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# THE EFFECTIVENESS OF CONCRETE-REPRESENTATIONAL-ABSTRACT INSTRUCTION STRATEGIES IN THE INSTRUCTION OF FRACTIONS TO STUDENTS WITH LEARNING DISABILITIES[i,ii]

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**Abstract:**
The present study aimed to determine the effectiveness of the concrete-representational-abstract instruction strategies employed in the direct instruction of fractions to students with learning disabilities. Furthermore, the generalization of the instruction to different settings and tools, the follow-up data for one and three weeks after the instruction, and the social validity data based on the views of the mothers on concrete-representational-abstract instruction strategies were analyzed. In the study, the inter-behavioral multiple probe model with a probe stage, a single-subject research model, was employed. The dependent variable was the level of identification of proper, half and quarter fractions by the participating students in the study, while the independent variable was the concrete-representational-abstract introduction strategies implemented with the direct instruction method. The study was conducted with three male students with learning disabilities, who resided in Izmir and attended an inclusive primary school. The study findings demonstrated that the concrete-representational-abstract instruction strategies were effective in the instruction of proper, half and quarter fractions to the students with learning disabilities, these skills acquired by the students could be permanent for one to three weeks after the instruction, and all students could generalize these skills to various

[i] ÖĞRENME GÜÇLÜĞÜ OLAN ÖĞRENCİLERE KESİRLERİN ÖĞRETİMİNDE SOMUT-YARI SOMUT-SOYUT ÖĞRETİM STRATEJİSİNİN ETKİLİLİĞİ

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settings and instruments, and the views of the mothers on concrete-representational-abstract instruction strategies were positive.

**Keywords**: instruction of fractions, mathematics instruction, concrete-representational-abstract instruction strategies, students with learning disabilities

## 1. Introduction

Learning disability is the tendency of an individual to experience inadequacy in one or more of the psychological states of reading, writing, listening, thinking, speaking or analyzing mathematical operations, making sense of them, or using written and spoken language (Scruggs & Mastropieri, 2004). Students with learning disabilities have problems in one or more of their reading, writing, or math skills. These skills, which we frequently use in our lives where students with learning disabilities have problems, are based on school life (Melekoğlu, 2017).

Individuals with learning disabilities are among the mathematical competencies and abilities that are aimed to be transferred to students with learning disabilities, such as four operations, calculus skills, numbers, problem-solving, clock skills, knowing and calculating money, and fractional expressions that they frequently encounter during their school life (Karabulut & Yıkmış, 2010). Mathematics skills find a place in the daily routines and other areas of children with learning disabilities, just as they make life easier in the daily routines of children whose development is within the limits of the norm and help their academic skills and professional development (Yıkmış, 2012). Fractional expressions in these skills are a concept that even children who do not have learning difficulties have difficulty in learning.

Fractions are one of the richest subjects in the primary school mathematics curriculum (MEB, 2015) and are included in all classes of primary school. The reason for this is the frequency of use of fractions in daily life and the necessity for future mathematical studies. As Van De Walle (2012) points out in his publications, it has an indispensable value because fractions form the basis of mathematical skills. However, the fact that fractions are shown with different symbols and shapes, with different definitions and types in the learning and instruction process causes confusion (Clarke, Roche, & Mitchell, 2008). Kids cookies etc.; they have the ability to divide things into two or multiple parts. They are aware that cakes and pizzas are falling into pieces (Mastropieri and Scruggs, 2016). However, although children frequently encounter the concepts of half and quarter in their daily lives, they may attribute a meaning outside of its meaning to fractions.

Especially in the first years of school, children may experience these conceptual confusions in whole, half and quarter issues. Clarke, Roche, and Mitchell (2008) stated that it would be better for children to realize fractions by living rather than giving them steps to be followed by the teacher. In addition, as İnce & Yıkmış (2021) stated, it is necessary to make adaptations in the education of students with special needs. For this

reason, it is recommended to use adaptive, multi-sensory methods and appropriate strategies in instruction fractions to children with learning disabilities.

Piaget (1971) suggested that students who move from concrete operations to the abstract operations period need concrete materials and experiences. He underlined that mathematics is a course in which abstract concepts are used intensively, and therefore, it is necessary to work with sufficiently concrete objects in the beginning so that the individual can make the abstractions required in the nature of the concepts. Bruner (2006), on the other hand, suggested that knowledge is represented imaginatively, actively and symbolically, and in this context, importance should be given to the use of concrete objects, pictures or drawings, and numbers and symbols in transferring knowledge to students. In addition to these views, it can be added that Cawley and Miller (1989) thought that instruction in which concrete objects are used brings abstract expressions to a stronger point.

Concrete-representational-abstract teaching strategy (CRA), which is one of the strategies used in the light of the approaches mentioned above, is the concrete stage in which objects and materials that can be moved and touched are included; It consists of three stages: the semi-concrete stage where the lines represented by these materials and different shapes are included, and the abstract stage where mathematical symbols and numbers are included, in addition to mathematical expressions. It is a strategy in which the number of senses used is reduced gradually so that the teaching appeals to more than one sense organ (Miller & Mercer, 1993). It is also defined as a supporting model in the sense of basic conceptuality, which is applied based on a certain stage, which allows to establish logical relations with concrete, semi-concrete and abstract forms of a skill or concept (Witzel, 2005; Ferreira, 2000; Flores, Hinton, & Schweck, 2014; Witzel , Mercer and Miller, 2003; Witzel, Riccomini and Schneider, 2008). In this direction, the abstraction of mathematical concepts with the help of concrete objects, depending on a certain stage, has a positive effect on the learning levels of students with special learning disabilities.

