grade	3	6.8	26	59.1	13	29.5	1	2.3	1	2.3
	Third Grade	7	15.9	22	50	14	31.8	1	2.3	-	-
	Fourth Grade	8	18.2	24	54.5	12	27.3	-	-	-	-
Q4	First Grade	4	9.1	9	20.5	23	52.3	8	18.2	-	-
	Second Grade	6	13.6	8	18.2	23	52.3	7	15.9	-	-
	Third Grade	3	6.8	12	27.3	25	56.8	4	9.1	-	-
	Fourth Grade	1	2.3	9	20.5	34	77.3	-	-	-	-
Q5	First Grade	-	-	8	18.2	24	54.5	8	18.2	4	9.1
	Second Grade	-	-	14	31.8	19	43.2	9	20.5	2	4.5
	Third Grade	-	-	16	36.4	22	50	6	13.6	-	-
	Fourth Grade	-	-	27	61.4	16	36.4	1	2.3	-	-
Q6	First Grade	-	-	8	18.2	19	43.2	10	22.7	7	15.9
	Second Grade	1	2.3	12	27.3	18	40.9	6	13.6	7	15.9
	Third Grade	2	4.5	16	36.4	23	52.3	3	6.8	-	-
	Fourth Grade	1	2.3	25	56.8	13	29.5	4	9.1	1	2.3
Q7	First Grade	16	36.4	17	38.6	5	11.4	2	4.5	4	9.1
	Second Grade	18	40.9	20	45.5	5	11.4	1	2.3	-	-
	Third Grade	27	61.4	13	29.5	2	4.5	2	4.5	-	-
	Fourth Grade	32	72.7	11	25	1	2.3	-	-	-	-

Appendix 3: Frequency-percentage distribution of each question

Number of Items	Scoring Categories	Completely true		Partially True (a)		Partially True (b)		Wrong Answer		No Answer	
		f	%	f	%	f	%	f	%	f	%
Q8	First Grade	1	2.3	3	6.8	20	45.5	17	38.6	3	6.8
	Second Grade	2	4.5	4	9.1	13	29.5	16	36.4	9	20.5
	Third Grade	2	4.5	4	9.1	17	38.6	19	43.2	2	4.5
	Fourth Grade	13	29.5	14	31.8	12	27.3	5	11.4	-	-
Q9	First Grade	-	-	-	-	25	56.8	10	22.7	9	20.5
	Second Grade	1	2.3	15	34.1	17	38.6	4	9.1	7	15.9
	Third Grade	2	4.5	4	9.1	20	45.5	16	36.4	2	4.5
	Fourth Grade	23	52.3	10	22.7	7	15.9	4	9.1	-	-
Q10	First Grade	-	-	-	-	1	2.3	4	9.1	39	88.6
	Second Grade	-	-	-	-	11	25	5	11.4	28	63.6
	Third Grade	-	-	19	43.2	12	27.3	9	20.5	4	9.1
	Fourth Grade	5	11.4	15	34.1	23	52.3	-	-	1	2.3
Q11	First Grade	-	-	4	9.1	10	22.7	19	43.2	11	25
	Second Grade	1	2.3	6	13.6	17	38.6	10	22.7	10	22.7
	Third Grade	-	-	9	20.5	14	31.8	17	38.6	4	9.1
	Fourth Grade	17	38.6	15	34.1	6	13.6	6	13.6	-	-
Q12	First Grade	2	4.5	19	43.2	6	13.6	3	6.8	14	31.8
	Second Grade	4	9.1	24	54.5	11	25	3	6.8	2	4.5
	Third Grade	8	18.2	23	52.3	8	18.2	5	11.4	-	-
	Fourth Grade	7	15.9	28	63.6	6	13.6	3	6.8	-	-
Q13	First Grade	34	77.3	8	18.2	2	4.5	-	-	-	-
	Second Grade	38	86.4	5	11.4	1	2.3	-	-	-	-
	Third Grade	38	86.4	4	9.1	-	-	2	4.5	-	-
	Fourth Grade	41	93.2	3	6.8	-	-	-	-	-	-