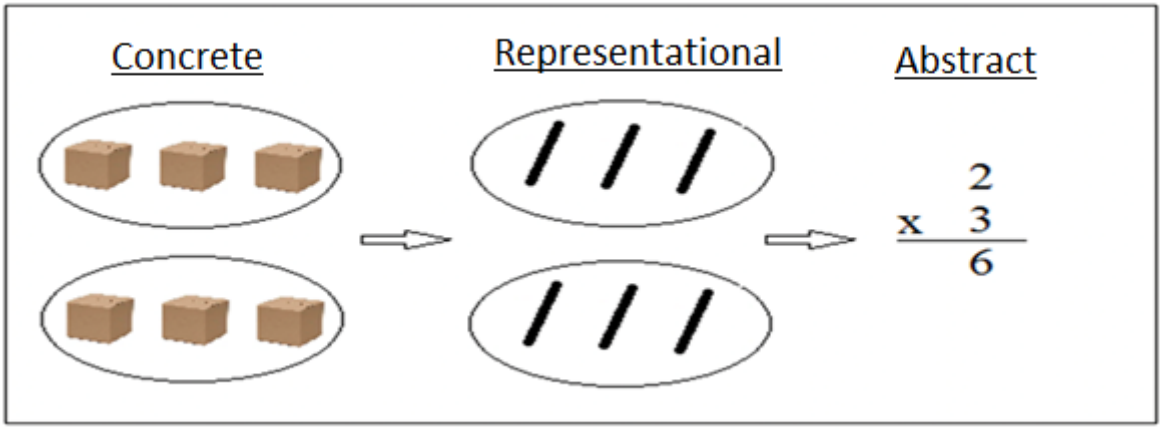


**Figure 1:** Concrete, representational and abstract stages according to the CRA instruction strategy (Özlü, 2016).

## 2. Literature review

In the related literature, there are many studies examining the effectiveness and efficiency of instruction mathematical skills and concepts by using the CRA instruction strategy to students with learning disabilities. Among these effectiveness studies; Peterson, Mercer, McLeod and Hudson (1988), in instruction the concept of place value to students with special learning disabilities; Harris, Miller, and Mercer (1995), in the instruction of multiplication skills and problem solving skills involving basic multiplication to second grade students with learning difficulties; Maccini and Hughes (2000), in instruction algebraic expressions to students with learning disabilities; Maccini and Ruhl (2000) teach students with learning disabilities the ability to solve algebraic problems; Ferreira (2002) teaches basic subtraction skills to students attending 5th grade with learning disabilities; Scheuermann, Deshler and Schumaker (2009), for instruction problem solving skills to students with learning disabilities; Flores (2009), in instruction hand subtraction skills to students with learning disabilities; Bouck, Park and Nickell (2017) teach the skill of solving problems that require exchanging money; Milton, Flores, and Moore (2018) teach students with learning disabilities the conceptual understanding of basic multiplication and division; there are studies examining the effectiveness of the CRA instruction strategy.

In addition to these studies, there are also studies comparing the effectiveness of two different instruction strategies. In the studies, CRA instruction strategy and other instruction methods and strategies were compared and examined. Peterson, Mercer, Tragash and O'Shea (1987) the effectiveness of CRA instruction strategy and traditional instruction methods in the instruction of the concept of place value to students with learning disabilities; Butler, Miller, Crehan, Babbitt, and Pierce (2003) determined the effectiveness of CRA instruction strategy and semi-concrete-abstract (RA) instruction strategy in instruction the concept of equivalence of fractions to students with mild and moderate learning difficulties; Witzel, Mercer, and Miller (2003) compared the effectiveness of the CRA instruction strategy and the traditional instruction method in instruction complex algebraic problems to middle school students with learning disabilities.

No study has been found in Turkey on teaching fractions to students with learning disabilities. In addition, in the first years of school, children must experience many conceptual confusions in whole, half and quarter subjects and appropriate strategies should be used in instruction fractions to children with learning difficulties. For this reason, it is thought that examining the effectiveness of the CRA instruction strategy in instruction fractions to students with learning disabilities will lead to other research and applications to be made in the related literature.

### 2.1 Purpose of the research

The purpose of this research is to determine the effectiveness of the CRA instruction strategy in instruction fractions to students with learning disabilities. Along with this basic objective, the research questions to be answered are given below:

1) Is the CRA instruction strategy effective in instruction fractions to students with learning disabilities?
2) Whether students with learning disabilities acquire fractional skills using the CRA instruction strategy, will this learning be a permanent and sustainable one and three weeks after the instruction process is completed?
3) Generalized across environments and tools?

## 3. Material and Methods

### 3.1 Model of the research

In this study, in order to determine whether the CRA instruction strategy presented with the direct instruction method is effective in instruction the ability to show whole, half and quarter in fractions to students with learning disabilities, the "multiple probe model with inter-behavioral probe phase" was designed and applied, which is one of the multiple probe models among the single-subject research approaches. Baseline data for all skills of the applied model are collected in the same time period. After the collected baseline data reaches a certain percentage of stability, the application phase is started for instruction of the first concept. After the criterion is met in the first concept, data is collected in three consecutive sessions for all concepts and the probe phase is included. At this stage, the expected first concept is that the probe data should meet the criteria, and in other skills, it should be similar to the baseline level.

After the probe phase, the second concept is started to be implemented in the instruction phase. If the criterion reaches the desired level in the second concept, the probe phase is applied to all concepts. At this time, it is expected that the probe data of the first two concepts will meet the criterion and that the other concept will be similar to the baseline level. After the probe phase of the research, the instruction phase of the third concept, which is the last, begins. In the last probe session, which is applied after the instruction of the third concept is over, it is expected that all three concepts will meet this criterion and have similar features. The repetition of all these stages continues for all participants (Gast, 2010).

### 3.2 The independent variable

The independent variable of the research is the CRA instruction strategy. The CRA instruction strategy consists of three stages: a) concrete instruction stage, b) representational instruction stage, c) abstract instruction stage. In this study, instruction with the CRA instruction strategy was carried out in the form of a one-to-one instruction arrangement for each stage.

### 3.3 The dependent variable

The dependent variable of the study is the level of showing whole, half and quarter from basic fraction skills of the students participating in the research. In this direction, when requested from the student, it is expected to show the whole, half and quarter at 100% accuracy.

### 3.4 Participants

The participants of this research were the researcher, students and observers. Three male students who attended a Special Education and Rehabilitation Center and were diagnosed with special learning disabilities participated in this study. The administration department of the institution was informed about the research and necessary permissions were obtained to carry out the research. Following the determination of the participants participating in the research, the families were informed about the research. Necessary work permits were obtained from the families on behalf of the four participants who were accepted to participate in the study. In addition, by interviewing the teachers and mothers of the selected participants, they were informed that they should not do any practice that would affect their ability to show the whole, half and quarter. Participants in this research are expected to have some characteristics: the individual who is a participant in the research was expected to meet prerequisite skills such as (a) answering questions, (b) being able to maintain attention for at least twenty minutes during the activity, (c) reading the written number, (d) visual perception and auditory perception skills.

Instead of the real names of the students participating in the research, code names that were not similar to those of the students in the institution where the study took place were used. The characteristics of the participants are listed below:

Ahmet is an eight-year-old male student with a learning disability who was diagnosed by the guidance research center. Ahmet has been studying in general education classes for two years. Ahmet is a strong social communicator who can follow the instructions given to him, read numbers and communicate with his peers. He is a student with the necessary preparation skills for reading and writing. In mathematical skills, he can count up to 100 independently, show the numbers up to 100 when asked, and can tell which numbers they are when the numbers are shown.

Mehmet is an eight-year-old male student with a learning disability who was diagnosed by the guidance research center. Mehmet has been studying in general education classes for two years. Mehmet is able to follow the instructions given to him, read the numbers, and communicate with his peers. He is a strong social communicator. He is a student with the necessary preparation skills for reading and writing. In mathematical skills, he can count up to 100 independently, show the numbers up to 100 when asked, and can tell which numbers they are when the numbers are shown.

Doğan is an eight-year-old male student with a learning disability who was diagnosed by the guidance research center. Doğan has been studying in general education classes for two years. Doğan is a strong social communicator who can follow

the instructions given to him, read numbers and communicate with his peers. He is a student with the necessary preparation skills for reading and writing. In mathematical skills, he can count up to 100 independently, show the numbers up to 100 when asked, and can tell which numbers they are when the numbers are shown.

Apart from the participants of the research, the researcher and the observer are the other participants who support the research. While the entire implementation process of the study was carried out by a single practitioner who has a doctorate in the special education department, the application reliability and inter-observer reliability data in the study were collected by two special education specialists who graduated from the special education department and completed their master's degree in the special education department.

**3.4 Environment**

Daily and collective attendance, monitoring and instruction sessions in the research were carried out in an individual classroom at the Special Education and Rehabilitation Center. There is a desk, two chairs and a book cabinet in each classroom. In order to obtain research data records for this class, a video camera system was kept ready.

**3.5 Tools**

reinforcers to be used in order to reinforce the correct responses of the participants in the research, a functional reinforcer determination form was created for the student and based on the opinion of the family and was used for each participant. In the concrete instruction phase, where the instruction starts from the concrete level, a material set consisting of 10 touchable and movable objects was created and used for instruction the whole. In the representational instruction stage, the drawings of objects representing the materials used in the concrete instruction stages were used. In the abstract instruction phase, fraction expressions corresponding to the whole were used. Expert opinion was sought about the instruction sets prepared at the stage before starting the application.

**3.6 Implementation process**

In this study, in which the effectiveness of the CRA instruction strategy applied with the direct instruction method in the instruction of the ability to show whole, half and quarter in fractions to individuals with learning disabilities, in the application phase; For all skills, the first collective probe sessions followed by daily probe sessions, instruction sessions and finally follow-up sessions were held with three students. The sessions in the study were held in an individual classroom in a Special Education and Rehabilitation Center, five days a week, between 10:00 and 12:00. The CRA instruction strategy implemented in the study consists of three stages: (1) The concrete stage where movable and tactile materials are used, (2) The representational stage where the lines in which the materials used are represented, and (3) The abstract stage where the numbers, symbols and mathematical expressions are used. The sessions at all stages of the concrete-semi-concrete-abstract instruction strategy implemented with the direct instruction method

were implemented by performing the steps of being a model, guided practices and independent practices. Baseline data were collected until all subjects participating in the study obtained stable data on three skills for three sessions. After obtaining stable data in the baseline sessions, the CRA instruction strategy, which was applied with the direct instruction method, was started to be applied for the first skill in each subject.

Participants were determined as the intermediate criterion required in the transitions to the concrete, representational and abstract instruction stages, and the students' reaching the 90% criterion in the probe sessions held at the end of the instruction sessions and obtaining stable data in three consecutive sessions. For all students, when the required intermediate criterion was reached in the concrete instruction stage, the representational instruction stage was started, and then the abstract instruction stage was started when the intermediate criterion was reached in the representational instruction stage. In the abstract instruction phase, the instruction was terminated after the students who participated in the study reached the 90% criterion in the probe sessions taken after the instruction sessions and showed determination for at least three consecutive sessions. Follow-up sessions were held one and three weeks after the instruction process was completed. Subsequently, generalization sessions were also taken right after the second, third and fourth full probe sessions. All sessions held during the study were organized in the practitioner one-to-one instruction model.

All practice sessions in the research were conducted by the practitioner and the data obtained were recorded in data recording forms these sessions were recorded with the help of video. In the research, instruction on the skill of showing the whole with Ahmet took five instruction sessions for concrete, five for representational and five for abstract, while instruction on the skill of showing half took six for concrete, four for representational and five for abstract. İnstruction the skill of showing the quarter lasted for six instruction sessions for concrete, five for representational and seven for abstract. It took five instruction sessions for concrete, five for representational and five for abstract, five for concrete, five for representational and six for abstract. Instruction on the skill of showing the quarter lasted for six instruction sessions for concrete, five for representational and six for abstract. The skill of showing the whole with Doğan took five instruction sessions, four for representational and five for abstract, five instruction sessions for concrete, five representational and six instruction sessions for abstract. İnstruction the skill of showing the quarter lasted for five instruction sessions for concrete teaching, five for representational and six for abstract teaching.

### 3.7 Polling sessions

In this study, before the implementation of the CRA instruction strategy, which was applied with the direct instruction method, probe evaluations were made to determine the performance of the participants. Probe sessions in the study; daily probe sessions, collective probe sessions and generalization probe sessions. With the CRA instruction strategy; Whether the instruction of the whole, half and quarter representation took place

was determined by the probe sessions held after the instruction sessions. Information on how the sessions in the study progressed is given below.

### 3.8 Mass polling sessions

Before starting the instruction phase of the studied skill, four-probe sessions were held in order to determine the performance of the participants on three skills after the criteria determined during the instruction phase were met and stable data was obtained. In the first stage, baseline data were collected. It was carried out with all the participants one by one, with the adaptation of the three skills.

In the first collective probe session, at least three sessions on the first skill, one by one, with three participants, were continued until stable data were obtained, and after the stable data were obtained, the instruction session of the first skill was started. After the participants performed at a rate that met the criterion in the first skill and after obtaining stable data on this subject, all participants were transferred to the second full probe session. Then, after the second full roll call was completed, the instruction phase of the second skill was started. After the instruction phase was completed, after the criterion was met in the second skill, the third collective probe sessions were started when stable data were obtained. When the stable data in the third collective probe sessions were obtained, the instruction phase for the third skill was started, and if the participant met the criteria and stable data were obtained, collective probe sessions were held for the last time in all skills.

The place where the collective probe sessions were held was the individualized education classroom. First, the implementer prepared the material, registration form and video camera to be used for the probe session. Afterwards, the participants were informed about the study. (“I will ask you to show the concept I asked when I ask which of the objects, I am going to show you now is whole, half, quarter”), certain clues were provided to motivate the participants to study (are you ready? Shall we start?), then the participants' reactions showing that they were ready for the study were reinforced verbally (well done, you are amazing etc.) at the end of this stage, the researcher presented the target stimulus to the student. (“Now show which of the objects I will show you is whole, half or quarter”), did not give any reaction to the correct or incorrect answers given by the participants to the instructions, and was unresponsive to the participants of the study, and after the study was completed, the subsequent trials were started.

### 3.9 Generalization sessions

In the study, generalization studies were carried out between tools and materials and environments. Generalization sessions and collective probe sessions in the study were continued together. Generalization sessions were held after all collective probes. The sessions were held in the form of pretest - posttest, this session; as soon as the first full probe phase was over, the posttest session was held as soon as the instruction activities and the criteria for each participant were met for the three skills. In the study, the correct

responses of the participants in the generalization probe sessions were reinforced with verbal reinforcements, similar to the phase in the instruction sessions. For the skills that the participants showed correctly in the generalization sessions, they were reinforced verbally with a continuous reinforcement schedule. In addition, at the end of all generalization sessions, food reinforcement was given to the participants (wafers, crackers, jelly beans, etc.).

**3.10 Instruction sessions**

The independent variable in this research was put into practice in the instruction sessions. After obtaining stable data in the baseline sessions for all participants in the study, the instruction phase was started with the first participant. In the instruction sessions, students with learning disabilities were taught the skill of showing whole, half and quarter with the CRA instruction strategy applied with the direct instruction method. The CRA instruction strategy applied in the research consists of three stages: (a) the concrete stage, which includes materials that can be touched and moved, (b) the representational stage, where drawings are used to represent objects, (c) finally, the abstract stage, where numbers and fraction expressions are used alone. Sessions at all stages of concrete-semi-concrete-abstract instruction strategy, modeling and guided practices in the direct instruction method, and all steps of independent practices were also presented.

In this research, the instruction sessions are shown in the following steps:

(a) the practitioner of the research made the instruction materials related to the instruction to be made ready at the place where the application will be made, (b) the practitioner of the research informed all the participants about the study, (c) in order to attract and motivate the participants for the study, “we will start when you are ready and Now, if you want to examine the objects in front of you, if you are ready, shall we start?, (d) the participants' feedback showing that they are ready to work has been verbally reinforced, "it's okay", etc., (e) the application phase has been started in accordance with the order that the researcher has prepared with the materials, (f) ) made explanations about the skill to be gained to the student participating in the study and showed how to do the skill, (g) helped the student at the points where the student needed, (h) the students were given the main instruction of the skill they studied, and the students were given the desired skill independently by using the materials. It was expected to show k, (k) after the subjects showed all the concepts in the independent practices step independently, "bravo, you are amazing", which was reinforced verbally, (l) At the end of the session, the participants were given verbal and food reinforcements for their willing participation in the study. At the end of the daily probe sessions, after the participants showed 90% performance in three consecutive sessions, the instruction of the first skill was terminated and the other skills were taught in turn to the same participant.

### 3.11 Monitoring sessions

Following the completion of the instruction phase, the follow-up sessions of the research were organized in the same way as the probe sessions in the first and third weeks. The follow-up sessions were organized to determine the level of retention of the skills acquired by the participants during the instruction sessions. In addition, they were reinforced verbally at the end of the study, among those who participated in the study. In all of the monitoring sessions, no interference was made with the participants' correct or incorrect representations of the concepts according to the instructions. Finally, at the end of the study, the participants were verbally reinforced and given the food reinforcers they chose.

### 3.12 Data collection

During the process of the research, three types of data were collected in this study, namely reliability, effectiveness and social validity. The following section contains information about the collection of this data.

### 3.13 Collection of efficacy data

The data that emerged in the research were collected by the researcher using the "Collective Probe, Daily Probe Generalization and Monitoring Sessions Data Collection Form". At this stage, the correct or incorrect responses of the participants in the ability to show whole, half and quarter were recorded. Then the calculation of the correct response percentage was made. There are two answers in the implementation phase of the instruction with the CRA instruction strategy. (1) Correct response: Have the student show the correct one of the presented concepts of the whole, half and quarter. (2) Incorrect response: It is accepted that the student shows the wrong one among the presented whole, half and quarter concepts or does not show any concept. "Single-Step Behavior Trial Record" was used in this study. In a single-step behavior trial record, the correct or incorrect response of the participant to the target stimulus is recorded and the correct behavior percentage and the number are calculated (Tekin-İftar and Kırcaali-İftar, 2004). The same type of data collection model was used in all phases of the study.

### 3.14 Reliability

Two reliability data were collected in this study. These are inter-observer reliability and application reliability.

During the inter-observer reliability data collection stage in the research, the recordings recorded by the observers with the video method were watched and the data contained in the content was recorded in the registration forms. Since the observers in the study had sufficient information about the CRA strategy presented through direct instruction, these observers were not informed about this issue. Subsequently, the inter-observer reliability calculation was calculated using the formula [(consensus) / (consensus + divergence)] X 100 (Tekin-İftar, 2012).

All probing, teaching, monitoring and generalization etc. in the study. Inter-observer reliability data were collected and recorded in 30% of the sessions. In research reliability calculations, and inter-observer reliability coefficient of 80% is considered sufficient, but 90% and above is accepted as the ideal reliability coefficient (Gast, 2010). The inter-observer reliability of this study was found to be 100% for each participant in showing whole, half and quarter in fractions.

In the research, in order to understand to what extent, the practitioner of the study carried out the instruction applied with the direct instruction method and with the concrete-semi-concrete-abstract instruction strategy, the practitioner behaviors that should be in the application plan were determined and then the application reliability form was prepared. For this study, application reliability data were collected in 30% of the collective probe, teaching, probe, generalization and follow-up sessions. To calculate the application reliability coefficient, the observed practitioner behavior was divided by the planned practitioner behavior and the percentage was calculated (observed practitioner behavior/planned practitioner behavior x 100) (Tekin-İftar, 2012). The people who made this application were selected using an unbiased assignment table and there were two special education experts, except for the researcher, who watched the recordings of the sessions. These observers marked as (+) if the implementer could successfully perform the steps in the application content presented, and (-) if he could not. The application reliability findings of this study were found to be 100% for each participant in showing whole, half and quarter in fractions.

### 3.15 Social validity data

In order to determine the social validity of the research, a social validity form was prepared by the researcher. In addition, a CD of video recordings during the implementation phase was prepared for all participants. After this CD was shown to the families of the subjects, the questions in the form were sent. The answers given by the families to each question in the social validity form were recorded. The obtained data were also analyzed qualitatively by calculating frequency and percentage, and the findings that emerged as a result of the analysis were interpreted. The questions in this form are as follows:

1) How do you think this study will contribute to your child's academic skills?
2) Do you think your child will be more accepted at school with this study?
3) Do you think this study will contribute to your child's daily life?
4) Would you like to continue such studies with your child?

### 3.16 Analysis of data

The collected data on the effectiveness of the CRA instruction strategy applied with the direct instruction method in instruction the ability to show whole, half and quarter in fractions to students with learning disabilities were analyzed by graphical analysis. The data obtained in all sessions during the implementation phase are processed one by one on this graph. The horizontal axis in the created graph represents the numerical

expressions of the number of sessions, and the vertical axis is the dependent variable. If there is an increase in the trend lines on the graphs created at the end of the research, this increase is an indication that the independent variable has an effect on the dependent variable (Tekin İftar, 2012). Data on permanence in the research were also created through graphical analysis. The data that emerged about the generalization sessions in the research were analyzed with the pretest-posttest method and displayed on a bar chart.

## 4. Results

Findings related to the effectiveness of the CRA instruction strategy applied with the direct instruction method in instruction of the skill of showing the whole, half and quarter in fractions to students with learning difficulties; Figure 2, Figure 3 and Figure 4 for Ahmet, Mehmet and Doğan, respectively. In the graph, C represents the concrete stage, R represents the representational stage, and A represents the abstract stage. The horizontal axis shows the number of sessions, and the vertical axis shows the correct response percentages of the subjects.

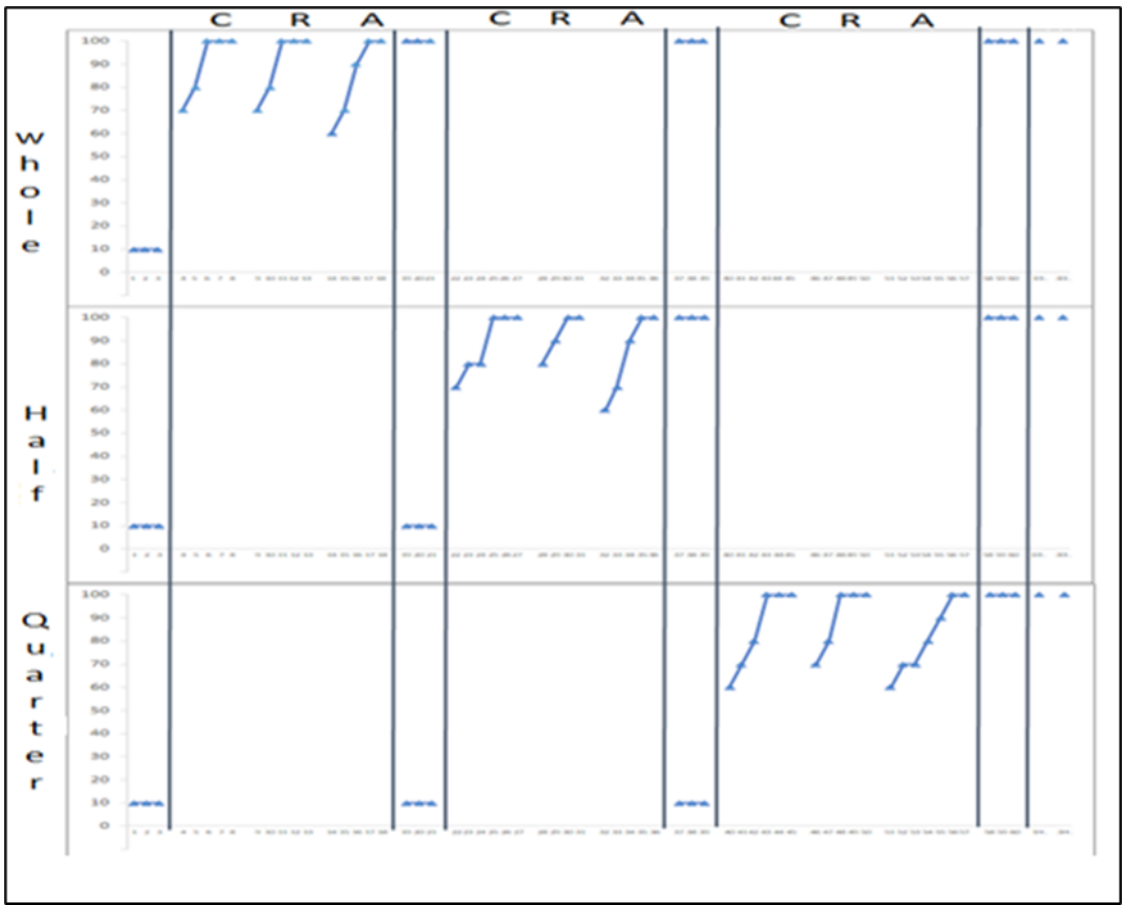


**Figure 2:** Baseline, Concrete Instruction (C), Representational Instruction (R), Abstract Instruction (A), Probe and Follow-up Data on Ahmet's Skills for Representing Whole, Half, and Quarter in Fractions

In Figure 2, there are data on Ahmet's ability to show the whole in the fractions in which the CRA instruction strategy applied with the direct instruction method is taught. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 5 representational and 5 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the whole concept in fractions. It is seen that Ahmet was successful at 100% and 100% in the follow-up sessions held one and three weeks after the practice sessions on showing the whole concept in fractions with the CRA instruction strategy were completed. The data obtained show that Ahmet maintains the skill he has acquired.

In the second graph in Figure 2, data on Ahmet's ability to show half in fractions taught with the CRA instruction strategy applied with the direct instruction method are given. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 6 concrete, 4 representational and 5 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the concept of half in fractions. It is seen that Ahmet was successful at the 100% and 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction the concept of half in fractions with the CRA instruction strategy were completed. The data obtained show that Ahmet maintains the skill he has acquired.

In the third graph in Figure 2, Ahmet's data on the CRA instruction strategy applied with the direct instruction method and his ability to show the quarter in the fractions in which he was taught. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 6 concrete, 5 representational and 7 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the baseline level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the concept of the quarter in fractions. It is seen that Ahmet was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction on how to show the concept of the quarter in fractions with the CRA instruction strategy. The data obtained show that Ahmet maintains the skill he has acquired.

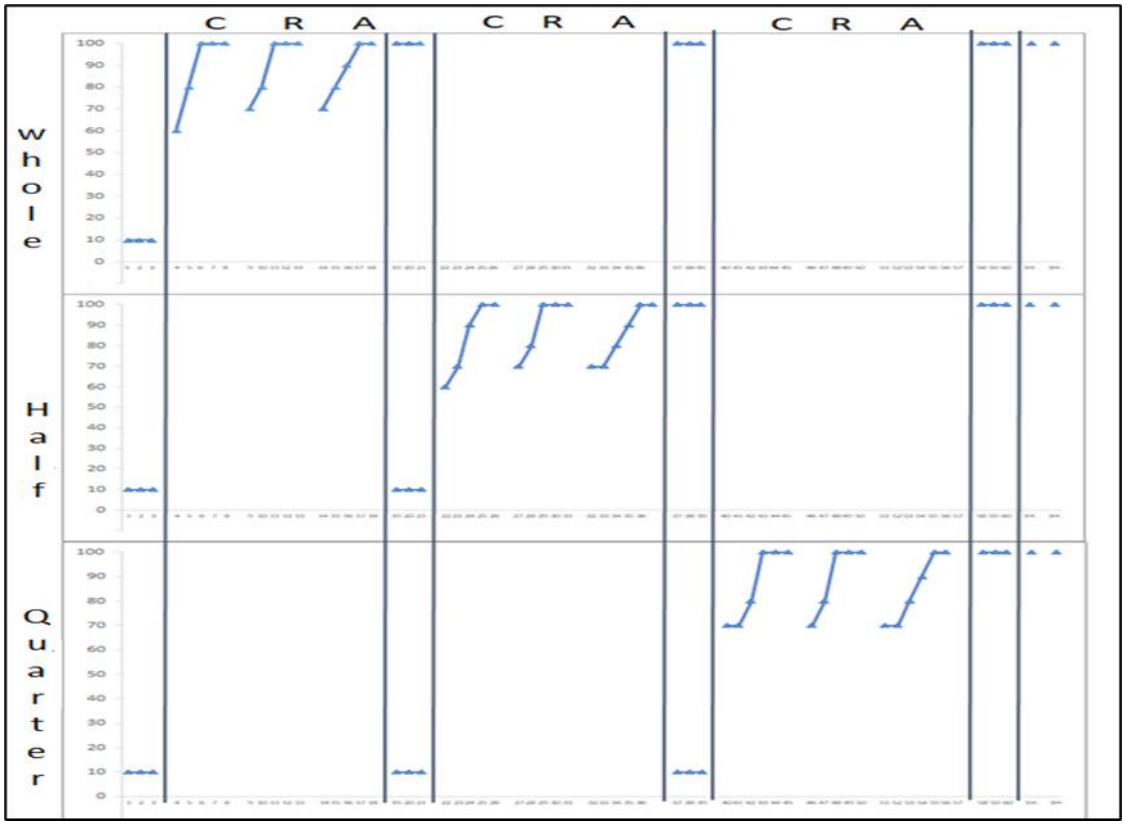


**Figure 3:** Baseline, Concrete Instruction (C), Representational Instruction (R), Abstract Instruction (A), Probe and Follow-up Data on Mehmet's Skills for Representing Whole, Half, and Quarter in Fractions

Figure 3 shows Mehmet's data on the CRA instruction strategy applied with the direct instruction method and his ability to show the whole in fractions that are taught. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 5 representational and 5 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the whole concept in fractions. It is seen that Mehmet was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on showing the whole concept in fractions with the CRA instruction strategy were completed. The data obtained show that Mehmet continues the skill he gained.

In the second graphic in Figure 3, there are data on Mehmet's ability to show half in fractions where he was taught with the CRA instruction strategy applied with the direct instruction method. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 5 representational and 6 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired

change in showing the concept of half in fractions. It is seen that Mehmet was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction on how to show the concept of half in fractions with the CRA instruction strategy. The data obtained show that Mehmet continues the skill he gained. In the third graph in Figure 3, there are data on Mehmet's ability to show quarters in the fractions in which he was taught with the CRA instruction strategy applied with the direct instruction method. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 6 concrete, 5 representational and 6 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the baseline level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the concept of the quarter in fractions. It is seen that Mehmet was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction on how to show the concept of the quarter in fractions with the CRA instruction strategy. The data obtained show that Mehmet continues the skill he gained.

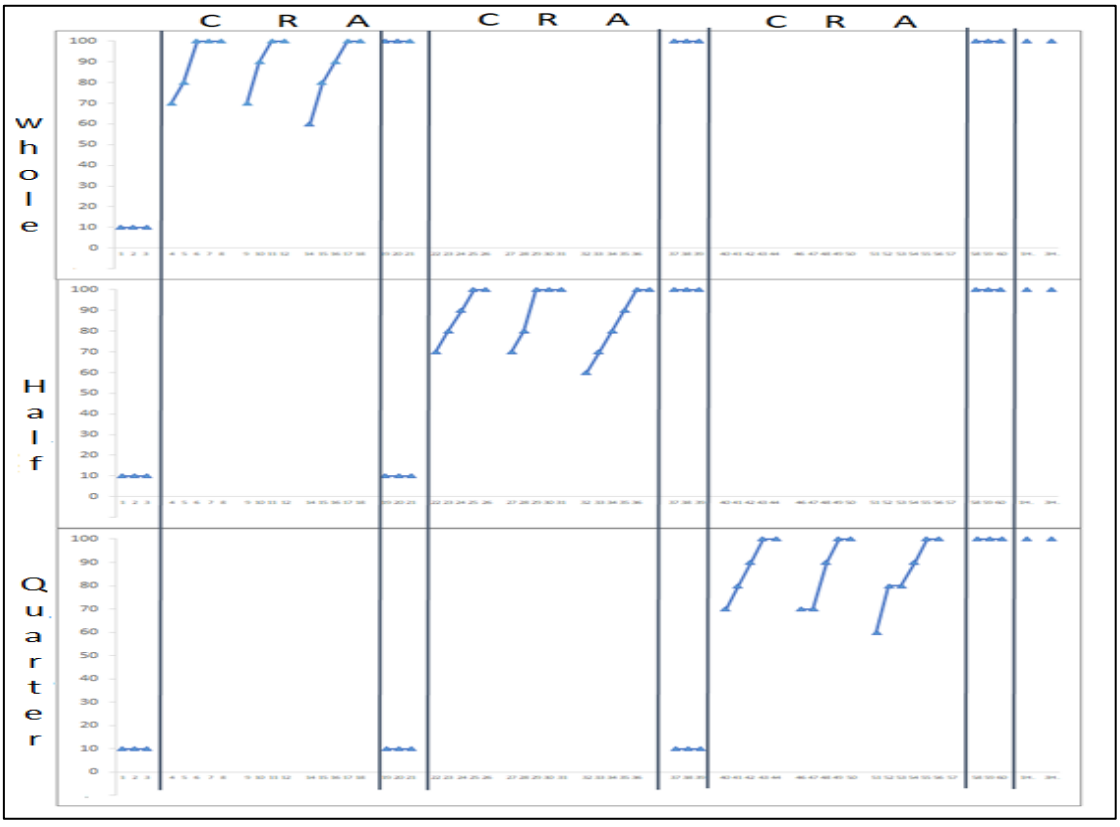


**Figure 4:** Initial, Concrete İnstruction (C), Representational İnstruction (R), Abstract İnstruction (A), Probe and Follow-up Data on Doğan's Skills for Representing Whole, Half, and Quarter in Fractions

Figure 4 shows Doğan's data on the ability to show the whole in the fractions in which the instruction is done with the CRA instruction strategy applied with the direct instruction method. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 4 representational and 5 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the whole concept in fractions. It is observed that Doğan was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction on the concept of the whole in fractions with the CRA strategy was completed. The data obtained show that Doğan maintains the skill he has acquired.

In the second graph in Figure 4, Doğan has data on the ability to show half in fractions taught with the CRA instruction strategy applied with the direct instruction method. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 5 representational and 6 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the initiation level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the concept of half in fractions. It is observed that Doğan was successful at the 100% level in the follow-up sessions held one and three weeks after the practice sessions on instruction the concept of half in fractions with the CRA instruction strategy was completed. The data obtained show that Doğan maintains the skill he has acquired.

In the third graphic in Figure 4, Doğan has data on the ability to show quarters in fractions that were taught with the CRA instruction strategy applied with the direct instruction method. After obtaining stable data in three consecutive sessions at the introductory level, instruction practice started with concrete-semi-concrete-abstract stages, respectively, in the CRA instruction strategy applied with the direct instruction method. A total of 5 concrete, 5 representational and 6 abstract instruction sessions were held with Ahmet. As a result of the analysis made between the baseline level and the implementation phase, it was seen that the CRA instruction strategy created the desired change in showing the concept of the quarter in fractions. It is observed that Doğan was 100% successful in the follow-up sessions held one and three weeks after the practice sessions on instruction on how to show the concept of the quarter in fractions with the CRA instruction strategy. The data obtained show that Doğan maintains the skill he has acquired.

### 4.1 Social validity findings

A social validity form for mothers was prepared in order to determine the social validity of the CRA instruction strategy applied with the direct instruction method in instruction the ability to show whole, half and quarter in fractions to students with learning disabilities. Interview forms were filled in by the researcher by personally interviewing the families. The findings are as follows: All of the families answered the question "What kind of contribution do you think this study will contribute to your child's academic skills?" in the social validity form, and they said that they thought that their children would be more enthusiastic about mathematics with this study. All of the mothers stated that they thought that after seeing the development and learning of their children at school, their friends would take them more with them. In response to another question, families stated that they think that the increase in their mathematics skills and success in the course will contribute to their children's daily work and that their self-confidence will increase. indicated that this should be done.

## 4. Discussion and Conclusion

In the study, the effectiveness of the CRA teaching strategy, which was applied to students with learning difficulties, with the direct teaching method in the teaching phase of whole, half and quarter fractions was determined. In addition, one and three weeks after the end of the education, the family's views on protection, generalization to different people and environments, and finally the CRA teaching strategy were included.

Findings of the research; (a) CRA instruction strategy is effective in instruction whole, half and quarter skills in fractions to students with learning difficulties, (b) the students who participated in the research can maintain the permanence of whole, half and quarter skills one week and three weeks after the end of the education, (c) shows that all of the students can generalize these skills to different environments and tools (d) families have positive views about the CRA instruction strategy.

When the findings of the study were examined, no decrease was observed in the correct responses of three subjects in the ability to show the whole in fractions after the start of the instruction process. It was observed that there was no change in Ahmet's correct response level for a while during the instruction phase of the skill of showing half in fractions, but the changes made afterwards were quite rapid. It can be said that the change in Ahmet's correct response level is due to the psychological problem that this situation created for him since Ahmet has loved ones around him who had Covid-19 in this process. It is thought that the decrease in the level of correct response in the second session during the instruction phase of Mehmet and Doğan's ability to show half is due to the psychological problems caused by the increase in the number of people who had this disease and the increase in cases due to Covid-19. While no decrease was observed in Mehmet and Doğan's correct response levels in instruction to show the quarter, a slight decrease was observed in Ahmet's correct response level. It is thought that before that instruction session, Ahmet had a boredom problem with participating in the study by

wearing a mask and reduced the level of correct response in the study. It was reported by his teacher at the institution that Ahmet had problems attending some classes while wearing a mask and this reduced his performance.

The CRA instruction strategy was found to be effective in instruction whole, half and quarter skills in fractions, and this finding is in line with other research findings examining the effectiveness of the CRA instruction strategy in instruction mathematical concepts and skills to students with special needs ( Peterson et al., 1989; Maccini and Ruhl, 2000; Maccini et al. Hughes, 2000; Scheuermann, Deshler and Schumaker, 2009; Ferreira, 2009; Carmack, 2011; Özlü, 2016; Bouck, Park and Nickell, 2017; Aydemir, 2017; Gibbs, Hinton and Flores, 2018; Nar, 2018; Flores and Hinton, 2019) this research shows consistency.

The findings obtained regarding the other purpose of the study show that all of the subjects were able to maintain their skills one week and three weeks after the end of the instruction. When the monitoring findings of the research are examined, it is seen that the whole half-quarter skill in the fractions acquired with the CRA instruction strategy was maintained at 100% accuracy in the two follow-up sessions of Ahmet, Mehmet, and Doğan, one week and three weeks after the instruction was completed. According to these findings, it can be said that the CRA instruction strategy is permanent in instruction of mathematical concepts and skills. These results are consistent with research findings examining the persistence of the CRA instruction strategy for students with special needs (Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Scheuermann, Deshler & Schumaker, 2009; Ferreira, 2009; Carmack, 2011; Özlü, 2016; Bouck, Park). and Nickell, 2017; Aydemir, 2017; Gibbs, Hinton & Flores, 2018; Nar, 2018; Flores & Hinton, 2019) show that the acquired skill continues after the completion of education. In addition, it can be said that CRA will make an important contribution to the relevant literature, as the present study examines the sustainability of the ability to represent whole, half and quarter in fractions, unlike other studies.

The third finding of the study shows that the CRA instruction strategy and the ability to show whole, half and quarter to individuals with learning disabilities can generalize at 100% level between different environments and tools. In the research, generalization sessions were carried out between tools and materials with different materials and objects, and generalization between environments was included in individual classes separate from the environment in which the application took place.

Examining the generalization effect of mathematical skills and concepts taught with the CRA instruction strategy in their research ( Peterson et al., 1989; Mercer & Miller, 1992; Ferreira, 2009; Carmack, 2011; Strickland & Maccini, 2012; Flores, Hinton & Schweck, 2014; Özlü, 2016; Bouck, Park & Nickell, 2017; Aydemir, 2017; Nar, 2018; Flores & Hinton, 2019) in research findings, it has been observed that the skills aimed to be taught are acquired and continued in the process after the completion of the instruction process. In this respect, it can be said that this finding of the research is consistent with the literature.

Social validity findings, the last finding of this study, is that the mothers of the students found the study useful. In the findings obtained with the social validity data collection tool for families in our study, mothers reported that instruction in the whole, half and quarter subject in fractions would facilitate their children's daily lives. However, when it is investigated whether a social validity analysis of the CRA instruction strategy has been carried out in the literature, it is thought that this research will contribute to the field in terms of being the first research to examine the views of families on the whole, half and quarter instruction using the CRA instruction strategy.

In the light of this information, it can be said that the research is important in the following aspects; (a) The findings of the study show that the CRA instruction strategy is effective in teaching fractions to students with learning disabilities, ensuring the permanence of these learned concepts, and generalizing them to different environments and tools. (b) Since the instruction of fractions is a gradual process from concrete to abstract, it can be said that more effective learning is achieved with the CRA instruction strategy. The fact that students see information in different ways through various representations in the learning process helps them to understand the relationship between these concepts and to transfer this information under necessary conditions. On the other hand, it can be stated that instruction carried out in the light of concrete materials and visual support is quite effective in terms of positive changes in students' interest in mathematics course and their adoption of this course. (c) This research is the first study in which whole, half and quarter fractions were taught to students with learning disabilities using the CRA instruction strategy, and it supports the studies carried out using the CRA instruction strategy in instruction mathematics skills and concepts to students with learning disabilities.

Based on the findings of this study, which yielded effective results, some suggestions for practice and future research are given. Teachers working in the field of special education can be advised to use the CRA instruction strategy for students with learning disabilities in their mathematical concepts and skills. For this purpose, teachers can be trained on the planning and implementation of the CRA instruction strategy. In the research, different tools and materials used in the concrete instruction phase and drawings used in the representational instruction phase for the whole, half and quarter, different tools and materials in the concrete instruction phase and different materials in representational instruction phase can be provided. drawings can be used, the curricula prepared for individuals with learning disabilities can be arranged with the CRA instruction strategy, the effectiveness of the CRA instruction strategy in instruction other academic skills in mathematics to other subjects with learning disabilities can be investigated, and the effectiveness of the CRA instruction strategy with individuals with different disabilities can be examined, and the CRA instruction strategy can be used together. Suggestions that this research can be reworked by making applications in other environments can be given for researches.

**Conflict of Interest Statement**
The authors declare no conflicts of interest.

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